Modern Physics

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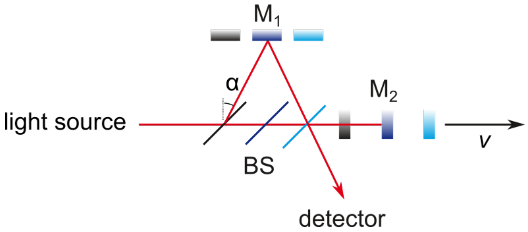
Michelson's idea: The interferometer moves with the earth through the "ether"



(michelsonmorleydinamic.mp4)

Michelson-Morley experiment 10 — Comment

The figure illustrates Michelson's idea.

The interferometer moves through the ether at the speed of the earth.

The position of the interferometer is displayed three times in a row.

Due to the movement relative to the aether, the beam from the beam splitter to the mirror 1 has to be tilted a little forward.

Since the speed of light is much greater than the speed of the earth, the effect is very small and results automatically for a special ray in a slightly divergent bundle of light.

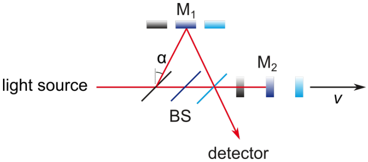
The video shows an animation of the expected effect.

Michelson-Morley experiment 11

a) time on path 1: $\overline{BS - M_1 - BS}$

$$\frac{t_1}{2} = \frac{\sqrt{\ell_1^2 + \left(\frac{vt_1}{2}\right)^2}}{c}$$

$$\rightarrow \boxed{t_1 = \frac{2\ell_1}{c}\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}}$$



Michelson-Morley experiment 11 Comment

The time required on the way from the beam splitter to the first mirror can be calculated using the Pythagorean theorem.

The total time is twice the time from the beam splitter to the first mirror.

Michelson-Morley experiment 12

b) time on path 2: $\overline{BS - M_2 - BS}$

time $\overline{BS - M_2}$

$$t_{2\rightarrow} = \frac{\ell_2 + v\, t_{2\rightarrow}}{c} \quad \text{and} \quad t_{2\rightarrow} = \frac{\ell_2}{c - v} = \frac{\ell_2}{c} \frac{1}{1 - \frac{v}{c}}$$

time $\overline{M_2 - BS}$

$$t_{2\leftarrow} = \frac{\ell_2}{c} \frac{1}{1 + \frac{v}{c}}$$

and

$$t_{2\rightarrow} + t_{2\leftarrow} = \boxed{t_2 = \frac{2\ell_2}{c} \frac{1}{1 - \left(\frac{v}{c}\right)^2}} > t_1 \quad \text{when} \quad \ell_1 = \ell_2$$

$$t_1 = \frac{2\ell_1}{c} \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Michelson-Morley experiment 12 Comment

For the necessary time on the way between the beam splitter and the second mirror, it must be taken into account that the mirror retracts from the beam splitter.

For the way back, the beam splitter approaches and the time required is shortened.

The sum of both times results in the equation framed in red.

The comparison shows that the time t_2 is larger than the time t_1 if the distance between the mirrors and the beam splitter is the same for both mirrors.

This is the expected effect shown in the last video.

Michelson-Morley experiment 13

phase diffence between ray 2 and 1

$$\Delta\varphi = \omega(t_2 - t_1)$$

with $\ell_1 = \ell_2 = \ell$ and $\omega/c = k = 2\pi/\lambda$

$$\Delta\varphi = \omega(t_2 - t_1) = \underline{\frac{2\pi}{\lambda}2\ell\left(\frac{1}{1-\left(\frac{v}{c}\right)^2} - \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right)}$$

- velocity of the earth $v_{\text{Earth}} \approx 3 \cdot 10^4\,\text{m}/\text{s}$
- speed of light $c \approx 3 \cdot 10^8\,\text{m}/\text{s}$

$$\boxed{\frac{v}{c} \approx 10^{-4}} \quad \rightarrow \quad \text{Taylor expansion}$$

Michelson-Morley experiment 13 Comment

Here, too, the phase difference results when the time delay between the two beams is multiplied by the angular frequency of the wave.

The phase difference also results from the effective path difference multiplied by the wave number.

Since the speed of the earth around the sun is so much smaller than the speed of light, a Taylor expansion can be used.

Michelson-Morley experiment 14

Maximum phase difference of the moving interferometer

$$\Delta\varphi = \omega(t_2 - t_1) = \frac{2\pi}{\lambda} 2\ell \left(1 + \left(\frac{v}{c}\right)^2 - 1 - \frac{1}{2}\left(\frac{v}{c}\right)^2\right) = \underline{2\pi \frac{\ell}{\lambda}\left(\frac{v}{c}\right)^2}$$

The factor ℓ/λ amplifies the small ratio of $(v/c)^2$

The interferogram should change when the moving interferometer is rotated. This should make it possible to determine the direction and magnitude of the earth's speed $\vec{v}$

The expected effect is not observed!

Michelson-Morley experiment 14 Comment 1

Applying the Taylor expansion gives the underlined formula.

The small ratio between the speed of the earth and the speed of light should be measurable through the large ratio between the light path and wavelength.

The rotation of the earth changes the orientation of the interferometer in relation to the earth's speed, so that the interference pattern should change over the course of the day.

The more stable and larger the interferometer became over time, the clearer it became that the expected effect could not be observed.

With the special theory of relativity published in 1905, the ether theory became obsolete, but this did not mean that Michlson's interferometric work lost its meaning.

Michelson-Morley experiment 14 Comment 2

Albert Abraham Michelson was awarded the Nobel Prize in 1907 ”for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid”.

Einstein’s post.

