Modern Physics

Contents:

Laser

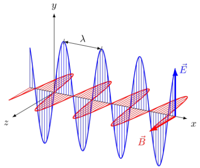
Wave-particle dualism

- Thermal radiation
- Planck’s radiation law
- Photoelectric effect
- Laser
- Compton effect
- Pair production
- Matter waves
- Uncertainty relations

Laser 1

Light **a**mplification by **s**timulated **e**mission of **r**adiation

Amplification of a wave with a wave vector $\vec{k}$ and a small spectral width $\Delta\lambda$



spectral energy density

$$u(\lambda) = \frac{1}{V}\frac{\Delta E_\lambda}{\Delta\lambda} = \frac{1}{V}\frac{1}{\Delta\lambda}\frac{hc}{\lambda}n_\lambda$$

Laser 1 Comment 1

The stimulated emission of radiation is the crucial process of a laser.

The stimulated emission for light amplification is possible from the microwave range to ultraviolet light.

The task of a laser is not to amplify light in general, but to amplify a certain wave, which is characterized by a wave vector and a narrow spectral width.

In some cases only one polarization of the wave is amplified.

The figure shows the classic image of an electromagnetic wave.

In order to amplify the wave, the number of photons that make up the wave must be increased.

Laser 1 Comment 2

For the discussion of thermal radiation, the spectral energy density $u(\lambda)$ was defined and it might be worth giving the formula $u(\lambda) = \frac{1}{V}\frac{\Delta E_\lambda}{\Delta\lambda}$ is words.

E_λ denotes the electromagnetic radiation energy for the wavelength λ in the wavelength interval $\Delta\lambda$ and V denotes the volume under consideration.

The number of photons n_λ is important for the laser.

Therefore, the spectral energy density can be expressed by the number of photons n_λ with wavelength λ in the wavelength interval $\Delta\lambda$.

(Obviously $\Delta\lambda$ must be small compared to λ.)

Laser 2

Adaptation of Einstein's equations to the laser

$$\frac{dN_2^{abs}}{dt} = +\,u(\lambda)B_{21}N_1 \quad \rightarrow \quad \frac{d{n_\lambda}^{abs}}{dt} = -n_\lambda W_{21}N_1$$

$$\frac{dN_2^{stim}}{dt} = -\,u(\lambda)B_{21}N_2 \quad \rightarrow \quad \frac{d{n_\lambda}^{stim}}{dt} = +n_\lambda W_{21}N_2$$

$$\frac{dN_2^{spon}}{dt} = -\,A_{21}N_2$$

with

$$W_{21} = \frac{1}{V}\frac{1}{\Delta\lambda}\frac{hc}{\lambda}B_{21}$$

Laser 2 Comment 1

Einstein's equation for absorption and stimulated emission can be adapted for the laser.

Instead of the thermal occupation of the energy levels, the photon number n_λ is now the decisive variable.

The first underlined equation describes the influence of absorption on the number of the photons.

It is obvious that the influence of the absorption is proportional to the population of the lower quantum state N_1 and the number of the available photons.

The equation outlined in red shows the formula for the proportionality constant W_{21}.

Laser 2 Comment 2

W_{21} results from the definition of the spectral energy density.

Likewise, the influence of the stimulated emission on the photon number is proportional to the population of the excited quantum state N_2 and the number of the available photons.

Absorption decreases the photon number, while stimulated emission increases the photon number.

The third equation is Einstein’s spontaneous emission equation.

Laser 3

Rate equation of the photon number n_λ

$$\frac{dn_\lambda}{dt} = \frac{dn_\lambda^{abs}}{dt} + \frac{dn_\lambda^{stim}}{dt} + \frac{dn_\lambda^{spon}}{dt} + \frac{dn_\lambda^{loss}}{dt}$$

$$\frac{dn_\lambda^{loss}}{dt} = -\frac{n_\lambda}{\tau}$$

$$\frac{dn_\lambda^{stim}}{dt} >> \frac{dn_\lambda^{spon}}{dt}$$

Laser 3 Comment 1

The formula outlined in red shows the rate equation for the photon number.

