Modern Physics

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Doppler effect

Special Relativity

Doppler effect 1

(DopplerEffectandShockWavesHD.mp4)

Doppler effect 1 Comment

This picture shows the Doppler effect for surface waves on water.

The distance between the wave fronts depends on the speed of the source and the direction from which the source is observed.

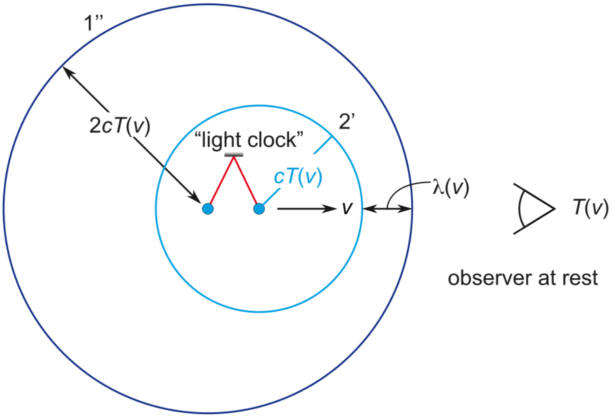
The smallest distance between the wave fronts is observed in front of the source.

The greatest distance between the wave fronts is observed behind the source.

If harmonic waves are excited, the wavelength for the different observation directions varies between these two cases.

The animation shows the Doppler effect of sound waves.

Doppler effect 2



The light source that emits spherical waves and approaches an observer at rest

$$\lambda(v) = 2cT(v) - vT(v) - cT(v) = \underline{(c - v)T(v)}$$

Doppler effect 2 Comment 1

In contrast to water or sound waves, there is no carrier medium for electromagnetic waves.

Therefore only the relative movement between source and observer is important.

The figure shows a source that moves relative to an observer and emits spherical electromagnetic waves.

The circles indicate the wave fronts of the spherical waves.

The wave fronts are emitted at different points due to the movement of the source.

The time between the emission of two wave fronts is the period of oscillation of the source.

Doppler effect 2 Comment 2

The period of oscillation is visualized by the light clock.

For the observer, the period of oscillation is longer than the proper period of oscillation of the source due to time dilation.

The geometry of the sketch indicates how to calculate the wavelength of an approaching source.

The formula underlined in red gives the wavelength of the moving source.

Doppler effect 3

with the formula for time dilation

$$\begin{aligned}\lambda(v) &= (c-v)T(v)\\ &= (c-v)\frac{T_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = cT_0\frac{1-\frac{v}{c}}{\sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}}\end{aligned}$$

and the proper wave length in the rest frame of the source $\lambda_0 = cT_0$ results the formula for the longitudinal Doppler effect for an approaching source

$$\boxed{\lambda(v) = \lambda_0\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}}$$

and with $\nu\lambda = c$

$$\boxed{\nu(v) = \nu_0\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}}$$

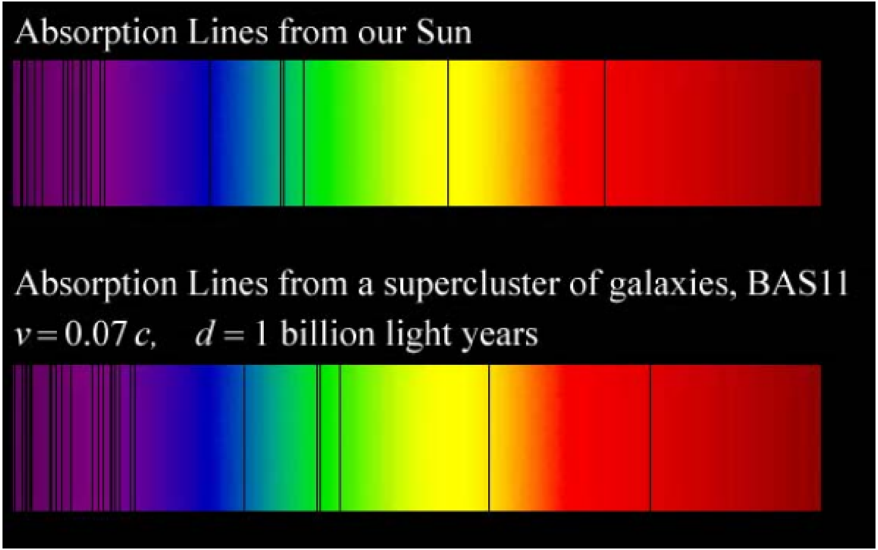
Doppler effect 3 Comment

The relationship between the wavelength in the rest frame of the source and the wavelength of a source approaching an observer can be calculated using the time dilation formula.