Special Relativity

Einstein's postulates

1. The laws of nature are the same in all inertial systems.
2. Observers always measure the value $c = 299\,792\,458$ m/s for the speed of light regardless of their speed.
3. There is no absolute state of rest

Einstein's postulates — Comment 1

The special theory of relativity is based on three postulates.

The first postulate says that all natural laws are the same in all inertial systems.

The second postulate says that the speed of light does not depend on the speed of the observer.

The third postulate says that there is no state of absolute rest.

Inertial reference systems were introduced by Newton.

An inertial frame is defined by the following conditions:

A body at rest always remains at rest in an inertial frame of reference, unless a force acts on the body.

Einstein's postulates — Comment 2

A body moving at a certain speed will always move in a straight line at that speed, unless there is a force acting on the body.

The idea of an inertial frame of reference is a mathematical idealization that is never fully realized in nature.

E.g. a frame of reference system rotating with the earth is not an inertia reference system, due to the centrifugal force and Coriolis force.

Nevertheless it is possible to approach the idealisation of an inertial frame of reference locally and during short times.

The general theory of relativity drops this restriction and applies in general frames of reference.

Einstein's postulates

Comment 3

The first postulate is a principle that has been used successfully since Newton's time.

The speed of light is a natural constant in Maxwell's theory of electrodynamics.

The second postulate says that Maxwell's electrodynamics correctly captures the essence of the speed of light.

Not Maxwell's theory of electrodynamics is an approximation, but Newton's theory of mechanics.

The second postulate demands that there must be a new theory of mechanics. Einstein formulated a new theory of mechanics. First for inertial reference systems (special relativity) and later for arbitrary reference systems (general relativity).

Einstein's postulates — Comment 4

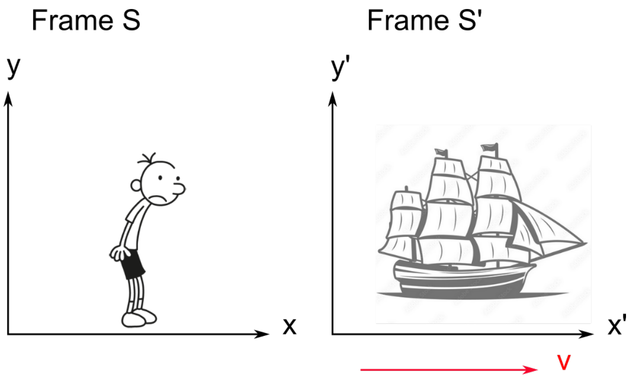
The third postulate confirms that all inertial systems are equivalent regardless of their relative speed.

In other words: there is no ether.

Lorentz Transf.

Special Relativity

Lorentz transformation 1



Lorentz transformation 1

Comment

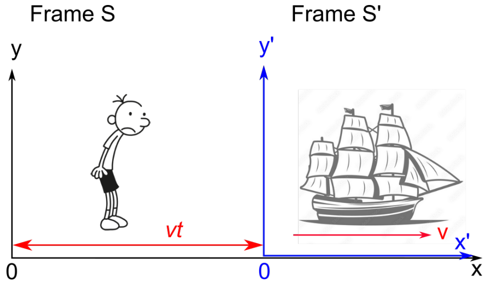
The sketch shows two reference systems.

For example, an observer in the reference system S looks at a ship that is moving away.

The ship is assigned a reference system S' that moves with the ship.

Lorentz transformation 2

$$
\begin{aligned}
x &= x' + vt' \\
y &= y' \\
z &= z' \\
t &= t'
\end{aligned}
$$



In classical physics, time is a universal quantity and therefore the same in both reference systems: $\boxed{t = t'}$

Lorentz transformation 2 Comment

The formulas give the coordinates of the ship in the reference system S if the direction of the speed $\vec{v}$ determines the x-axis and if the coordinates of the reference system S' are denoted by x', y' and z'.

This assumes that at time $t = t' = 0$ the origin of reference system S coincides with the origin of reference system S'.

This transformation is called the Galilean transformation.

The Galilean transformation has proven itself in classical mechanics.

For example, if a force acts on a mass, the result is in both reference systems same value for acceleration.

Newton's equation of motion does not change when the reference system is changed.

Lorentz transformation 3

The speed of light does not change in the two coordinate systems due to the relative speed between frame S and frame S '

$$\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial x^2}$$

and

$$\frac{1}{c^2}\frac{\partial^2 \psi'}{\partial t'^2} = \frac{\partial^2 \psi'}{\partial x'^2}$$

The wave function ψ denotes either the electric or the magnetic field of an electromagnetic wave

ct and x must be transformed similar

Lorentz transformation 3 Comment 1

Einstein's first postulate does not hold true when the Galilei transform is used to transform Maxwell's equations of electrodynamics.

In a vacuum, Maxwell's equations are reduced to simple wave equations for the electric and magnetic field.

The electromagnetic waves propagate at the speed of light.

If the Galileo transform is used, the speed of light would depend on the speed of the coordinate system.

This result is correct if, for example, sound waves are considered.

Finding the correct transformation equations for the Maxwell equations is a mathematical problem that was solved by Hendrik Lorentz in 1895.

Lorentz transformation 3 Comment 2

Hence the transformation is called the Lorentz transformation.

In the wave equations framed in red, the wave functions ψ and ψ' indicate the electric and magnetic field of a plane electromagnetic wave in the coordinate system S and S', which propagates along the x-axis.

Since the wave equations must be the same in both reference systems, the product of time and the speed of light ct must be transformed in the same way as the coordinate x.

It is a direct consequence of the Lorentz transformation that time is no longer absolute, but depends on the frame of reference.

The clocks in the two coordinate systems show different times, even if the clocks in the two coordinate systems were synchronized at the beginning of the experiment.

