Modern Physics

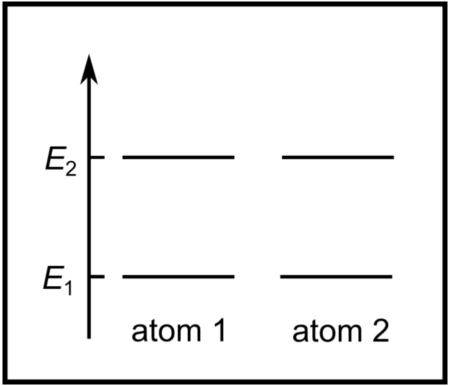
Contents:

Temperature

Essentials of Thermodynamics

Temperature 1

Consider a system with two atoms that have two energy levels



Temperature 1 Comment

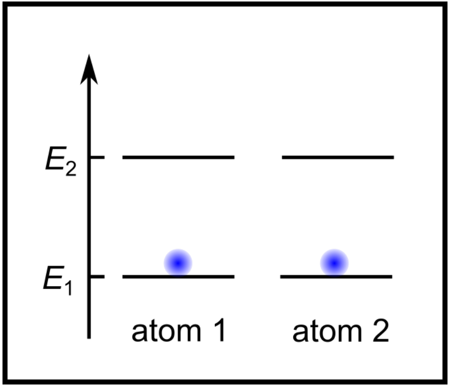
Temperature is a quantity in everyday life.

In physics, temperature is defined in the context of statistical physics.

The figure shows a simple statistical system made up of two atoms with two energy levels.

Temperature 2

There are different configurations, i. e. possibilities of the atoms to occupy the energy levels



Configuration I: $E_\mathrm{I} = 2E_1$ and $\Gamma(E_\mathrm{I}) = 1$

Temperature 2

Comment

The energy of this model system can have three different values.

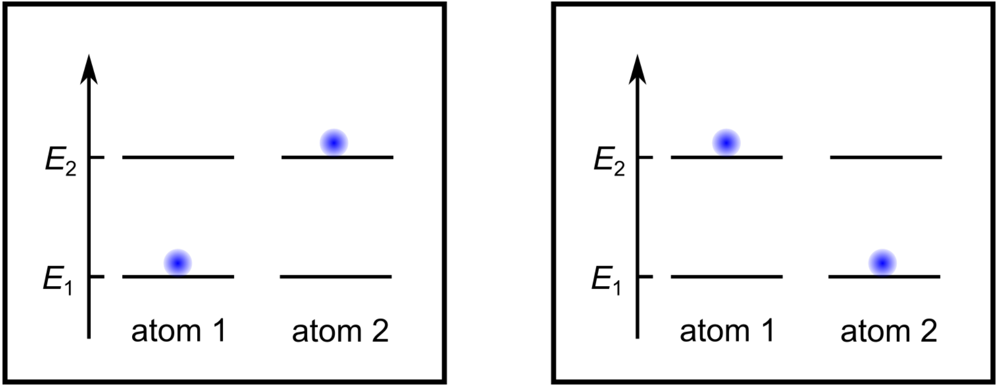
The first figure shows the ground state.

There is only one possibility to realise the ground state.

The number of possibilities for realizing an energy in a system is denoted by the capital letter Γ.

For the ground state, Γ takes the value one.

Temperature 3



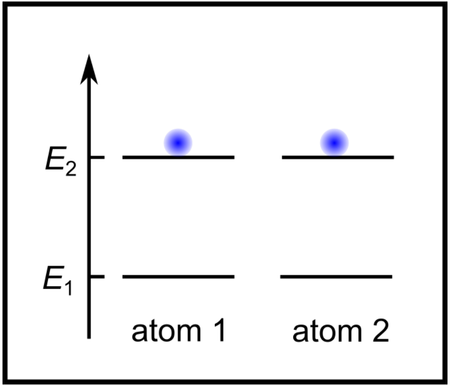
Configuration II: $E_{\text{II}} = E_1 + E_2$ and $\Gamma(E_{\text{II}}) = 2$

Temperature 3 Comment

When one atom is excited there are two options.

The energy is $E_1 + E_2$ and Γ takes the value 2 for this energy.

Temperature 4



Configuration III: $E_{\mathrm{III}} = 2E_2$ and $\Gamma(E_{\mathrm{III}}) = 1$

Temperature 4 Comment

When both atoms are excited, the energy is $2E_2$.

There is only one way to realize this energy and the value of Γ is 1.

Temperature 5

$\Gamma(E)$ denotes the possibilities for realizing the energy E

The definition of entropy is

$$\boxed{S(E) = k_B \ln \Gamma(E)}$$

Boltzmann constant

$$\underline{k_{\mathrm{B}} = 1.38 \cdot 10^{-23}\,\mathrm{J/K}}$$

Temperature 5

Comment

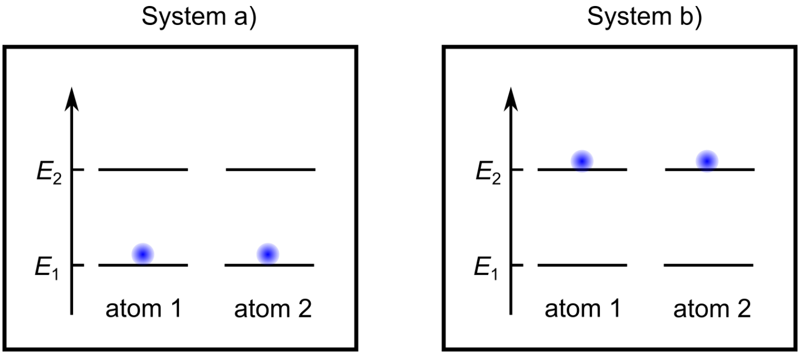
The entropy of a system is determined by the number of possibilities for realizing the energy of a statistical system.

Entropy is denoted by the capital letter S and is defined by the equation outlined in red.

The Boltzmann constant is added for practical reasons, which later become clear.

Temperature 6

For two systems a) and b), the total energy and entropy is the sum of the two sub-systems, e.g.



$$E_{ab} = E_a + E_b, \;\; \Gamma_{ab} = \Gamma_a(E_a)\Gamma_b(E_b) = 1, \;\; S_{ab}(E) = k_B \ln \Gamma_a(E_a) + k_B \ln \Gamma_b(E_b) = 0$$

Temperature 6

Comment

With two systems, the total energy is the sum of the energy of the two subsystems.