The first equation framed in red shows the result.

The wavelength of the approaching source becomes shorter and the frequency higher.

Doppler effect 4



Doppler effect 4 Comment 1

The opposite effect is usually observed in astronomy.

Almost all galaxies are withdrawing from our galaxy (not the Crab Nebula).

The figure shows absorption spectra of the sun and the supercluster of galaxies BAS11.

The electromagnetic spectrum of the stars that make up a galaxy is due to thermal radiation and is therefore broad and smooth.

Sharp absorptions can be observed due to the atoms and molecules that form a gas cloud around the stars.

The upper figure shows the absorption spectrum of our sun.

The figure below shows the absorption spectrum of a supercluster of galaxies.

Doppler effect 4 Comment 2

Both spectra show the same type of spectral lines.

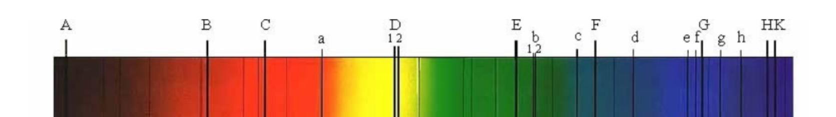
Compared to the absorption spectrum of the sun, the sharp absorption lines of the supercluster are shifted to longer wavelengths.

The galaxy cluster is moving away from our galaxy.

Historical remark:

Long before the spectra of atoms were studied and understood, the spectrum of the sun was observed .

Joseph von Fraunhofer simply labeled 1814 the absorption lines by letters.



Doppler effect 4 Comment 3

The sodium D lines, which lead to striking absorption in the yellow region of the sun’s spectrum, are reminiscent of the work of J. Fraunhofer.

Doppler effect 5

longitudinal Doppler effect of a receding source

$$\boxed{\lambda(v) = \lambda_0 \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}} \quad \text{and} \quad \boxed{\nu(v) = \nu_0 \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}}$$

red shift

$$z = \frac{\lambda(v) - \lambda_0}{\lambda_0} = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} - 1$$

small velocity approximation $v << c$ (note: $\frac{1}{1-\frac{v}{c}} \approx 1 + \frac{v}{c}$)

$$\underline{z = \frac{\lambda(v) - \lambda_0}{\lambda_0} = \frac{v}{c}}$$

Doppler effect 5 Comment

The equations outlined in red indicate the longitudinal Doppler effect for a receding source.

Often the relative change in wavelength is used.

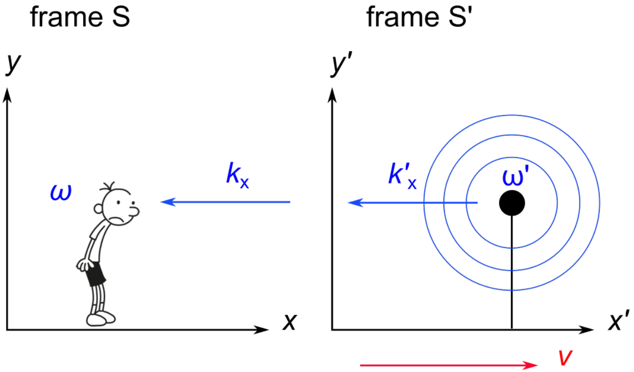
This is known as redshift for a receding source.

If the speed is much less than the speed of light, then the redshift is simply given by the ratio between the speed and the speed of light.

It is worth noting that the relative Doppler effect of electromagnetic waves for small speeds is always proportional to the ratio of speed and the speed of light.

Doppler effect 6

Doppler effect and Lorentz transformation



$$\vec{k} = (\underline{k_x = -\omega/c}, \quad k_y = 0 \quad k_z = 0) \quad \text{and} \quad \vec{k}\,' = (\underline{k'_x = -\omega'/c}, \quad k'_y = 0 \quad k'_z = 0)$$

Doppler effect 6 Comment

To obtain the wavelength or frequency of a moving source observed at an angle with respect to the relative speed between the frames of reference, the Lorentz transform can be used.

As a simple example, the longitudinal Doppler effect can first be calculated using the Lorentz transformation.

The figure shows a source resting in the coordinate system S'.

The coordinate system S' moves away from the observer with the velocity v.

The k-vectors in the two coordinate systems S and S' are given in the line below the figure.

A plane wave is observed that propagates in the negative x-direction.