The photon number is reduced by absorption and increased by stimulated emission.

The spontaneous emission leads to incoherent light and disrupts the coherent light of the laser.

The last term describes the loss of photons in the active laser area.

The underlined equation describes the loss term more precisely.

The loss is proportional to the photon number and is determined by a loss rate τ^{-1}.

The rate of loss is determined by the construction of the laser.

Laser 3 Comment 2

When the amplification process is turned, off the number of photons decreases exponentially, i.e. $n_\lambda(t) = n_\lambda^0 \exp(-t/\tau)$.

Spontaneous emission is important for starting the laser.

The inequality written in blue says that the contribution of the spontaneous emission must be very much smaller than the contribution of the stimulated emission when the laser is operated.

Laser 4

Comparison between stimulated and spontaneous emission in thermal equilibrium

$$\frac{dN_2^{stim}}{dt} = -u(\lambda)B_{21}(\lambda)N_2$$
$$\frac{dN_2^{spon}}{dt} = +A_{21}(\lambda)N_2$$

with

$$u(\lambda) = \frac{A_{21}(\lambda)}{B_{21}(\lambda)} \frac{1}{e^{hc/\lambda k_B T} - 1}$$

and

$$\frac{dN_2^{stim}}{dN_2^{spon}} = \frac{B_{21}(\lambda)}{A_{21}(\lambda)} u(\lambda) = \frac{1}{e^{hc/\lambda k_B T} - 1}$$

Laser 4 Comment

The first lasers were build in 1960.

To understand the challenge of building a laser, it is useful to consider the relationship between stimulated and spontaneous emission in thermal equilibrium.

The equations framed in red give the Einstein equations for the occupation numbers of the excited state 2.

The underlined equation indicates the spectral energy density in thermal equilibrium (compare Planck's radiation law).

The last line gives the quotient between stimulated and spontaneous emission.

The following example shows that stimulated emission is completely irrelevant in thermal equilibrium!

Laser 5

Example: $\lambda = 600$ nm and $T = 300$ K

$$\frac{dN_2^{stim}}{dN_2^{spon}} = \frac{1}{\exp\left(\frac{6.626\cdot 10^{-34}\,\mathrm{Ws}^2\; 3\cdot 10^8\,\mathrm{ms}^{-1}}{6\cdot 10^{-7}\,\mathrm{m}\; 1.38\cdot 10^{-23}\,\mathrm{WsK}^{-1}\; 300\,\mathrm{K}}\right) - 1} = \frac{1}{\exp(80)}$$

$$= \frac{1}{\underline{5.5\cdot 10^{34}}}$$

Laser 6

rate equation for the relevant photon number n_λ

$$\frac{dn_\lambda}{dt} = (N_2 - N_1)W_{21}n_\lambda - \frac{n_\lambda}{\tau} = \left((N_2 - N_1)W_{21} - \frac{1}{\tau}\right) n_\lambda$$

solution of the differential equation (N_1, N_2 and τ are constant)

$$n_\lambda(t) = n_\lambda^0 \, e^{At}$$

$$A = \left((N_2 - N_1)W_{21} - \frac{1}{\tau}\right)$$

condition for amplification

$$A = (N_2 - N_1)W_{21} - \frac{1}{\tau} > 0 \quad \rightarrow \quad \text{population inversion} \quad N_2 - N_1 \geq \frac{1}{W_{21}\tau}$$

Laser 6 Comment 1

The formula outlined in red gives the rate equation of the photon number.

Only the photon number n_λ of the laser laser light has to be taken into account.

The spontaneous emission is not included in this equation.

Spontaneous emission is only important to start the laser process.

Once started, the laser process leads to an exponential increase in the photon number n_λ and the spontaneous emission can be neglected.

The solution of the rate equation is a simple exponential function as long as the occupation numbers and the loss rate do not depend on the photon number.

Laser 6 Comment 2

Due to the exponential increase, the laser photons dominate and the contribution of the spontaneous emission can be neglected.

The last equation gives the condition for the laser process.