The total number of possibilities for realizing a certain total energy is given by the product of the possibilities for realizing the energy of the individual systems $\Gamma_{\mathrm{a}}(E_{\mathrm{a}})$ and $\Gamma_{\mathrm{b}}(E_{\mathrm{b}})$.

The total entropy is therefore given by the sum $S_{\mathrm{a}} + S_{\mathrm{b}}$.

The figure illustrates this idea.

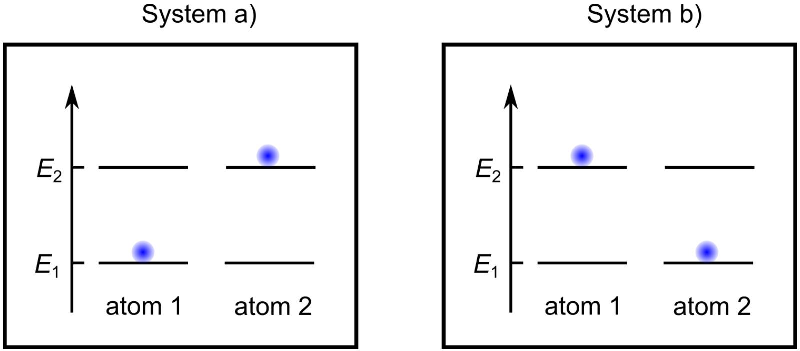
Subsystem a) is in the ground state and subsystem b) is in the state with the largest energy.

The number of possibilities is 1 for each subsystem.

The number is equally one for the combined system. The entropy of each system is zero and the total entropy is also zero.

Temperature 7

another example



$$E_{ab} = E_a + E_b \quad \text{and} \quad \Gamma_{ab} = \Gamma_a(E_a)\Gamma_b(E_b) = 4 \quad \text{and}$$

$$S_{ab} = S_a + S_b = k_B \ln 2 + k_B \ln 2 = k_B \ln 4$$

Temperature 7

Comment

The figure shows the situation that only one atom is excited in each of the two subsystems.

There are two ways of realizing the excited energy for each subsystem.

There are four ways of realizing the energy for the entire system.

The total entropy is twice the logarithm of two.

Temperature 8

For two macroscopic systems a) and b) is the total energy

$$\boxed{E_{ab} = E_a + E_b}$$

the number of configurations

$$\underline{\Gamma_{ab}(E_{ab}) = \Gamma_a(E_a) \cdot \Gamma_b(E_b)} \quad \text{and} \quad \ln \Gamma_{ab}(E_{ab}) = \ln \Gamma_a(E_a) + \ln \Gamma_b(E_b)$$

the total entropy

$$\boxed{S_{ab} = S_a + S_b}$$

Temperature 8 Comment

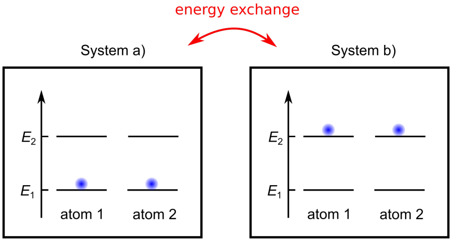
The equations outlined in red summarize these results.

The total energy is the sum of the energies of all subsystems.

The total entropy is the sum of the entropies of all subsystems.

Temperature 9

Two systems in thermal contact can exchange energy



The total energy is conserved

$$E = E_a + E_b = 2E_1 + 2E_2$$

There are different possibilities to distribute the energy within the two subsystems

Temperature 9

Comment

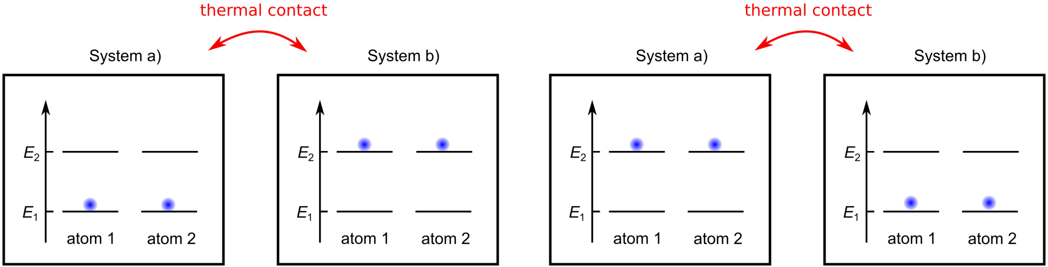
The figure shows the two subsystems again, with one subsystem in the ground state and the other in the most excited state.

The two subsystems can exchange their energy.

There are various ways of distributing the energy within the two subsystems.

Temperature 10

There are 2 configurations if one of the two systems takes all of the excitation energy



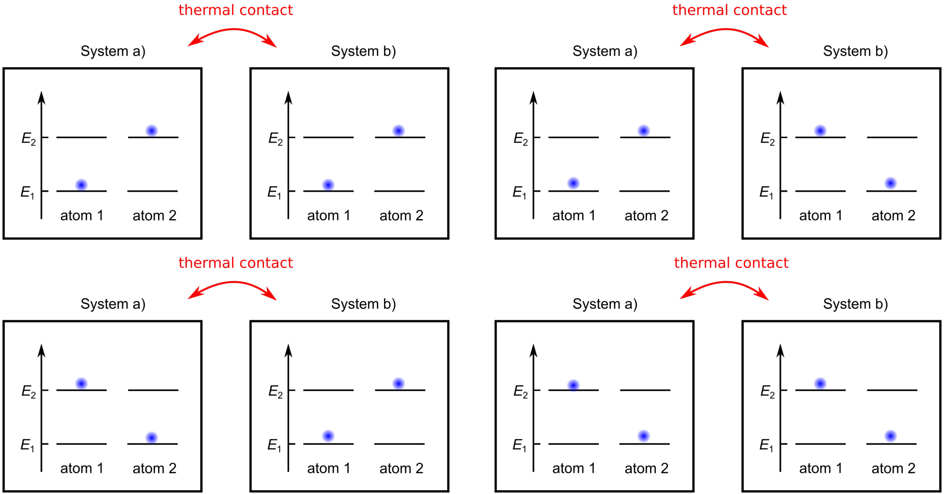
Temperature 10

Comment

This figure shows the two possibilities that either system a) or system b) takes the entire excitation energy.