Doppler effect 7: Lorentz transformations between frame S and S'

Lorentz Transformation

$$\boxed{\begin{aligned} \omega &= \gamma(\omega' + v{k_x}') \\ k_x &= \gamma({k_x}' + \frac{v}{c^2}\omega') \end{aligned}}$$

$$\underline{\omega = \omega(v)} = \gamma(\omega' + v\,k'_x) = \gamma\left(\omega' - v\,\frac{\omega'}{c}\right) = \omega'\frac{1-\frac{v}{c}}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \underline{\omega_0\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}}$$

$\omega' = \omega_0$ is the proper frequency in the reference system S' and $\omega = \omega(v)$ is the frequency of the moving source in the reference system S.

Doppler effect 7 Comment

The equations in the frames show the Lorentz transformations between the reference systems S and S ’.

The Doppler effect can be calculated using all of these equations.

The calculation is particularly easy with the first equation marked in blue.

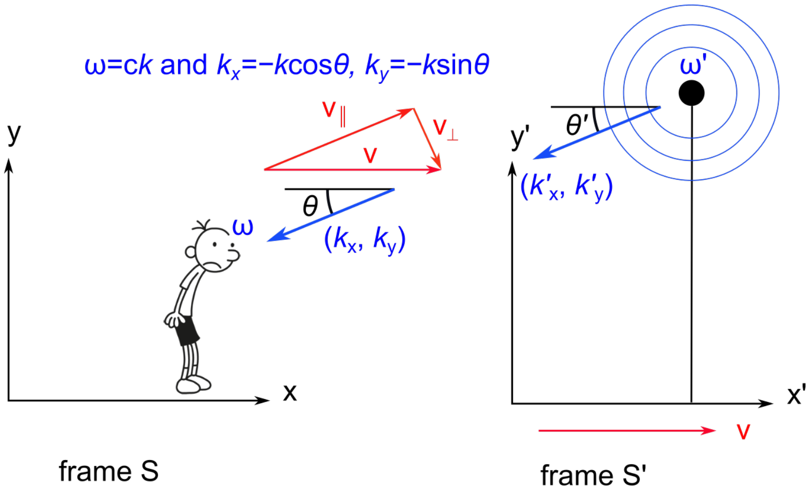
The calculation assumes that the source is moving away to the right of the observer, as shown in the sketch.

This gives: $k_x = -\omega/c$ or $k_x{}' = -\omega'/c$ and the observed frequency of the source decreases as the source moves away from the observer.

If the source moves to the left, then the *k*-vectors point in the direction of the positive coordinate axis and are therefore positive quantities. The speed *v* is then negative in this situation.

Doppler effect 8

Doppler effect at any viewing angle



Doppler effect 8 Comment

The figure shows an observer in the reference system S who perceives a wave at the angle θ relative to the velocity of the source.

The radiation source rests in the reference system S’, which in the sketch is moving away from the observer with the velocity v.

The formulas written in blue give the frequency and the components of the wave number vector in the observer’s frame of reference.

In the sketch, the reference system S’ has passed the observer.

At an earlier point in time it must have been on the left side of the sketch, approaching the observer.

The observer had to look to the left to see the frame of reference S’ and $\theta > 90°$ applies to the observation angle.

Doppler effect 9

$$\boxed{\begin{aligned} \omega &= \gamma(\omega' + v{k_x}') \\ k_x &= \gamma({k_x}' + \frac{v}{c^2}\omega') \end{aligned}}$$

with

$$k_x = -k\cos\theta = -\frac{\omega}{c}\cos\theta \quad \text{and} \quad vk_x = \gamma v{k_x}' + \gamma\frac{v^2}{c^2}\omega'$$

follows

$$\omega = \gamma\omega' + vk_x - \gamma\frac{v^2}{c^2}\omega' = \gamma\omega' - \frac{v}{c}\omega\cos\theta - \gamma\frac{v^2}{c^2}\omega'$$

$$\boxed{\omega = \omega'\frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{\left(1+\frac{v}{c}\cos\theta\right)}}$$

with the proper frequency $\omega' = \omega_0$

Doppler effect 9 Comment

To calculate the Doppler effect, the two Lorentz transformation formulas outlined in red must now be used.

Here ω' is the natural frequency of the source ω_0.

With the second formula, the wave vector component $k_x{}'$ can be expressed by the wave vector component k_x, which can be measured by the observer.

A small calculation then gives the frequency ω in the reference system S.

Doppler effect 10

$$\boxed{\omega = \omega' \frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{\left(1+\frac{v}{c}\cos\theta\right)}}$$

with the proper angular velocity $\omega' = \omega_0$

$\theta = 0°$ gives the longitudinal Doppler effect for a source that is receding

$$\omega = \omega_0 \frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{1+\frac{v}{c}} = \omega_0 \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

$\theta = 180°$ gives the longitudinal Doppler effect for a source that is approaching

$$\omega = \omega_0 \frac{\sqrt{1-\left(\frac{v}{c}\right)^2}}{1-\frac{v}{c}} = \omega_0 \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

Doppler effect 10 Comment

The formula outlined in red once again gives the formula for the general Doppler effect.