Laser 7

$\rightarrow$ population inversion

$$N_2 - N_1 > \frac{1}{W_{21}\tau}$$

with

$$\frac{1}{W_{21}} = \frac{V\lambda\Delta\lambda}{hcB_{21}}$$

- small volume, i.e. high photon density
- small spectral width $\Delta\lambda$
- the wavelength λ should not be to small due $B_{21} \propto \lambda^2$
- for a continuous wave laser $W_{21}(N_2 - N_1) = \tau^{-1}$

Laser 7 Comment

In order to be able to amplify light through stimulated emission, the occupation number of the excited state 2 must be greater than the occupation number of the ground state 1 and one speaks of population inversion.

Population inversion never occurs in thermal equilibrium.

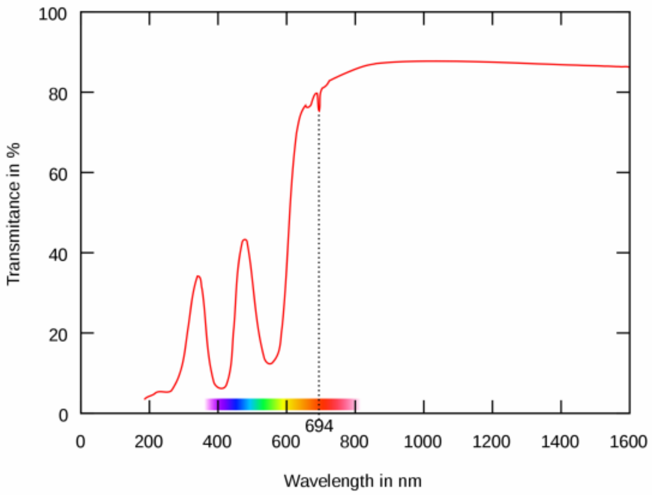
The underlined equation shows that the width of the transition used for the stimulated emission should be small.

At small wavelengths, amplification by stimulated emission becomes difficult because the Einstein coefficient B_{21} is proportional to the square of the wavelength.

Therefore, it becomes more difficult to generate coherent light by using the stimulated emission as the wavelength becomes shorter.

Laser 8: Ruby-Laser ($Cr^{3+}:Al_2O_3$)

Absorption spectrum of ruby



Laser 8 Comment

The first lasers were realized in 1960.

One laser was the ruby laser, the other the He-Ne laser.

Ruby is an Al_2O_3 crystal, doped with Cr^{3+} ions.

The spectrum shows the absorptions of the Cr^{3+} ions.

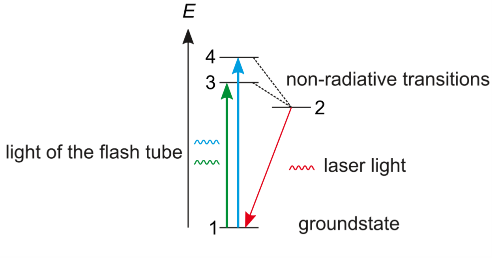
There are two strong absorptions in the blue and green regions of the spectrum and a tiny absorption in the red region.

The illustration on the right shows the ruby laser.

The ruby crystal is surrounded by a flash tube.

Laser 9: Ruby-Laser ($Cr^{3+}:Al_2O_3$)

energy level scheme of Cr^{3+} in Al_2O_3



Disadvantage: More than half of the atoms have to be excited from the ground state into state 2 for population inversion

→large excitation power

Laser 9 Comment 1

The figure shows the relevant energy levels of the Cr^{3+} ion.

The flash tube excites Cr^{3+} ions to energy levels 3 and 4.

The strong absorption lines indicate that this can be done efficiently through the flash tube.

The excited Cr^{3+} ions change quickly from energy level 3 and 4 to energy level 2.

The transitions are not due to electromagnetic radiation, but to lattice vibrations.

Instead of photons, the quantum particles of the lattice vibrations are emitted.

These quantum particles are called phonons in analogy to the photons of electromagnetic waves.

Laser 9 Comment 2

The transition between energy level 2 and the ground state only leads to a very weak absorption in the spectrum.

The transition probability between level 2 and the ground state is very small.