Temperature 11

There are 4 configurations when each system takes part of the excitation energy



Temperature 11

Comment

This figure shows the four possibilities if each system only takes part of the excitation energy.

Temperature 12

If two macroscopic systems a) and b) are isolated from the rest of the world in thermal contact, both systems have the same temperature in thermal equilibrium

The total energy is constant

$$\boxed{E = E_a + E_b}$$

and the number of possibilities

$$\boxed{\Gamma(E_\mathrm{a} + E_\mathrm{b}) = \Gamma_\mathrm{a}(E_\mathrm{a})\Gamma_\mathrm{b}(E_\mathrm{b})} \quad \text{is maximal}$$

i.e. the entropy $S = k_\mathrm{B} \ln \Gamma$ is maximal

$$\mathrm{S}(E_\mathrm{a}, E_\mathrm{b}) = \mathrm{S}_\mathrm{a}(E_\mathrm{a}) + \mathrm{S}_\mathrm{b}(E_\mathrm{b}) \quad \text{and} \quad \boxed{\frac{\partial S}{\partial E_a} = 0, \qquad \frac{\partial S}{\partial E_b} = 0}$$

Temperature 12 Comment 1

Two macroscopic systems eventually reach the same temperature in thermal contact.

The total energy of the two systems is conserved when the two systems are well isolated from the surrounding world.

The first fundamental assumption of thermodynamics is that a macroscopic system is in thermal equilibrium in a state in which the number of possibilities for realizing the total energy is maximal.

Temperature 12

Comment 2

Since the logarithm is a monotonic function of the number of possibilities, the entropy in thermal equilibrium is also maximal.

The total energy is conserved and therefore constant.

However, the energy of the subsystems can change.

The mathematical criterion for thermal equilibrium is therefore that the partial derivative of the entropy with respect to the energy of each subsystem must be zero.

Temperature 13

With $E =$ constant

$$dE_a = -dE_b$$

therefore

$$\frac{\partial S}{\partial E_a} = 0 = \frac{\partial (S_a + S_b)}{\partial E_a} = \frac{\partial S_a}{\partial E_a} + \frac{\partial S_b}{\partial E_b}\frac{\partial E_b}{\partial E_a} = \frac{\partial S_a}{\partial E_a} - \frac{\partial S_b}{\partial E_b}$$

and

$$\underline{\frac{\partial S_a}{\partial E_a} = \frac{\partial S_b}{\partial E_b}}$$

Since the temperatures of all subsystems are the same in thermal equilibrium, the definition of temperature is

$$\boxed{\frac{1}{T} = \frac{\partial S}{\partial E}}$$

Temperature 13 Comment 1

A small calculation leads to the underlined formula.

The derivation of the entropy of each subsystem with respect to the energy of the subsystem is the same for all subsystems in thermal equilibrium.

Since the temperature of all subsystems must be the same in thermal equilibrium, the reciprocal temperature is defined as the partial derivative of the entropy with respect to the energy.

To understand that it makes sense to use the reciprocal temperature, one can consider two special cases.

At very high temperatures, both the energy and entropy are high.

Temperature 13

Comment 2

The entropy hardly changes when the energy is increased further.

Therefore the derivation of the entropy with respect to the energy tends to zero for very high temperatures and the reciprocal temperature will likewise tend towards zero.

On the other hand, only the ground state is occupied at low temperatures.

The entropy of the ground state is usually zero.

Therefore, the entropy increases enormously when the energy is increased.

The reciprocal temperature will be large and consequently the temperature will be very small.

Boltzmann factor

Essentials of Thermodynamics

1. Temperature
2. Boltzmann factor
3. Maxwell’s velocity distribution function
4. Ideal gas laws

Boltzmann factor 1

In thermal equilibrium, the Boltzmann factor gives the occupation probability w_n of an energy level E_n of a microscopic system that is in thermal contact with a macroscopic system

Boltzmann factor 2

The energy of the macroscopic system is E_M and that of the microscopic system is E_n.The total energy is

$$\underline{E = E_M + E_n}$$

If there is only one possibility to realize the energy E_n of the microscopic system, the total entropy is equal to the entropy of the macroscopic system
$S(E_M) = k_B \ln \Gamma(E_M)$

The number of possibilities to realize the energy E_M of the macroscopic system is

$$\Gamma(E_M) = \exp\left(\frac{S(E_M)}{k_B}\right) = \exp\left(\frac{S(E - E_n)}{k_B}\right)$$

Boltzmann factor 2

Comment 1

The system is divided into a macroscopic system and a microscopic system.

The macroscopic system could be a crystal and the microscopic system an atom in the crystal.

The total energy is the sum of the energy of the macroscopic and microscopic system.

For the sake of simplicity, it is assumed that there is only one possibility of realizing the energy E_n of the microscopic system, i.e. for each energy in the microscopic system there is only one energy level.

Therefore, the microscopic system does not contribute to the total entropy of the system.

Boltzmann factor 2

Comment 2

The formula written in blue indicates the number of possibilities to realize the energy of the macroscopic system.

The number of possibilities of realizing a state with the energy E_M depends on the energy E_M and changes when the energy of the microscopic system changes.

The energy of the macroscopic system corresponds to the difference between the total energy and the energy of the microscopic system.

Boltzmann factor 3

Taylor expansion ($E_n << E$)

$$S(E - E_n) = S(E) - \frac{\partial S}{\partial E} E_n = \underline{S(E) - \frac{E_n}{T}}$$

and

$$\Gamma(E_M) = \exp\left(\frac{S(E-E_n)}{k_B}\right) = \exp\left(\frac{S(E)}{k_B}\right) \underline{\exp\left(-\frac{E_n}{k_B T}\right)}$$

With the assumption: $\exp\left(\frac{S(E)}{k_B}\right) =$ constant is the occupation probability of energy level E_n

$$\boxed{w_n \propto \Gamma(E_M) \propto \exp\left(-\frac{E_n}{k_B T}\right)}$$

Boltzmann factor 3 Comment 1

The energy of the microscopic system is certainly much smaller than the energy of the macroscopic system.