Lorentz transformation 3 Comment 3

In addition, the electric and magnetic fields are different in the two coordinate systems.

This is obvious if, for example, a charge is considered that rests in the coordinate system S'.

In this case there is only an electric field in the coordinate system S', but no magnetic field.

According to Ampere's law, there is also a magnetic field in the coordinate system S, since the charge moves in this coordinate system.

The Lorentz transformation does not only have to be applied to the coordinates, but also to the electric and magnetic fields.

Lorentz transformation 3 Comment 4

The transformation of electromagnetic fields is more complicated and is not covered in this lecture.

Lorentz transformation 4

ansatz:

$$\begin{aligned} x &= x' + vt' & \rightarrow \quad & x = \gamma\left(x' + \frac{v}{c}ct'\right) \\ y &= y' & \rightarrow \quad & y = y' \\ z &= z' & \rightarrow \quad & z = z' \\ t &= t' & \rightarrow \quad & ct = \gamma\left(ct' + \frac{v}{c}x'\right) \end{aligned}$$

With the wave equation one gets for the factor γ

$$\boxed{\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$

Lorentz transformation 4 Comment 1

Starting from the Galileo transformation, the Lorentz transformation can easily be guessed.

The Galileo transformation must be extended in such a way that the coordinate x and the product ct are treated the same.

In the formula for the coordinate x, the product of speed and time is expanded to include the speed of light.

It should also be noted that the time in the S' coordinate system is no longer the time in the S coordinate system.

The formula for transforming the time is extended analogously to the formula for transforming the coordinate x.

Lorentz transformation 4 — Comment 2

Only the coordinates x' and ct' have to be swapped.

When these formulas are used to transform the wave equation, it turns out that an additional factor γ is needed.

The formula outlined in red shows γ.

The wave equation does not change form when the x and ct coordinates are transformed with these formulas.

This can easily be checked if a scalar wave function is used .

The complicated transformation of the electromagnetic fields is not required in this lecture.

Lorentz transformation 5

Lorentz Transformation:

$$x = \gamma(x' + \frac{v}{c}ct')$$

$$y = y' \quad \text{und} \quad z = z'$$

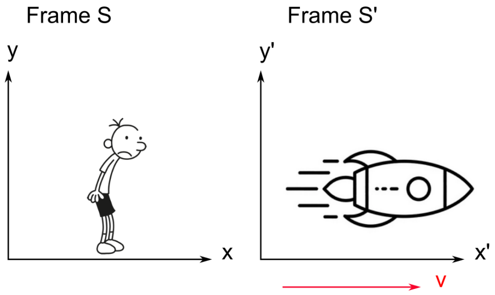
$$ct = \gamma(ct' + \frac{v}{c}x')$$

or simply

$$x = \gamma(x' + vt')$$

$$y = y' \quad \text{und} \quad z = z'$$

$$t = \gamma(t' + \frac{v}{c^2}x')$$



Time is no longer a universal quantity and is therefore different in both reference systems: $\boxed{t \neq t'}$

Lorentz transformation 5 Comment 1

The formulas outlined in red summarize the Lorentz transformation.

Since the Lorentz transformation leaves the speed of light unchanged, the Lorentz transformation according to Einstein's postulates is the correct transformation between inertial systems.

The Maxwell equations are invariant when the Lorentz transform is applied.

Lorentz transformation 5 — Comment 2

Newton's equation of motion is not invariant when the Lorentz transform is applied.

Einstein's postulates state that Newtonian mechanics is an approximation that is only valid if the speeds involved are much smaller than the speed of light.

For macroscopic objects on earth, this limitation is generally not a problem.

But in cosmic space and in the microcosm of atoms, molecules and solids the speeds are often comparable to the speed of light.

The main challenge of the theory of relativity was therefore the reformulation of mechanics.

Lorentz transformation 5 Comment 3

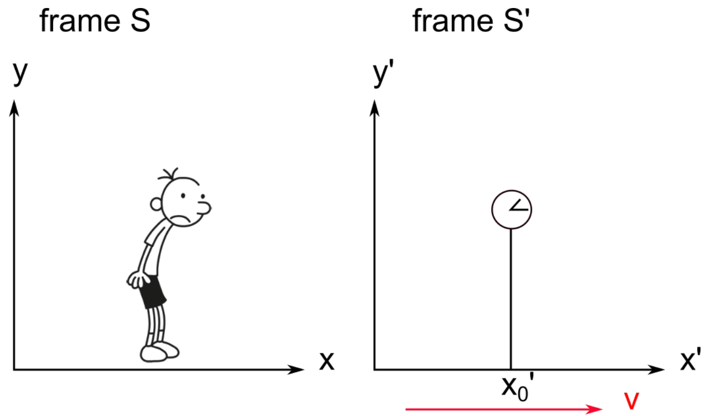
As long as no accelerations are involved, this problem can be solved within the framework of the special theory of relativity.

General relativity solves the problem when accelerations and large distances are important, which is usually the case with cosmological problems.

Time dilation

Special Relativity

Time dilation and length contraction: time dilation 1



With $t = \gamma(t' + \frac{v}{c^2} x_0{}')$ follows for time intervals $\boxed{\Delta t = \gamma \Delta t'}$

Time dilation and length contraction 1 Comment 1

Due to the Lorentz transformation, time is no longer an absolute quantity that can be defined for the entire cosmic space.

A distinction must be made between the time of a clock that is resting in relation to an observer and the time of a clock that is moving in relation to an observer.

The sketch shows an observer in the coordinate system S.

The person in the coordinate system S observes a clock that is moving away with the speed v and the coordinate system S'.

There is one fundamental problem with the sketch, however.

Time dilation and length contraction 1 Comment 2

The information between the watch and the viewer is always transmitted by electromagnetic waves.