Therefore, population inversion can be achieved by a strong light pulse from the flash tube.

As soon as the population inversion is achieved, a short laser pulse is triggered by photons due to spontaneous emission and the Cr^{3+} ions relax back to the ground state.

The main disadvantage of this laser is that more than half of the Cr^{3+} ions have to be excited in order to produce population inversion.

Laser 9 Comment 3

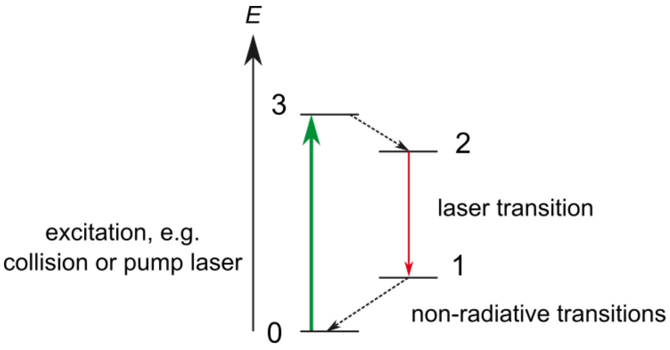
Therefore, the excitation power is large compared to the power of the laser pulse.

In addition, the energy level scheme cannot be used to build a permanent laser.

If a strong permanent light source is used to excite energy levels 3 and 4, the system becomes saturated, i.e. the occupation number is the same for all energy levels and a population inversion cannot be achieved.

Laser 10

4 level scheme



Laser 10 Comment 1

Usually a four-step scheme is used, as shown in the figure.

The laser medium (atoms, molecules, etc.) is excited from the ground state to the excited state 3.

There is a large transition rate from state 3 to state 2.

The laser transition between state 2 and 1 is indicated by a red arrow.

The laser transition does not connect state 2 with the ground state, but with the excited state 1.

If the lifespan in state 1 is very long, a laser pulse results, as with the ruby laser.

With a four-level scheme, however, half of the atoms or molecules no longer have to be excited if a population inversion is to be achieved.

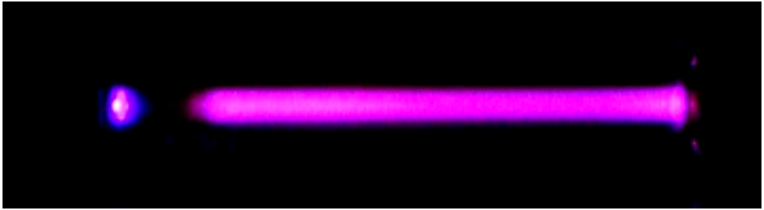
Laser 10 Comment 2

Therefore, a small excitation power is sufficient to generate laser light.

If the lifespan of state 1 is very short, then energy level 1 is practically always unoccupied and the population inversion between states 1 and 2 can be set permanently.

In this case, a continuous wave laser can be implemented with the four-level scheme.

Laser 11: He-Ne Laser



(Gasentladungsroehre.mp4)

Laser 11 Comment

The He-Ne laser is a gas laser and is based on a discharge tube.

In a gas discharge tube, a high voltage is applied to two electrodes located in the tube.

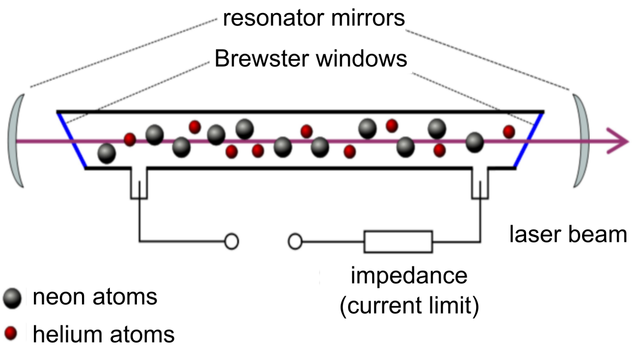
Ionized atoms or molecules accelerate and collide with other gas particles, causing an avalanche of charged gas particles and free electrons.