Even in a small crystal there are about 10^{20} atoms and the energy of a single atom is negligibly small compared to the energy of the whole crystal.

Therefore a Taylor expansion can be used.

Since the derivative of the entropy with respect to the energy is the reciprocal temperature, the underlined formula results for the entropy of the system.

The number of possibilities of the macroscopic system to realize a state with the energy E_M is given by the product of two exponential functions.

Boltzmann factor 3 Comment 2

The exponent of the first exponential function is the entropy divided by the Boltzmann constant.

The exponent of the second exponential function results from the quotient of the energy of the microscopic system and the thermal energy, which is given by the product of the Boltzmann constant with the temperature.

This exponential function underlined in red is the Boltzmann factor.

It is assumed that the first exponential is a constant and that the energy dependence of $\Gamma(E_\mathrm{M})$ is completely determined by the Boltzmann factor.

Boltzmann factor 3 Comment 3

Then the number of possibilities to realize the energy E_M of the macroscopic system is proportional to the Boltzmann factor.

$\Gamma(E_M)$ depends on the excitation energy of the microscopic system.

$\Gamma(E_M)$ decreases as the energy of the microscopic system increases.

It is therefore assumed that the occupation probability w_n of a microscopic energy level E_n is also proportional the $\Gamma(E_M)$. This assumption has been well confirmed experimentally.

For example, consider the case where the microscopic system takes all the energy and the macroscopic system is in its ground state.

Then $\Gamma(E_M) = 0$ and hence, according to the assumption, the probability that the microscopic system takes all the energy is zero.

Boltzmann factor 4

$$\underline{w_n \propto \exp(-\frac{E_n}{k_B T})}$$

Constant of proportionality: $\underline{\text{partition sum } Z}$

$$w_n = \frac{1}{Z}\exp(-\frac{E_n}{k_B T})$$

and

$$\sum_n w_n = 1 = \frac{1}{Z}\sum_n \exp(-\frac{E_n}{k_B T})$$

i.e.

$$\boxed{Z = \sum_n \exp(-\frac{E_n}{k_B T})}$$

Boltzmann factor 4 Comment

The proportionality constant between the occupation probability and the Boltzmann factor is determined by the partition sum.

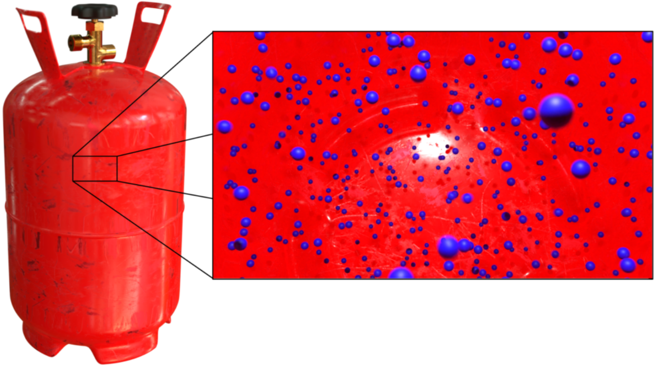
The equation outlined in red gives the definition of the partition sum.

Maxwell distribution function

Essentials of Thermodynamics

Maxwell’s velocity distribution function 1

The ideal gas



(IdealesGas.mp4)

Maxwell’s velocity distribution function 1 Comment

A simple application of Boltzmann’s theory is the velocity distribution function of an ideal gas.

The distribution function was derived by Boltzmann and Maxwell in 1860.

The video shows an animation of an ideal gas.

The atoms of the gas move randomly at different speeds.

The atoms collide with each other and the wall of the container.

Maxwell's velocity distribution function 2

The Boltzmann factor for a particle of the gas with the kinetic energy $E_{\text{kin}} = mv^2/2$ is

$$w(E_{kin}) \propto \exp\left(-\frac{E_{kin}}{k_B T}\right) = \exp\left(-\frac{mv^2}{2k_B T}\right)$$

The probability that the velocity of a gas particle lies in an interval $v, v + dv$ is

$$w(v) \propto 4\pi v^2 dv$$

and the number of particles with velocity v in an interval dv is

$$dN(v) = C\, 4\pi v^2 dv \exp\left(-\frac{mv^2}{2k_B T}\right)$$

C is the constant of proportionality

Maxwell’s velocity distribution function 2 Comment

The first formula gives the Boltzmann factor for a particle that moves with a certain kinetic energy.

The exponent of the exponential function results from the quotient of the kinetic energy and the thermal energy.

The probability that a particle moves at a particular speed results from the second formula written in blue.

The probability is proportional to the volume of a spherical shell with the radius v and the thickness dv.

The factor $4\pi v^2$ gives the surface of a sphere with the radius v.

The number of particles with the speed in the interval $v, v + dv$ is proportional to the product of the geometric factor $4\pi v^2 \mathrm{d}v$ and the Boltzmann factor.

Maxwell's velocity distribution function 3

The constant of proportionality C is determined by the integration over all speeds from zero to infinity

$$N = \int_{v=0}^{\infty} dN(v) = \int_{v=0}^{\infty} C\, 4\pi \exp\left(-\frac{mv^2}{2k_BT}\right) v^2 dv \quad \to \quad \underline{C = \frac{N}{\left(\frac{2\pi k_B T}{m}\right)^{3/2}}}$$

Maxwell's velocity distribution function

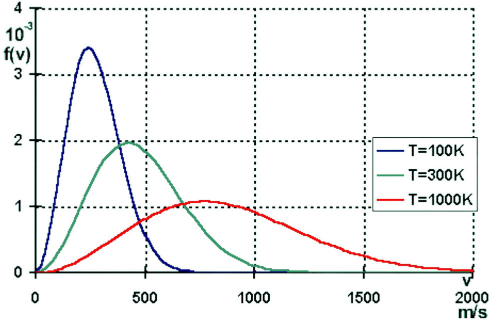
$$\boxed{f(v) = \frac{1}{N}\frac{dN(v)}{dv} = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2k_BT}\right)}$$

Maxwell’s velocity distribution function 3 Comment

The constant of proportionality is determined by the integration over all speeds from zero to infinity.

The formula outlined in red is Maxwell’s velocity distribution function.