I will completely ignore this aspect in this section.

The effects caused by the finite transit time of the light waves are discussed in the section: „Visible" effects due to the Lorentz transformation.

In this section only the direct consequences of the Lorentz transformation are discussed.

With the underlined formula of the Lorentz transformation one finds the equation framed in red, which relates the corresponding time intervals in the coordinate systems S and S '.

Time dilation and length contraction 1 — Comment 3

Please note that the location of the clock x'_0 in the coordinate system S' does not change.

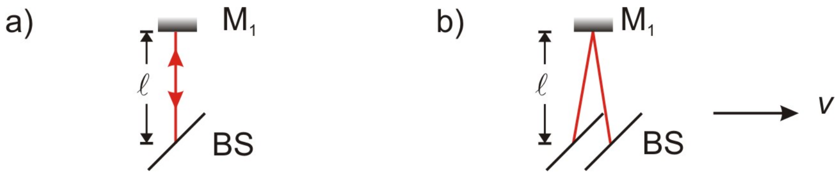
If the interval between the two times t_1 and t_2 is calculated, the position of the clock disappears and only the difference between the two times remains.

The time in the coordinate system S' in which the clock rests is called proper time.

With $\gamma = 1/\sqrt{1 - \mathrm{v}^2/\mathrm{c}^2}$ is Δt always larger than the proper time $\Delta t'$.

Time dilation and length contraction 2

time dilation and the Michelson interferometer



a) The interferometer at rest relative to an observer

necessary time for a light pulse $\boxed{t_0 = \frac{2\ell}{c}}$

Time dilation and length contraction 2 Comment

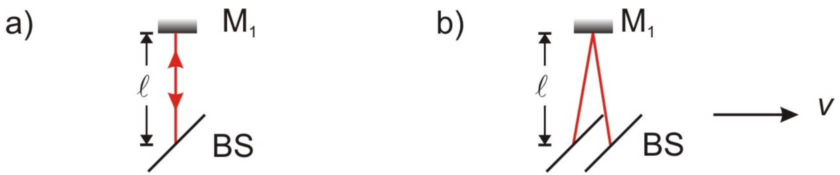
The effect of time dilation can be illustrated with the Michelson interferometer.

The time that the light needs to pass the distance between the beam splitter and the mirror 1 defines the proper time when the interferometer is at rest with respect to an observer.

Figure a) shows this situation.

The formula outlined in red indicates the proper time.

Time dilation and length contraction 3



b) the interferometer moves with respect to an observer

necessary time for a light pulse

$$\boxed{t(v) = \frac{2\ell}{c}\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = t_0\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}}$$

t_0: proper time and $t(v)$: time of the moving clock $\rightarrow$ time dilation

Time dilation and length contraction 3 Comment

Figure b) illustrates the situation when the interferometer moves relative to an observer.

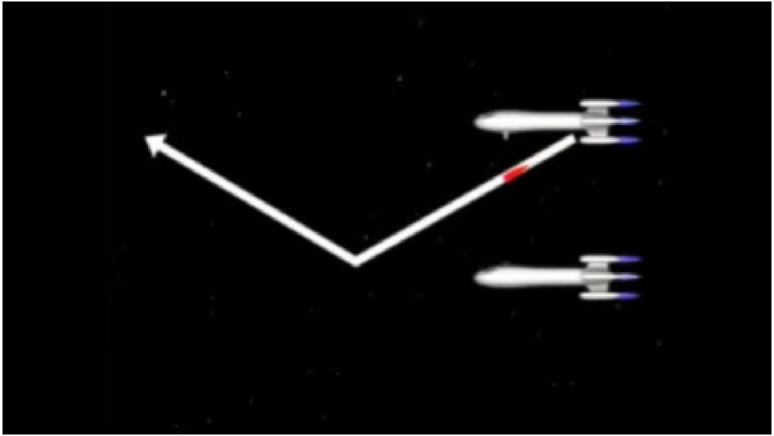
The light pulse between the beam splitter and the mirror M_1 has to travel a longer distance and therefore needs more time (this is due to the aberration effect, which will be discussed in the next lecture).

The time required can be calculated using the formula outlined in red.

The same formula was obtained with the Lorentz transformation.

Clocks that move relative to an observer run slower for the observer.

Time dilation and length contraction 4



(TimeDilationTwoSpaceships.mp4)

Time dilation and length contraction 4 — Comment

The video illustrates the effect.

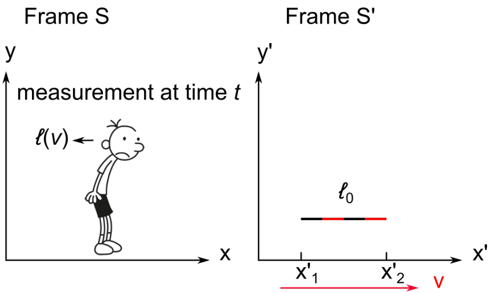
The video suggests that the effect of time dilation affects both the pilots of the spaceships and the observer on the asteroid.

Time dilation is not just the math of a complicated theory, it affects living organisms.

Since time is sacred to many people, many people felt challenged in their beliefs and tried to contradict the special theory of relativity.

In this context, the so-called twin paradox has become very popular.

Time dilation and length contraction: length contraction 5



$$\boxed{\ell(v) = \ell_0\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

Time dilation and length contraction 5 — Comment 1

The sketch shows an observer in the reference system S who is looking at a scale that is moving away with the speed v.

In its rest frame S' the scale has the length $\ell_0 = x'_2 - x'_1$.

If the length of the moving scale $\ell(v) = x_2 - x_1$ is to be calculated using the Lorentz transformation, then the coordinates x_1 and x_2 must be used at the same time and the formula outlined in red results.