The gas particles are excited by the collision. The gas particles give off the excitation energy again through collisions and through the emission of light

The video shows the light effects of a gas discharge tube.

Laser 12: He-Ne Laser

the basic set-up of a He-Ne laser



Laser 12 Comment

The figure shows the basic structure of a He-Ne laser.

The He-Ne laser is a continuous wave laser.

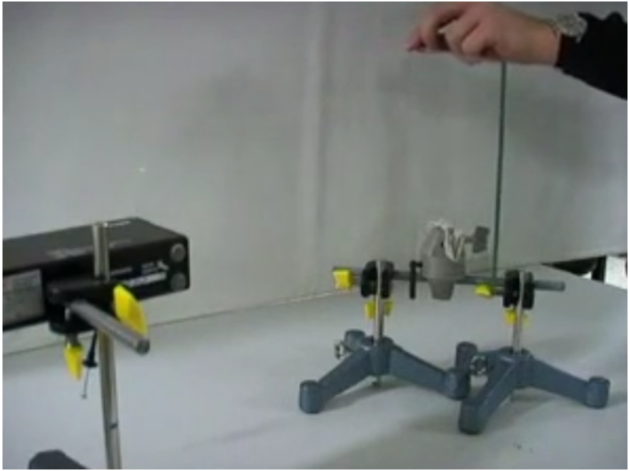
A gas discharge tube is filled with He and Ne atoms.

In the illustration, the gas discharge tube is located between two mirrors and the tube itself is closed off by so-called Brewster windows.

The gas discharge tube can also be closed directly by mirrors.

One of the mirrors is semi-transparent so that laser light can leave the discharge tube.

Laser 13: He-Ne Laser



(brewster.mp4)

Laser 13 Comment

The video demonstrates the effect of the Brewster windows.

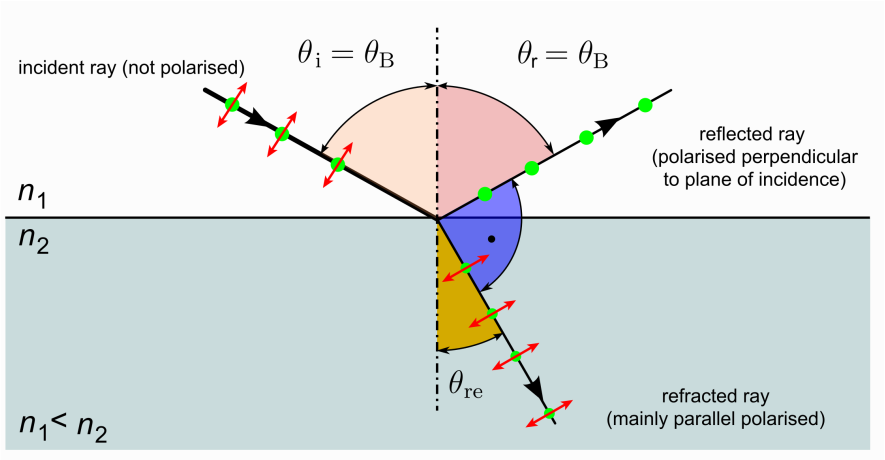
Only light that is polarized perpendicular to the direction of the beam and to the normal of the glass plate is reflected if the Brewster condition (Brewster angle) is met.

Light that is polarized in the plane that is formed by the laser beam and the normal of the window can pass through the window without loss.

The perpendicular polarization is greatly weakened by the reflection and can therefore not be amplified by stimulated emission.

The suppression of one polarization direction can be avoided if the mirrors are built directly into the end caps of the gas discharge tube.

Laser 14: Brewster's angle (1815)



Laser 14 Comment

The sketch illustrates the Brewster effect.

If the reflected beam and the refracted beam are perpendicular to each other, then only the component of the light that is polarized perpendicular to the plane of incidence is reflected.

The Brewter effect is described by the Maxwell equations and in particular by the Fresnel equations.