Maxwell’s velocity distribution function 4



The speed for which Maxwell’s velocity distribution function is maximum is

with $\frac{df(v)}{dv} = 0 \quad \rightarrow \quad v_{max}^2 = \frac{2k_BT}{m}$ and $\underline{E_{kin,max}} = \frac{1}{2}mv_{max}^2 = \underline{k_BT}$

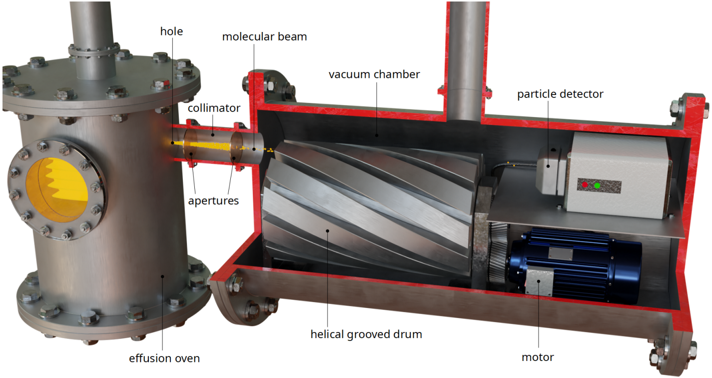
Maxwell’s velocity distribution function 4 Comment

The figure shows Maxwell’s velocity distribution function for three different temperatures.

A calculation shows that the kinetic energy of the particles in the maximum of Maxwell’s velocity distribution function is given by the thermal energy, i.e. the product of Boltzmann’s constant and temperature.

Maxwell’s velocity distribution function 5

Experimental test



(Geschwindigkeitsfilter.mp4)

see also: (https://www.tec-science.com/thermodynamics/kinetic-theory-of-gases/determination-of-the-velocity-distribution-in-a-gas/)

Maxwell's velocity distribution function 5 Comment

The animation shows how the velocity distribution function can be measured.

The particles are vaporized in an oven at a certain temperature.

The vaporized particles are guided through the spiral-shaped grooved roller and counted by the particle detector.

The speed of rotation of the roller determines the speed of the particles reaching the particle detector.

The Maxwell velocity distribution can be determined by counting the number of particles for different rotational speeds of the roller.

Ideal gas laws

Essentials of Thermodynamics

Ideal gas laws 1

Law of Boyle-Mariotte (1662 and 1676)

$$\boxed{p \propto \frac{1}{V}}$$

Laws of Gay-Lussac (1802)

$$\boxed{\begin{array}{c} V \propto T \\ p \propto T \end{array}}$$

Law of Avogadro (1811)

$$\boxed{V \propto N} \quad \text{and} \quad T = \text{constant}, \; p = \text{constant}$$

Ideal gas laws 1

Comment 1

Measuring the temperature of a gas using the velocity distribution function is tedious.

This task can be accomplished much more conveniently when the ideal gas laws are used.

Boyle and Mariotte discovered in the second half of the 17th century that the product of pressure and volume is a constant for a gas.

Gay-Lussac discovered in 1802 that the volume and pressure of a gas are proportional to temperature.

With these laws it became possible to use gases to measure temperature.

Since there is no negative volume, it is immediately clear that there must be absolute zero temperature.

Ideal gas laws 1

Comment 2

A discovery by Avogadro in 1811 opened the door to a scientific understanding of matter.

Avogadro showed that the volume of a gas at constant pressure and temperature is proportional to the number of particles in the gas.

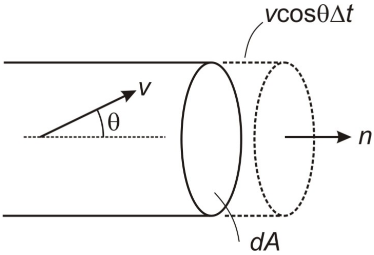
This result does not depend on the type of particles that are in the gas and mixtures of different particles are also possible.

With Avogadro’s law it became possible to determine the relative masses of various chemical compounds in the gas phase.

These measurements finally led to the periodic table of the elements, which was first published in 1869 by Dmitri Ivanovich Mendeleev in St. Petersburg.

Ideal gas laws and Maxwell's velocity distribution function 2

Kinetic gas theory



$$\boxed{pV = \frac{2}{3} N \langle E_{kin} \rangle}$$

The ideal gas law results when the mean kinetic energy $\langle E_{kin} \rangle$ is calculated with the Maxwell velocity distribution function

$$\langle E_{kin} \rangle = \frac{1}{2} m \int_{v=0}^{\infty} v^2 f(v) dv = \frac{3}{2} k_B T \quad \text{and} \quad \boxed{pV = N k_B T}$$

Ideal gas laws 2

Comment 1

The kinetic gas theory shows that the product of pressure and volume is proportional to the mean kinetic energy of the particles.

The kinetic gas theory is based on the assumption that the particles collide elastically with one other and with the walls.

In order to calculate the pressure, the force that the gas particles exert on the wall of the container must be determined.

The figure shows how the number of particles that hit the container wall in the time interval Δt can be determined.

The number of particles results from the density of the gas particles and the volume shown in dashed lines.

Ideal gas laws 2

Comment 2

The momentum transfer per time interval is the force and the force per area is the pressure.

The mean kinetic energy of the gas can be calculated with Maxwell's velocity distribution function by integrating over the velocities.

The calculation results in the well-known ideal gas law.

The temperature measurable with the ideal gas law is the temperature defined by the statistical arguments of Boltzmann, i.e. $\frac{1}{T} = \frac{\partial S}{\partial E}$.

Ideal gas laws 2

Details of calculation

The number of particles with velocity v is

$$dN(v) = Nf(v)dv$$

The number of particles with velocity v in direction θ is

$$dN(v,\theta) = Nf(v)dv \cdot \frac{2\pi \sin\theta d\theta}{4\pi} = \frac{1}{2}Nf(v)dv \sin\theta d\theta,$$

i.e. the fraction between the solid angle $2\pi \sin\theta d\theta$ and the total solid angle 4π.