Calculation:

$$x = \gamma(x' + vt') \quad \text{and} \quad t = \gamma(t' + \frac{v}{c^2}x')$$

with

$$\gamma t' = t - \gamma \frac{v}{c^2} x'$$

Time dilation and length contraction 5 — Comment 2

results

$$x = \gamma x' + vt - \gamma \frac{v^2}{c^2} x' \quad \rightarrow \quad x - vt = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} x' (1 - \frac{v^2}{c^2}) = x' \sqrt{1 - \frac{v^2}{c^2}}$$

and

$$\ell(v) = x_2(t) - x_1(t) = (x'_2 - x'_1) \sqrt{1 - \frac{v^2}{c^2}} = \ell_0 \sqrt{1 - \frac{v^2}{c^2}}$$

In the difference of the coordinates, the term $-vt$ is eliminated because the two coordinates have to be determined at the same time t.

Time dilation and length contraction 6

length contraction and the Michelson interferometer

the time t_1 between the beam splitter and mirror 1 is

$$t_1 = \frac{2\ell}{c}\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

the time t_2 between the beam splitter and mirror 2 is

$$t_2 = \frac{2\ell}{c}\frac{1}{1-\left(\frac{v}{c}\right)^2}$$

In order for the two times to be the same, the light path in the forward direction must be shortened

$$\boxed{\ell(v) = \ell\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

Time dilation and length contraction 6 — Comment

Here, too, the comparison with the Michelson interferometer is instructive.

The first formula specifies the time that the light needs between the beam splitter and the mirror M_1.

The second formula specifies the time that the light needs between the beam splitter and the mirror M_2.

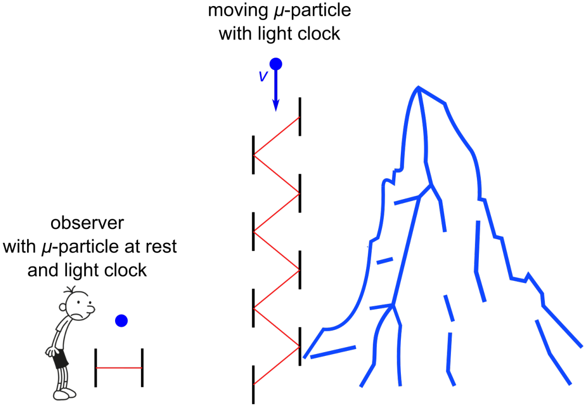
Since the interference between the two partial beams cannot depend on the relative speed between the interferometer and an observer, the two times t_1 and the time t_2 must be the same.

Due to the length contraction, no movement-related effect can be observed with the Michelson interferometer.

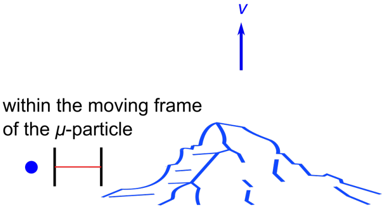
Hall and Rossi

Special Relativity

Experiment of Hall and Rossi



time dilation



length contraction

Experiment of Hall and Rossi — Comment 1

The special theory of relativity was very important for the development of quantum mechanics.

In 1927 Dirac used the special theory of relativity to derive the Schrödinger equation of an electron.

The optical spectrum of the hydrogen atom can be described almost completely using the Dirac equation.

Despite the enormous importance of the special theory of relativity, it was not until 1941 that Hall and Rossi succeeded in experimentally testing the fundamental effects of time dilation and length contraction.

Hall and Rossi carried out their experiments with μ-particles, which were discovered in 1936.

Experiment of Hall and Rossi

Comment 2

The lifetime of a μ-particle at rest is around 2 μs.

After a certain time, the μ-particle decays into an electron and another particle called the electron neutrino.

To measure the lifespan, the μ-particles in the matter are stopped and the electrons are counted, which are created by the decay of the μ-particles with high kinetic energy.

The number of counts decreases exponentially according to the lifetime of the μ-particle.

For Hall and Rossi's experiment it is important that the number and the speed of the μ-particles can be measured.

Experiment of Hall and Rossi — Comment 3

The μ-particles are generated by cosmic radiation in the earth's atmosphere and move at high speeds towards the ground.

The number of μ-particles with a certain speed was measured on a mountain and at sea level.

To better understand what is going on, one can imagine that each μ-particle is connected to a light clock.

The same number of μ-particles is observed on the mountain and at the base of the mountain.

The μ-particles on the mountain move towards the surface of the earth at a certain speed.

The μ-particles examined at the foot of the mountain are at rest.

Experiment of Hall and Rossi — Comment 4

The figure on the left shows that a light pulse can travel back and forth between the two mirrors of the light clock faster with the resting μ-particles than with the moving particles.

The clock of the moving μ-particles runs slower than the clock of the resting μ-particles.

At the foot of the mountain there are more moving μ-particles than one would expect based on the observation of the resting μ-particles.

The lifetime of the μ-particles increases due to the movement of the particles.

This effect is known as time dilation.

Experiment of Hall and Rossi

Comment 5

The figure on the right illustrates the situation when the observer moves with the μ-particles.

The height of the mountain is reduced in the frame of reference of the μ-particle by length contraction.

This shortens the time that the μ-particles need to get from the top to the base of the mountain.

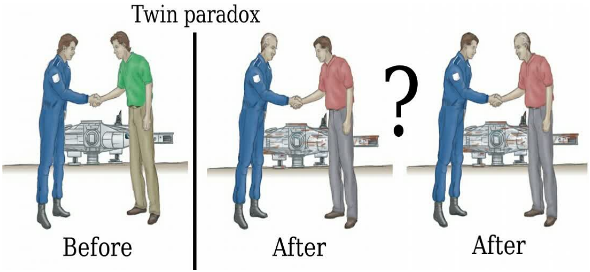
For this reason, a larger number of μ-particles reaches the foot of the mountain than can be expected from observing the μ-particles at rest.