$\theta = 0°$ applies in case the source moves away and $\theta = 180°$ in case the source approaches.

In both cases one obtains the well-known formula for the longitudinal Doppler effect.

Doppler effect 11

$$\boxed{\omega = \omega_0 \frac{\sqrt{1 - \left(\frac{v}{c}\right)^2}}{\left(1 + \frac{v}{c}\cos\theta\right)}}$$

Approximation $v << c$, $\gamma \to 1$ and $\theta \neq 0°$:

$$\omega = \omega_0 \frac{1}{\gamma\left(1 + \frac{v}{c}\cos\theta\right)} \quad \overset{v<<c}{\longrightarrow} \quad \underline{\omega = \omega_0\left(1 - \frac{v}{c}\cos\theta\right)}$$

$$\frac{\omega - \omega_0}{\omega_0} = \boxed{\frac{\Delta\omega}{\omega_0} = -\frac{v}{c}\cos\theta} \quad \text{or} \quad \frac{\Delta\omega}{\omega_0} = \frac{v_{\parallel}}{c}$$

Doppler effect 11 Comment

In many applications, the velocities v are very small compared to the speed of light.

If the root of the numerator in the formula framed in red is replaced by one and the remainder by a Taylor expansion ($1/(1+x) = 1-x$), then the formula underlined in red results.

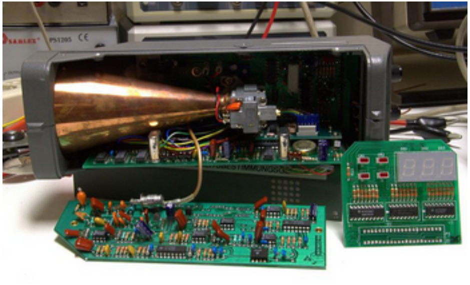
The frequency shift is simply proportional to the projection of the velocity v onto the direction of observation, i.e. $\Delta\omega/\omega = v_{\|}/c$.

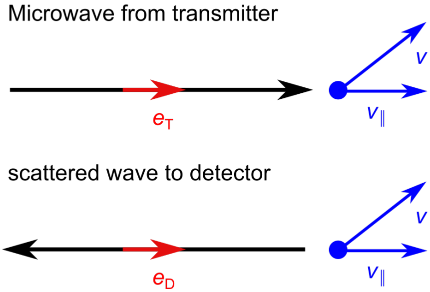
The frequency increase is maximum when the source approaches the observer directly.

No frequency shift results when the source is observed at 90° and a frequency lowering results when the source moves away.

Doppler effect 12: Applications

- ‘Radar’ speed traps





$$\omega_1 = \omega_0\left(1 - \frac{\vec{v}\vec{e}_T}{c}\right) \quad \text{and} \quad \omega_2 = \omega_1\left(1 - \frac{\vec{v}\vec{e}_D}{c}\right)$$

$$\omega_2 = \omega_0\left(1 - \frac{\vec{v}\vec{e}_T}{c}\right)\left(1 - \frac{\vec{v}\vec{e}_D}{c}\right) \approx \omega_0\left(1 - \frac{\vec{v}}{c}(\vec{e}_T + \vec{e}_D)\right) = \underline{\omega_0\left(1 - 2\frac{v_\parallel}{c}\right)}$$

Doppler effect 12 Comment 1

The figure on the left shows the radar.

The horn is necessary to match the characteristic impedance of the gun diode (microwave source) to the characteristic impedance of the free space.

The sketch on the right shows the basic principle.

ω_1 denotes the angular velocity of the wave hitting the moving object.

In the situation outlined, the car moves away from the microwave source. Consequently, the observed frequency ω_1 decreases.

ω_2 denotes the elastically scattered wave emitted by to moving object towards the detector.

The car continues to move away from the microwave source and the frequency ω_2 is again smaller than the frequency ω_1 that the car receives from the speed trap.

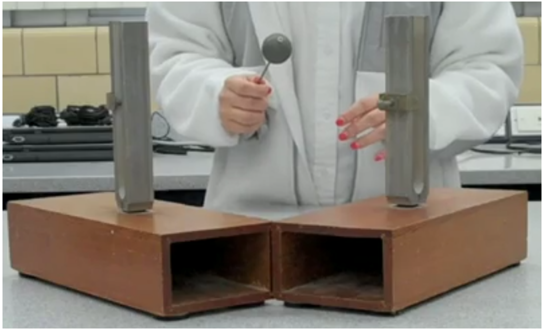
Doppler effect 12 Comment 2

The Doppler shift of the angular velocity is $\omega_0 2v_{||}/c$.