The momentum transfer of one particle to the wall is

$$\Delta p = 2mv\cos\theta$$

Ideal gas laws 2

Comment 4

The density of particles with velocity v in direction θ is

$$\frac{dN(v,\theta)}{V} = \frac{1}{2}\frac{N}{V} f(v) dv \sin\theta d\theta,$$

The number of particles which transfer momentum during the time interval Δt on the area dA of the wall is

$$\frac{dN(v,\theta)}{V} dA v \cos\theta \Delta t$$

The force due to the momentum transfer on area dA is ($F = \Delta p / \Delta t$)

$$dF = 2mv\cos\theta \cdot \frac{dN(v,\theta)}{V} dA v \cos\theta$$

Ideal gas laws 2

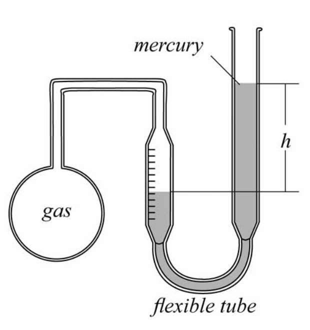
Comment 5

The pressure dF/dA times volume is

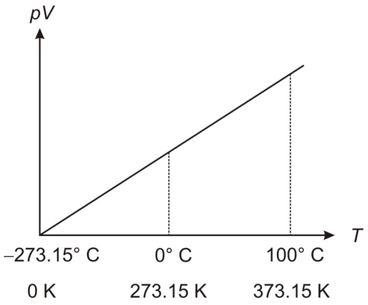
$$\begin{aligned}
pV &= \int 2mv^2 \cos^2\theta dN(v,\theta) \\
&= mN \int_{v=0}^{\infty} dv v^2 f(v) \int_{\theta=0}^{\pi/2} d\theta \cos^2\theta \sin\theta \\
&= mN \int_{v=0}^{\infty} dv v^2 f(v) \cdot \frac{1}{3} \\
&= \frac{2}{3} N \int_{v=0}^{\infty} dv \frac{1}{2} m v^2 f(v) \\
&= \frac{2}{3} N \langle E_{kin} \rangle
\end{aligned}$$

Ideal gas laws 3

Gas thermometer



Kelvin and the degree Celsius temperature scale



Ideal gas laws 3 Comment

The left figure shows a gas thermometer.

To measure the temperature, you have to keep the volume constant with the mercury column and determine the pressure of the gas.

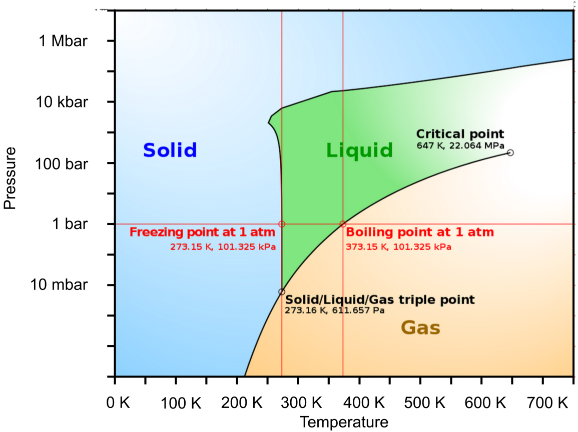
The height of the mercury column is then proportional to the temperature.

The right figure shows the definition of the degree Celsius temperature scale.

Ice water and boiling water define the two fixed points on the temperature scale.

Ideal gas laws 4

Definition of the degree Celsius temperature scale



Ideal gas laws 4 Comment

The figure shows the phase diagram of water, with which the fixed points of the degree Celsius temperature scale are defined.

The absolute temperature scale can be defined with the degree Celsius temperature scale and the ideal gas law.

The absolute temperature is measured in Kelvin.

The absolute zero of the temperature corresponds to -273.15 ° C when the degree Celsius temperature scale is used.

The experimental definition of the temperature scale explains why the Boltzmann constant has to be introduced into the definition of entropy.

If the Boltzmann constant is replaced by 1, the temperature is measured in units of energy.

Modern Physics

Contents:

Michelson Experiment

Special Relativity

1. Michelson-Morley experiment
2. Einstein’s postulates
3. Lorentz transformation
4. Time dilation and length contraction
5. Twin paradox
6. Experiment of Hall and Rossi
7. „Visible“ effects due to the Lorentz transformation
8. Invariant distance
9. Relativistic invariants
10. Doppler effect
11. Relativistic mechanics

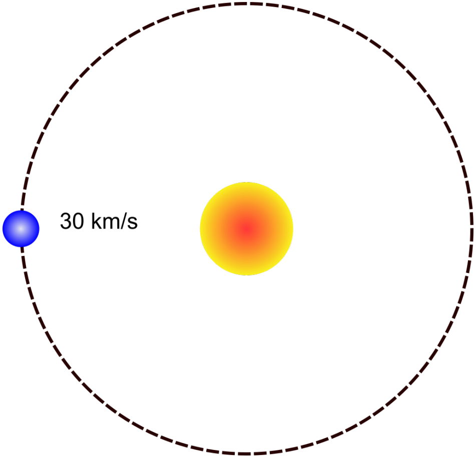
Michelson-Morley experiment 0

Stellar aberration (discovered and explained by James Bradley in 1727)

(Aberration.mp4)

Michelson-Morley experiment 1

Michelson’s project: Measurement of the earth’s speed relative to the ether



Michelson-Morley experiment 1 Comment 1

The aberration of light from stars was discovered by James Bradley in 1727.

The angle of observation of a star varies periodically over the course of the year.

With Newton’s corpuscle theory of light, James Bradley was able to trace the effect back to the movement of the earth around the sun.

Since the speed of the earth is known, the speed of light can be determined from the aberration angle.

He discovered that the speed of light is the same for all the stars studied.

This astonishing result was confirmed more and more precisely in the following years.

Michelson-Morley experiment 1

Comment 2

The result is astonishing, since in Newton's corpuscular theory of light it can be expected that the speed of light also depends on the speed of the star.

At the beginning of the 19th century it was no longer possible to ignore the wave properties of light.

Similar to sound waves or waves on the water surface, the speed of light should be determined by the medium in which the waves propagate.

The hypothetical medium in which light waves propagate was called ether.

Unfortunately, it is not trivial to explain the aberration effect in a classical wave theory.

The inclination of the wave fronts and thus the direction of propagation of the wave is independent of the speed of an observer.

Michelson-Morley experiment 1 Comment 3

A possible solution to this problem was provided by Fresnel’s ether theory in 1819.

The predictions of Fresnel’s ether theory could be confirmed experimentally in first order in v/c.