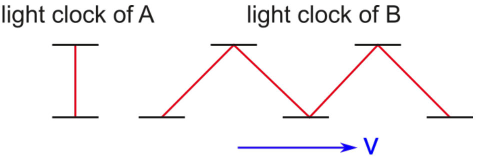
The square root behavior of time dilation and length contraction can be observed if the effect is measured for different velocities of the μ-particles.

Twin paradox

Special Relativity

Twin paradox





Twin Paradox

Comment 1

The figures illustrate the twin paradox.

Twin A stays on Earth while twin B embarks on a journey to a distant star.

When twin A watches twin B's clock, he finds that twin B's clock is running slower than his own.

Twin A therefore expects that his brother will be younger than himself when he returns to earth.

The light clocks in the picture below illustrate the effect.

The paradox assumes that the situation is symmetrical for both twins.

Therefore, when twin B returns to Earth, he expects his brother to have aged less than himself.

Twin Paradox — Comment 2

The paradox takes the symmetry of the frames of reference of twin A and twin B for granted.

But that is not true.

The frame of reference of twin A is an inertial system during the entire journey of twin B, while there must be accelerations in the frame of reference of twin B, since otherwise he could not return to earth.

The reference system of twin B cannot be an inertial system during the entire journey.

The defined properties of an inertial system can be used to objectively test whether a reference system is an inertial system or not.

Twin Paradox

Comment 3

It is therefore not surprising that the situations of twin A and twin B are different.

By exchanging radio signals, the twins can check each other's watch.

Therefore, it will come as no surprise to either twin A or twin B that twin A will be older than twin B.

The effect was tested in 1971 with a famous experiment.

Atomic clocks were used in the Hafele-Keating experiment.

The clocks traveled the earth twice in airplanes and were compared to an atomic clock in the United States Naval Observatory.

Twin Paradox

Comment 4

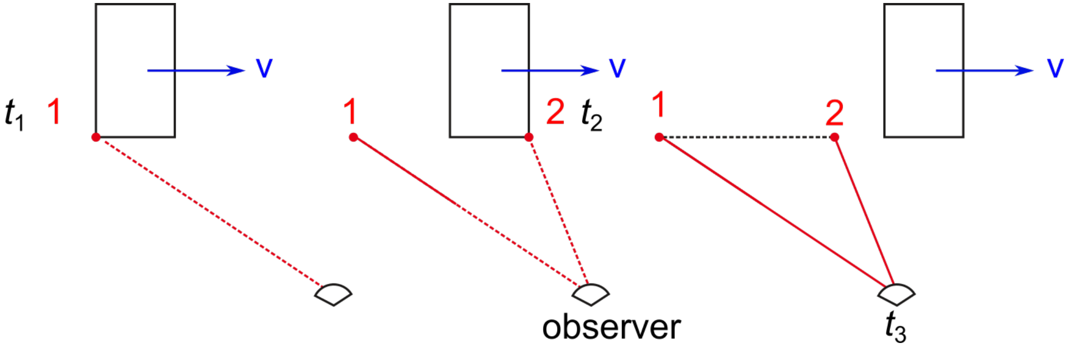
The evaluation of the experiment is complex, since the general theory of relativity must also be used to analyze the experimental data.

Special Relativity

Apparent effects

Apparent effects due to the Lorentz transformation 1

a) apparent length



Apparent effects 1

Comment 1

A moving object can only be seen if the observer receives electromagnetic radiation that is emitted by the moving object.

The image that a viewer perceives at a certain time is generated by the radiation that penetrates his eye at that particular time.

The figure shows a cube moving with the velocity v.

In the direction of speed, the cube is shortened due to length contraction.

The light rays are drawn in red and reach the viewer's eye at a certain time.

Apparent effects 1 Comment 2

But these rays are not emitted at the same time.

In the direction of speed, the viewer does not see a shortened cube but, on the contrary, an elongated side.

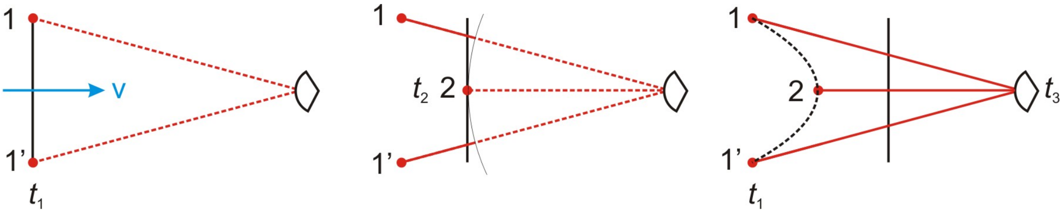
The length of the edge of the cube changes with the position of the cube.

The actual edge length can only be seen when the cube is directly facing the observer.

If the cube moves further to the right, the edge is seen even shorter.