Doppler effect 13: Applications

The Doppler shift is measured by wave mixing

$$\exp(i\omega_1 t) + \exp(i\omega_2 t) = \exp(i\frac{\omega_1 + \omega_2}{2} t) \cdot 2\cos(\frac{\omega_1 - \omega_2}{2} t)$$



(TuningFork.mp4)

Doppler effect 13 Comment

The horn of the radar transmitter also serves as a receiver of the scattered radar wave.

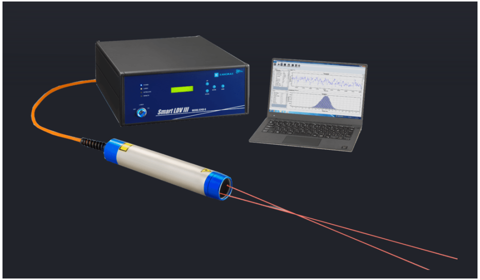
The outgoing and incoming waves can be decoupled (e.g. by a circulator).

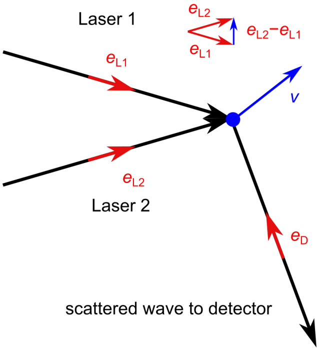
The incoming wave is mixed with the wave from the microwave source and a beat signal is produced.

The beat frequency is $\omega_0 v_{\parallel}/c$.

Doppler effect 14: Applications

- Laser Doppler velocimetry or anemometry





$$\omega_1 = \omega_0\left(1 - \frac{\vec{v}}{c}(\vec{e}_{L1} + \vec{e}_D)\right) \quad \text{and} \quad \omega_2 = \omega_0\left(1 - \frac{\vec{v}}{c}(\vec{e}_{L2} + \vec{e}_D)\right)$$

$$\omega_1 - \omega_2 = \omega_0 \frac{\vec{v}}{c}(\vec{e}_{L2} - \vec{e}_{L1})$$

Doppler effect 14 Comment

The figure on the left shows a laser Doppler velocimeter.

The speed of an object is measured at the crossing point of the two laser beams.

The figure on the right shows the principle.

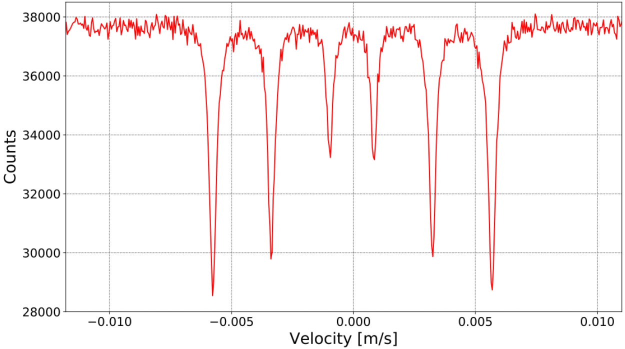
The scattered waves of the two laser beams are superimposed in the detector and the beat frequency is measured.

The calculation shows that the beat frequency is independent of the position of the detector, i.e. the detector can e.g. be integrated into the device that emits the two laser beams.

The component of the measured velocity lies in the plane of the two laser beams and perpendicular to the direction towards the source of the laser beams.

Doppler effect 15: Applications

- Mössbauer effect, or recoilless nuclear resonance absorption (1958)



^{57}Fe Mössbauer spectrum

Doppler effect 15 — Comment 1

So far it has been assumed that the scattering of light is elastic, i.e. no energy is transferred to the moving object.

This is an excellent approximation for microwaves and visible light, but not for X-rays or γ rays.

γ rays are emitted from nuclei and are extremely sharp.

Since the energy of γ rays is large, recoil in an emission or absorption process cannot usually be neglected.

Therefore, the γ radiation emitted by one nucleus cannot normally be absorbed by another nucleus of the same type.

In 1958 Rudolf Mössbauer discovered that there is no recoil in a γ emission or absorption when the nucleus is part of a solid.

Doppler effect 15 Comment 2

The energy of the nuclear transitions depends somewhat on the environment of the nucleus.

Because the spectral width of γ rays is very small, the tiny energy shifts due to the electronic environment of the nucleus can be probed by the Mössbauer effect.

The figure shows the Mössbauer spectrum of the ^{57}Fe nucleus.