With this success of Fresnel’s ether theory, the wave theory of light was generally accepted.

Mickelson designed his famous experiment to test the ether theory in second order of v/c.

In contrast to measurements of the aberration of stars, which can be easily understood in a classical particle picture of light, Michelson tried to measure the influence of the earth’s speed on the speed of light directly through the interference of waves.

Michelson-Morley experiment 1 Comment 4

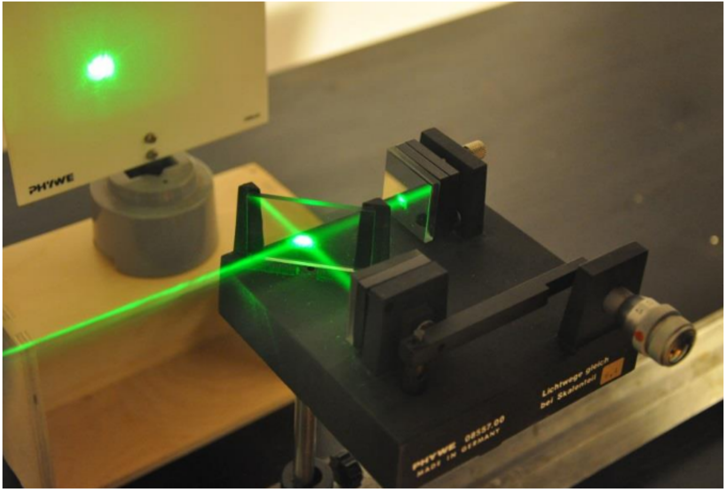
The Michelson-Morley experiment is an experiment designed to test a historical theory of the ether.

The Michelson-Morley experiment is not at all necessary to find or understand the theory of relativity.

However, the Michelson-Morley experiment nicely illustrates two important consequences of the Lorentz transform: time dilation and length contraction.

It is therefore worth taking a closer look at the historical experiment.

Michelson-Morley experiment 2



Michelson-Morley experiment 2 Comment

The central part of a Michelson interferometer is a beam splitter.

This is a glass plate that is coated on one side with a thin metal layer.

Part of the incident beam is reflected to one mirror and the remainder is transmitted to a second mirror.

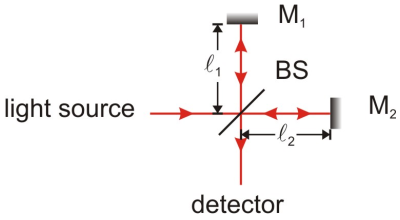
This process is repeated for the rays that are reflected by the two mirrors.

Now there are two rays that propagate towards the observation screen and interfere with each other.

Michelson-Morley experiment 3

Michelson Interferometer

- BS: beam splitter
- M1 and M2: mirror 1 and 2



(michelsonmorleystatic.mp4)

time on the way $\overline{BS - M_1 - BS}$

$$\underline{t_1 = \frac{2\ell_1}{c}}$$

time on the way $\overline{BS - M_2 - BS}$

$$\underline{t_2 = \frac{2\ell_2}{c}}$$

phase diffence between ray 1 and 2

$$\Delta\varphi = \omega(t_1 - t_2) = \frac{\omega}{c}(2\ell_1 - 2\ell_2) = \frac{2\pi}{\lambda}(2\ell_1 - 2\ell_2)$$

Michelson-Morley experiment 3 Comment

The sketch shows the schematic setup of the interferometer.

The phase difference between the two beams reaching the detector can easily be calculated.

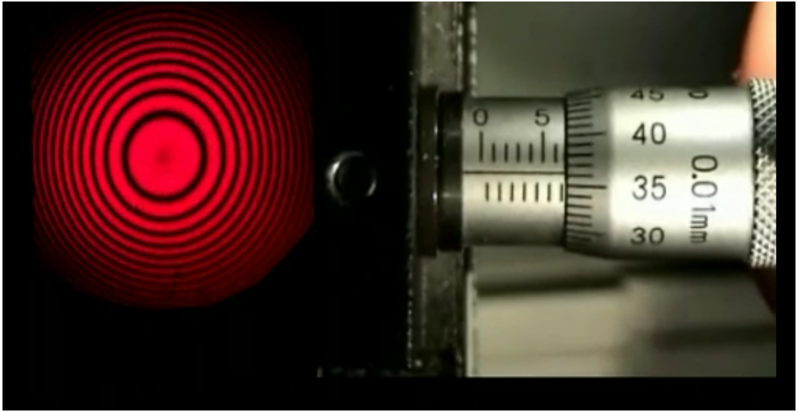
The time it takes for light to travel the distance between the beam splitter and the two mirrors is given by the underlined formulas.

The phase difference results when the time difference between t_1 and t_2 is multiplied by the angular frequency of the light wave.

Alternatively, the path difference between the two beams that reach the detector can be multiplied by the wavenumber.

The animation shows the situation when the interferometer is resting in the ether.

Michelson-Morley experiment 4



(MichelsonInterferometerPathVariations.mp4)

Michelson-Morley experiment 4

Comment

The video shows the interference pattern of a Michelson interferometer.

A slightly divergent Laser beam is used.

The interference pattern changes when the distance between one of the mirrors and the beam splitter is changed.

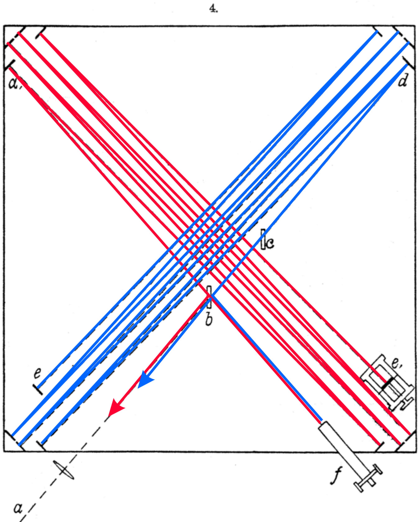
Michelson Morley Experiment (1881) 5

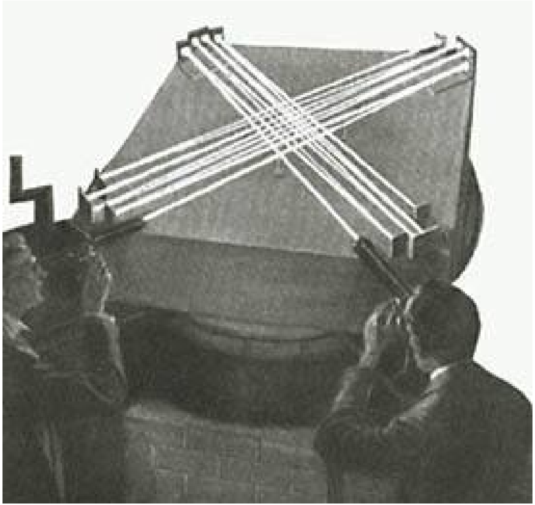
Michelson-Morley experiment 5

Comment

The picture shows the first Michelson interferometer, which was built in 1881.