Apparent effects due to the Lorentz transformation 2

b) apparent curvature of a straight line



Apparent effects 2 Comment

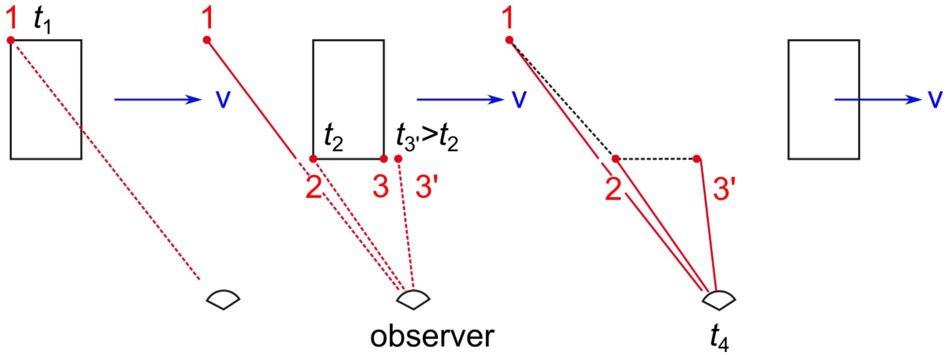
The figure shows a stick moving directly towards an observer.

The rays of light that start at the ends of the stick take longer than a ray that starts in the middle of the stick.

The resulting image is a bent rod.

Apparent effects due to the Lorentz transformation 3

c) apparent rotation



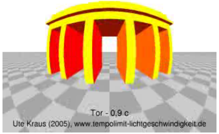
Apparent effects 3 Comment

Because of its motion, the side of the cube between points 1 and 2 can be seen by an observer if the speed of the cube is high enough.

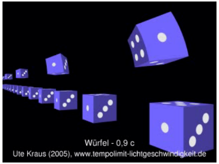
This effect is known as apparent rotation.

Apparent effects due to the Lorentz transformation 4

1. Video: gitter.mp4



2. Video: BrandenburgerTor.mp4



3. Video: wuerfelkette.mp4

4. Video: tuebingen.mp4
5. Video: rundfahrttuebingen.mp4

Apparent effects 4 — Comment 1

The animations illustrate these effects.

The first video shows a grid approaching the viewer.

You can see the change in length and curvature of the lines of the grid.

The 2nd video shows a model of the Brandenburg Gate.

It moves towards the viewer, although intuitively one has the opposite impression.

You can see the changes in length and curvature of the edges.

For a brief moment, the actually hidden back of the Brandenburg Gate becomes visible through his movement.

Apparent effects 4 — Comment 2

The third video shows the effect of the apparent rotation by comparing standing and moving cubes.

The 4th and 5th videos each show a relativistic journey through the old town of Tübingen.

Of course, the animations do not take into account the change in wavelength of light caused by movement, i.e. the Doppler effect.

Invariant dist.

Special Relativity

Invariant distance 1

Definition of the invariant distance s

$$\boxed{s^2 = (ct)^2 - \vec{r}^2 = (ct)^2 - x^2 - y^2 - z^2}$$

with the Lorentz transformation

$$\boxed{\begin{aligned} ct &= \gamma(ct' + \frac{v}{c}x') \\ x &= \gamma(x' + \frac{v}{c}ct') \\ y &= y' \\ z &= z' \end{aligned}}$$

follows for s^2

$$\underline{s^2 = \gamma^2(ct' + \frac{v}{c}x')^2 - \gamma^2(x' + vt')^2 - y'^2 - z'^2}$$

Invariant distance 1 Comment

The first formula outlined in red gives the definition of the invariant distance.

"s" is called a distance because it has the dimension of a length and is measured in meters.

The underlined equation results from the application of the Lorentz transformation.

Invariant distance 2

and with

$$\boxed{\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}}$$

results

$$s^2 = \frac{1}{1-\left(\frac{v}{c}\right)^2}\left((ct')^2 + (v/c)^2 x'^2 - x'^2 - (vt')^2\right) - y'^2 - z'^2$$

$$\boxed{s^2 = (ct')^2 - \vec{r}'^2 = s'^2}$$

The value of s^2 is the same in all inertial systems

Invariant distance 2

Comment

The calculation given shows that the invariant distance is independent of the respective coordinate system.

The value of the quantity “s” is the same in all inertial systems and is independent of the speed.

Rel. invariants

Special Relativity

Relativistic invariants 1

The following equation always applies to an electromagnetic wave in a vacuum

$$\boxed{\nu\lambda = c}$$

or

$$\omega = ck$$

or

$$\omega^2 = c^2\vec{k}^2$$

or

$$\boxed{\left(\frac{\omega}{c}\right)^2 - \vec{k}^2 = 0}$$

$$(ct)^2 - \vec{r}^2 = s^2$$

Relativistic invariants 1 — Comment 1

Only two relativistic invariants are discussed with in this lecture, although there are many more in nature.

In this section the relativistic invariant is discussed, which is formed by the angular frequency and the wave vector.

The first formula outlined in red shows the relationship between frequency and wavelength of an electromagnetic wave in a vacuum.

An electromagnetic wave in a vacuum always propagates at the speed of light.

A small rearrangement of this equation results in the second equation outlined in red.

Relativistic invariants 1 Comment 2

The angular frequency divided by the speed of light squared minus the square of the wave vector is always zero.

This equation applies regardless of the relative velocities in all inertial systems.

The last line shows the similarity between the invariant distance and the relationship between the angular frequency and the wave vector.

In the case of angular frequency and wave vector, the quantity corresponding to the “s” is always zero.