The γ radiation is emitted by ^{57}Fe nuclei in the reference source and absorbed by ^{57}Fe in the substance under investigation.

The resonance condition is established by the Doppeler effect.

For this purpose, the γ source is moved with a Mössbauer transducer at a constant speed relative to the substance to be examined.

Relativistic mechanics

Special Relativity

Relativistic mechanics 1

Max Planck 1900

$$\boxed{E = h\nu = \hbar\omega}$$

Louis de Broglie 1924

$$\boxed{p = \frac{h}{\lambda} \quad \text{and} \quad \vec{p} = \hbar\vec{k}}$$

Planck's constant

$$\underline{h = 6.626 \cdot 10^{-34}\,\text{Js}}$$

and $\hbar = h/2\pi$

Relativistic mechanics 1 Comment 1

Newton’s second law is of particular importance in mechanics.

A mass is accelerated by a force, and the product of mass and acceleration is equal to the force acting on the mass.

The special theory of relativity is based on the Lorentz transformation, which does not describe any accelerated movements.

Accelerations are therefore not a topic of the special theory of relativity and a complete description of mechanics with the special theory of relativity is not possible.

This is the subject of general relativity.

Relativistic mechanics 1 Comment 2

Nevertheless, there are important aspects of mechanics that are already visible in the special theory of relativity and that are very important in the context of quantum mechanics.

The basic laws of quantum mechanics offer the easiest access to the mechanics of the special theory of relativity.

However, this is not the historical approach that I do not want to discuss in this lecture.

The first law is Planck’s law, which was discovered in 1900.

The energy of electromagnetic waves is quantized.

The energy of the quanta is given by the first equation outlined in red.

Relativistic mechanics 1 Comment 3

If Planck's constant is divided by 2π, the angular frequency can be used.

The Planck constant divided by 2π was introduced by Nils Bohr and is known as the reduced Planck constant or simply as h-bar.

The second basic law of quantum mechanics was found by de Broglie in 1924.

He conjectured that all particle form waves and that waves are always formed by particles.

The wavelength is related to the momentum of the particles.

The second formulas outlined in red give the relationship between the momentum and the wavelength or the momentum vector and the wave number vector.

Relativistic mechanics 1 Comment 4

Based on de Broglie’s theory, Planck’s law is not limited to electromagnetic waves, but applies to all types of waves.

The energy of the particles is proportional to the angular frequency and the momentum is proportional to the wave vector.

Relativistic mechanics 2

ct and $\vec{r}$

$$\boxed{\begin{array}{l} ct = \gamma(ct' + \frac{v}{c}x') \\ x = \gamma(x' + \frac{v}{c}ct') \quad \text{and} \quad y = y' \quad \text{and} \quad z = z' \end{array}}$$

ω/c and $\vec{k}$

$$\boxed{\begin{array}{l} \omega = \gamma(\omega' + vk_x{}') \\ k_x = \gamma(k_x{}' + \frac{v}{c^2}\omega') \quad \text{and} \quad k_y = k_y{}' \quad \text{and} \quad k_z = k_z{}' \end{array}}$$

E/c and $\vec{p}$

$$\boxed{\begin{array}{l} E = \gamma(E' + vp_x{}') \\ p_x = \gamma(p_x{}' + \frac{v}{c^2}E') \quad \text{and} \quad p_y = p_y{}' \quad \text{and} \quad p_z = p_z{}' \end{array}}$$

Relativistic mechanics 2 Comment

Since the angular frequency and the components of the wave vector are subject to the Lorentz transformation, energy and momentum must also be transformed using the Lorentz transformation when the inertial system is changed.

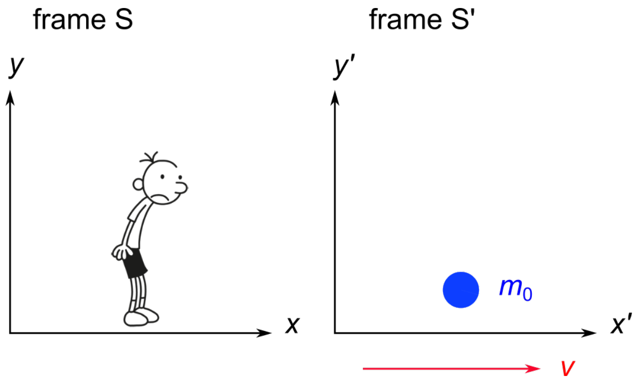
The first set of equations gives the Lorentz transformation for time and space.

The second set of equations provides the Lorentz transformation for the angular frequency and the components of the wave vector.

The third set of equations shows the Lorentz transformation of energy and momentum.