Michelson Morley (1887) Experiment 6





Michelson-Morley experiment 6 Comment

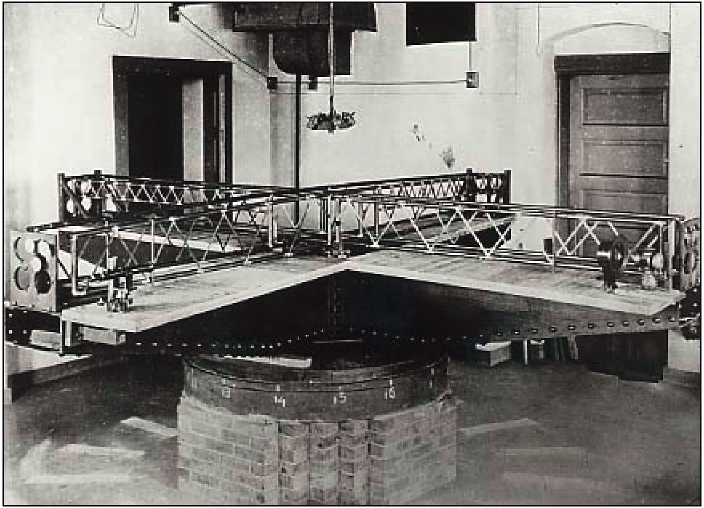
These pictures show the improved Michelson interferometer built in 1887.

The experiment consists of a granite block that is placed in a mercury bath.

This greatly improves the mechanical stability.

The left picture shows that the light path has also been lengthened by at least a factor of ten.

Michelson Morley (1905) Experiment 7



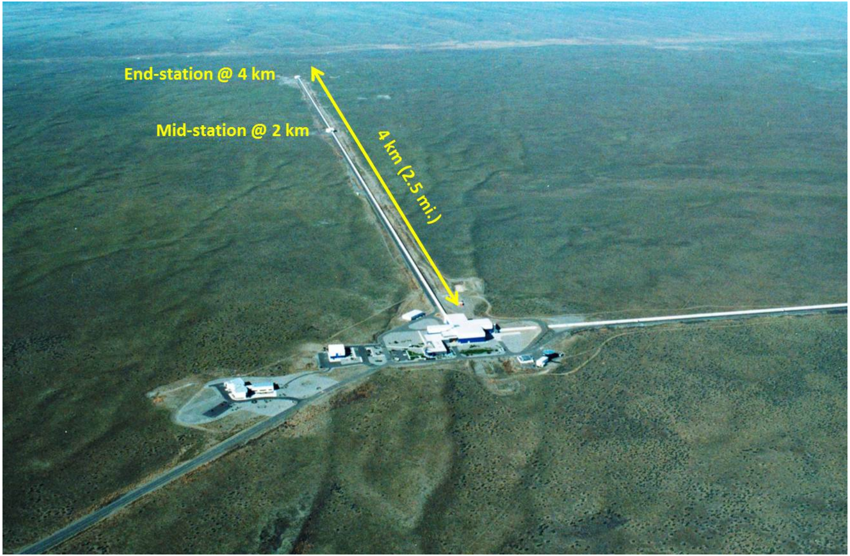
Michelson-Morley experiment 7

Comment

This picture shows the refined interferometer that was built in 1905.

The experiments by Michelson and Morley did not show the expected effect.

Michelson-Morley experiment: Ligo-Detector 8

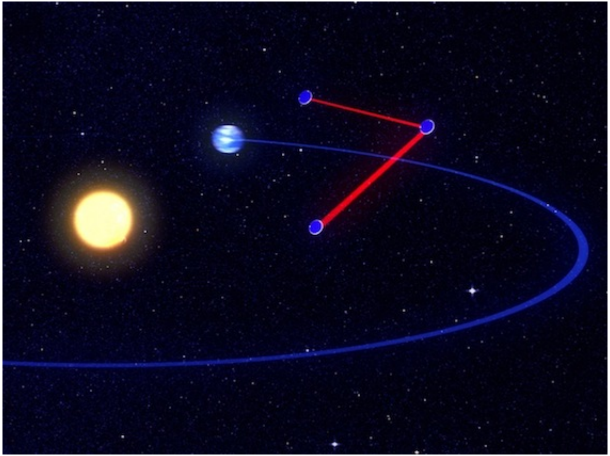


Michelson-Morley experiment 8

Comment

Nowadays, Michelson interferometers are used to detect gravitational waves.

Michelson-Morley experiment 9



additional information: (https://www.elisascience.org/)

Michelson-Morley experiment 9 Comment

Gravitational waves were successfully detected for the first time in 2015.

This opened a new window for the exploration of the cosmos that is not based on electromagnetic waves.

2023 five gravitational wave detectors are working.

This allows the source of a gravitational wave to be localized and associated with electromagnetic signals.

These gravitational wave detectors can be used to investigate sources of gravitational waves that are in our galaxy.

To study sources outside of our galaxy, much larger gravitational wave detectors are needed.

Therefore, a large interferometer in space is planned.

Revision

Summary in questions

1. What is entropy in physics?
2. How is temperature defined in physics?
3. Write down the Boltzmann factor.
4. Write down the partition sum.
5. How is Maxwell's velocity distribution function defined?
6. Write down Maxwell's velocity distribution function without pre-factors.
7. Sketch Maxwell's velocity distribution function for two different temperatures.
8. What is the kinetic energy of the gas particles in the maximum of the velocity distribution function?
9. Write down the ideal gas law.
10. Explain the definition of the Kelvin and the degree Celsius temperature scale.