Relativistic invariants 2

$\frac{\omega}{c}$ and $\vec{k}$ have to be transformed with the Lorentz transformation

$$ct = \gamma(ct' + \frac{v}{c}x')$$
$$x = \gamma(x' + \frac{v}{c}ct')$$
$$y = y'$$
$$z = z'$$

$$\boxed{\begin{aligned} \omega &= \gamma(\omega' + v k_x') \\ k_x &= \gamma(k_x' + \frac{v}{c^2}\omega') \\ k_y &= k_y' \\ k_z &= k_z' \end{aligned}}$$

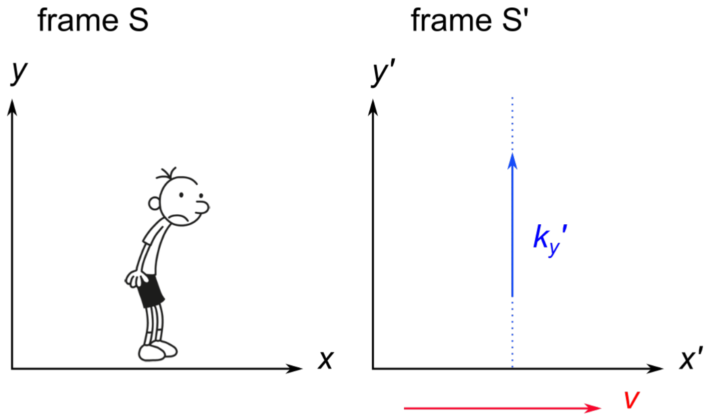
Relativistic invariants 2 Comment

This similarity means that if the reference system is changed, the Lorentz transformation must be used to transform the angular frequency and the wave vector.

The equations outlined in red show the Lorentz transformation of the angular frequency and the wave vector.

These equations correspond to the usual Lorentz transformation of time and space.

Relativistic invariants 3



$$k_x{}' = 0, \quad k_y{}' = k' = \omega'/c, \quad k_z{}' = 0$$

Relativistic invariants 3 Comment

As an example, consider a plane electromagnetic wave that propagates in the coordinate system S 'along the y' direction.

This can be, for example, the light path between the beam splitter and the mirror 1 of a Michelson interferometer, which moves relative to an observer.

The lower line gives the wave vector of the electromagnetic wave in the coordinate system S '.

Relativistic invariants 4

with the Lorentz transformation

$$\boxed{\begin{aligned} \omega &= \gamma(\omega' + v k_x') \\ k_x &= \gamma(k_x' + \frac{v}{c^2}\omega') \\ k_y &= k_y' \\ k_z &= k_z' \end{aligned}}$$

$$\boxed{\begin{aligned} ct &= \gamma(ct' + \frac{v}{c}x') \\ x &= \gamma(x' + \frac{v}{c}ct') \\ y &= y' \\ z &= z' \end{aligned}}$$

and $k_x' = 0, \quad k_y' = k' = \omega'/c, \quad k_z' = 0$

$$\boxed{\begin{aligned} \omega &= \gamma\omega' \\ k_x &= \gamma\frac{v}{c^2}\omega' = \gamma\frac{v}{c}k' \\ k_y &= k' \\ k_z &= 0 \end{aligned}}$$

Relativistic invariants 4

Comment

The Lorentz transformation can be applied to this situation.

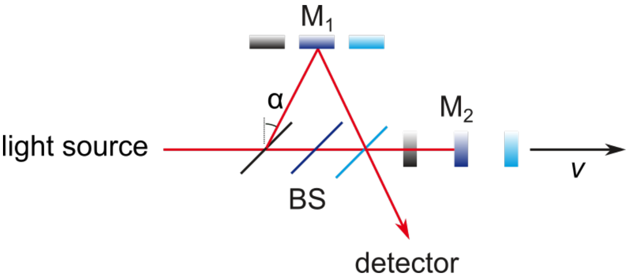
The first equation describes the Doppler effect, i.e. the change of the frequency with velocity.

The following equations show that the wave vector in frame S is no longer parallel to the y-axis.

The wave vector in the coordinate system S is tilted in the direction of the speed of the coordinate system S'.

Relativistic invariants 5

Observing a moving Michelson interferometer



Relativistic invariants 5 Comment

The beam to mirror M1 tilts in the forward direction as assumed in the classical discussion of the Michelson interferometer.

The cause of the tilt is not an experimental imperfection that is exploited to advantage, but the aberration effect discovered by James Bradley in 1725.

Relativistic invariants 6

the tilt angle with respect to the y-axis is

$$\tan\alpha = \frac{k_x}{k_y} = \gamma\frac{v}{c} \qquad \text{and when} \qquad v << c: \qquad \boxed{\alpha = \frac{v}{c}}$$

Relativistic invariants 6 Comment

When the speed is small compared to the speed of light, the angle of inclination is given by the ratio between the speed and the speed of light.

This effect is the aberration of light discovered by the astronomer James Bradley in 1725.

In 1818 Fresnel introduced the complicated and contradicting theory of partial ether-drag to explain this effect within the classical wave picture.

Einstein showed in 1905 that the aberration of electromagnetic waves is a simple consequence of the Lorentz transform.

Revision

Summary in questions

1. The difference in transit time between the light pulses to mirrors 1 and 2 expected by Michelson is not observed. How can this effect be explained by Einstein's special theory of relativity?
2. What is an inertial system?
3. What is the Galilean transformation between two inertial frames that move relative to each other?
4. Extend the Galilean transformation to the Lorentz transformation. What idea motivates the modification of the Galilean transformation?
5. What is the Lorentz transformation between two inertial frames that move relative to each other?
6. Write the formula for time dilation and explain the meaning of the symbols used.
7. Write the formula for length contraction and explain the meaning of the symbols used.

Summary in questions

8. What is the definition of “invariant distance”?
9. Wie ändert sich der invariante Abstand, wenn das Bezugssystem gewechselt wird?
10. Wieso ändert sich $\omega^2/c^2 - \vec{k}^2 = 0$ nicht, wenn das Bezugssystem gewechselt wird?
11. Schreiben Sie die Lorentz-Transformation für Frequenz und Wellenzahlvektor auf.
12. In der Animation zum Aberrationseffekt von Sternenlicht bewegt sich das Teleskop nach rechts und das Licht fällt von oben ein. Vergewissern Sie sich mit der Lorentz-Transformation, dass das Teleskop nach rechts geneigt werden muss.