Relativistic mechanics 3

$$\boxed{\begin{aligned} E &= \gamma(E' + v p_x{}') \\ p_x &= \gamma(p_x{}' + \frac{v}{c^2}E') \end{aligned}}$$



$$\boxed{p_x = \gamma \frac{v}{c^2} E'} \qquad v << c: \quad p_x = m_0 v \quad \rightarrow \quad m_0 = \frac{E'}{c^2}$$

rest energy : $\boxed{E_0 = m_0 c^2} = E'$

Relativistic mechanics 3 Comment 1

The figure shows an observer in the coordinate system S and a mass m_0, which rests in the coordinate system S'.

The mass m_0 moves with the reference system S'.

The momentum of the mass is zero in the reference system S'.

With the Lorentz transformation, the momentum of the mass in the coordinate system S can be calculated and the first equation outlined in red results.

If the relative speed between the reference systems S and S' is much smaller than the speed of light, the factor $\gamma = 1$.

In Newtonian mechanics, the momentum is calculated from the product of mass and speed.

Relativistic mechanics 3 Comment 2

This results in the formula for the mass m_0 written in blue.

The mass is equal to the energy of the mass at rest divided by the square of the speed of light.

The energy of the mass at rest is called rest energy.

The second equation outlined in red gives the formula for the rest energy.

Relativistic mechanics 4

energy of a mass m_0 moving with frame S'

$$\boxed{E = \gamma E' = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$

Taylor series

$$\underline{E = m_0 c^2 + \frac{1}{2} m_0 v^2} + \ldots$$

relativistic mass

$$\boxed{m = \gamma m_0 = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$

Relativistic mechanics 4 Comment

The energy of the moving mass can be calculated using the Lorentz transformation.

The energy is equal to the rest energy multiplied by the factor γ.

This result is given by the first equation outlined in red.

When the velocity of the mass is small, the Taylor series of the square root gives the equation underlined in red.

The energy is equal to the sum of rest energy and classical kinetic energy.

The relativistic mass results from the product of the mass m_0 multiplied by the factor γ.

m_0 is usually referred to as the rest mass.

Relativistic mechanics 5

Energy of a moving mass: $E = \gamma m_0 c^2$, bzw. $E = mc^2$

$$\boxed{E = \frac{m_0 c^2}{\sqrt{1 - \vec{v}^2/c^2}}}$$

Momentum of a moving mass:

$$p_x = \gamma m_0 v \quad \text{or} \quad \boxed{\vec{p} = m\vec{v}}$$

or

$$\boxed{\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1 - \vec{v}^2/c^2}}}$$

Relativistic mechanics 5 Comment

The energy of a moving mass is equal to the product of the relativistic mass and the square of the speed of light.

The formula for the relativistic momentum corresponds to the formula for the classical momentum.

The rest mass must be replaced by the relativistic mass.

Relativistic mechanics 6

(crashtest.mp4)

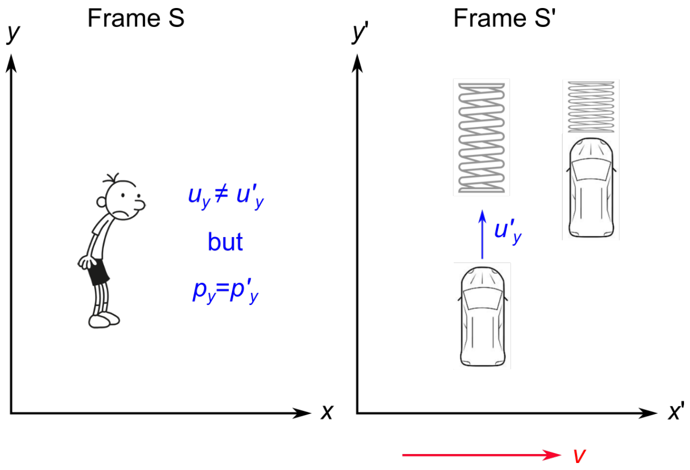
Relativistic mechanics 6 Comment

Since the introduction to the basic formulas of relativistic mechanics is very abstract, I will now discuss a crash test to show that Einstein’s formulas provide meaningful results in everyday situations.

The video shows an elementary mechanical experiment: a vehicle drives into a barrier, deforms and stops.

In order to establish a reference to Einstein’s theory of relativity, this experiment is observed on the one hand by a person in the experimental hall and on the other hand by a person passing by outside.

Relativistic mechanics 7



Relativistic mechanics 7 — Comment 1

The figure shows the crash test in the reference system S’. The car drives into a barrier illustrated by a spring at speed u'_y and gets stuck. The deformation of the barrier and the car are a measure of the car’s momentum, which is indicated by the compressed spring.

An observer who moves past with his reference system S observes the experiment. He sees the same deformation of the car and the barrier and thus observes the same impulse p_y that an observer can also measure in the reference system S’.

According to the Lorentz transformation, the momentum does not change transversely to the direction of movement of the two reference systems, which is consistent with the observation.

Relativistic mechanics 7 Comment 2

Due to time dilation, the velocities in the y direction do not match:
$u_y = dy/dt \neq u'_y = dy'/dt'$.

This means that the Newtonian momenta $p_y = mu_y \neq p'_y = mu'_y$ do not match in the two reference systems, which contradicts observation.

Relativistic mechanics 8

Speed of the car in the frame of reference S

$$\boxed{\begin{aligned} u_x &= v \\ u_y &= \frac{dy}{dt} = \frac{dy}{\frac{dt'}{\sqrt{1-v^2/c^2}}} = \frac{dy'}{\frac{dt'}{\sqrt{1-v^2/c^2}}} = u'_y/\gamma = u'_y\sqrt{1-v^2/c^2} \end{aligned}}$$

Einstein's momentum in the frame of reference S'

$$p'_y = \frac{m_0 u'_y}{\sqrt{1-{u'_y}^2/c^2}}$$

Einstein's momentum in the frame of reference S

$$p_y = \frac{m_0 u_y}{\sqrt{1-(v^2+(u_y)^2)/c^2}} = \frac{m_0 u'_y\sqrt{1-v^2/c^2}}{\sqrt{1-v^2/c^2-{u'_y}^2(1-v^2/c^2)/c^2}} = p'_y$$

Relativistic mechanics 8 Comment 1

The first formulas outlined in red give the speed of the car in the observer's reference system S. In the x direction it is the relative speed at which the observer moves to the left. The reference system S' of the experiment moves to the right with the speed *v*.

For the speed in the *y* direction, the effect of time dilation must be taken into account.

For the observer in frame S, the velocity in *y* direction is a little slower than in frame S'.

Since the coordinates in the *y* direction do not change due to the Lorentz transformation, the second formula in the red border results for the connection between the velocities in the two reference systems.

Relativistic mechanics 8 Comment 2

In the observer's reference system, the car has a smaller speed in the y-direction by a factor of $1/\gamma = \sqrt{1 - v^2/c^2}$.

However, since the observers detect the same deformations in both reference systems, this result cannot be described with Newtonian momenta.

With Einstein's momentum formula, the momentum in the y direction is the same in both reference systems, as the calculation in the last line shows.

Relativistic mechanics 9

relativistic invariant of angular frequency and wave number

$$\underline{\left(\frac{\omega}{c}\right)^2 - \vec{k}^{\,2} = 0}$$

relativistic invariant of energy and momentum

$$\begin{aligned}\left(\frac{E}{c}\right)^2 - \vec{p}^{\,2} &= \gamma^2 m_0^2 c^2 - \gamma^2 m_0^2 v^2 \\ &= \frac{m_0^2}{1-\left(\frac{v}{c}\right)^2}\left(c^2 - v^2\right) = \frac{m_0^2 c^2}{1-\left(\frac{v}{c}\right)^2}\left(1 - \frac{v^2}{c^2}\right) \\ &= m_0^2 c^2\end{aligned}$$

$$\boxed{E^2 - c^2\vec{p}^{\,2} = m_0^2 c^4}$$

Relativistic mechanics 9 Comment 1

The underlined equation gives the relativistic invariant of the angular frequency and the wave number vector for an electromagnetic wave in a vacuum.

The invariant is simply zero, since electromagnetic waves in a vacuum always travel at the speed of light.

The angular frequency can be replaced by energy and the wave number vector by the momentum.

A small calculation gives the equation outlined in red.

The relativistic invariant of energy and momentum results from the square of the rest mass multiplied by the fourth power of the speed of light.

Relativistic mechanics 9 Comment 2

From this equation it follows that the rest mass of the quantum particles that form electromagnetic waves must be zero. ($E^2 - c^2p^2 = 0$ and $E = \hbar\omega = cp = c\hbar k$ or $\omega = ck$).

These quantum particles are called photons.

Revision

Summary in questions 1

1. Give the formulas for the longitudinal Doppler effect of electromagnetic waves.
2. Give the formulas for the Doppler effect at low speeds and at any angle of observation.
3. Give the Lorentz transformation of energy and momentum.
4. Give the formula for the relativistic energy of a free particle.
5. Give the formula for the momentum.
6. Give the formula for the relativistic mass.
7. Give the relationship between energy and momentum for particles with rest mass m_0.