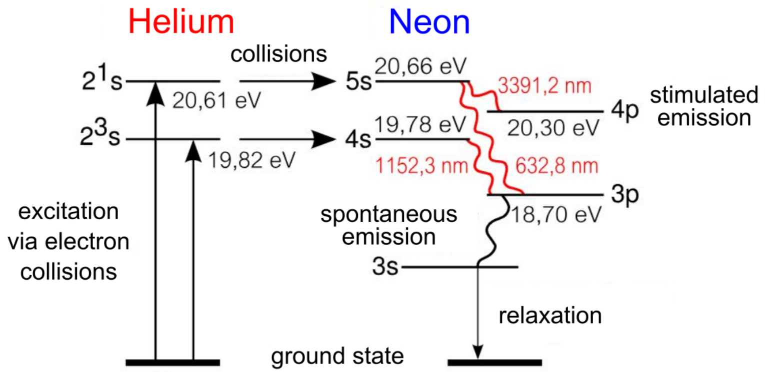
The heuristic explanation is based on the Hertzian dipole.

With a Hertzian dipole, the emitted radiation disappears along the dipole axis and is maximal in the vertical direction.

If the Brewster condition is fulfilled, no light polarized in the plane of incidence can be reflected.

Laser 15

relevant energy levels of the He and Ne atoms



Laser 15 Comment 1

The figure shows the relevant energy levels of the He atom and some energy levels of the Ne atom.

There are many more energy levels in the neon atom than in the helium atom because the electron configuration of helium is made up of 2 electrons while the configuration of neon is made up of 10 electrons.

The two excited energy levels of the helium atom can be excited directly by collisions with free electrons.

The excitation energy of the helium atoms is transferred to neon atoms when helium atoms collide with neon atoms.

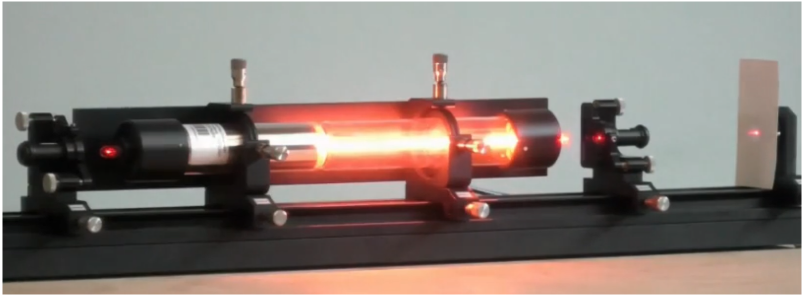
Laser 15 Comment 2

The 4s and 5s levels of the neon atom are excited, so that the population inversion condition is met for many transitions to lower energy states.

The transition between the 5s and 3p levels is often used in the He-Ne laser, although other transitions can also be amplified.

The wavelength of the transition is in the red area of the visible spectrum.

Laser 16: He-Ne Laser



(HeNeFreiburg.mp4)

Laser 16 Comment

The video shows how a He-Ne laser works.

If the gas discharge tube is placed between two mirrors, the light amplification starts through stimulated emission.

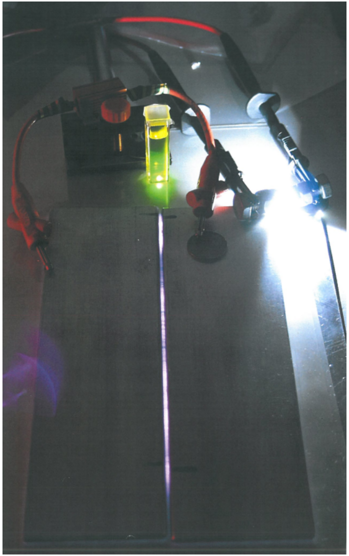
The video also shows that many spectral lines are excited by the collisions of the atoms.

Only one transition is amplified by the stimulated emission.

Which transition this is can be set by adjusting the mirrors.

The emission of the red laser light is particularly robust, but the emission of other wavelengths is also possible.

Laser 17: N_2 Laser



(N2(Stickstoff)Laser.mp4)

Laser 17 Comment

The video shows the simple construction of a N_2 laser that can be operated directly with air.

It consists of two aluminum plates, a spark gap and a high voltage source.

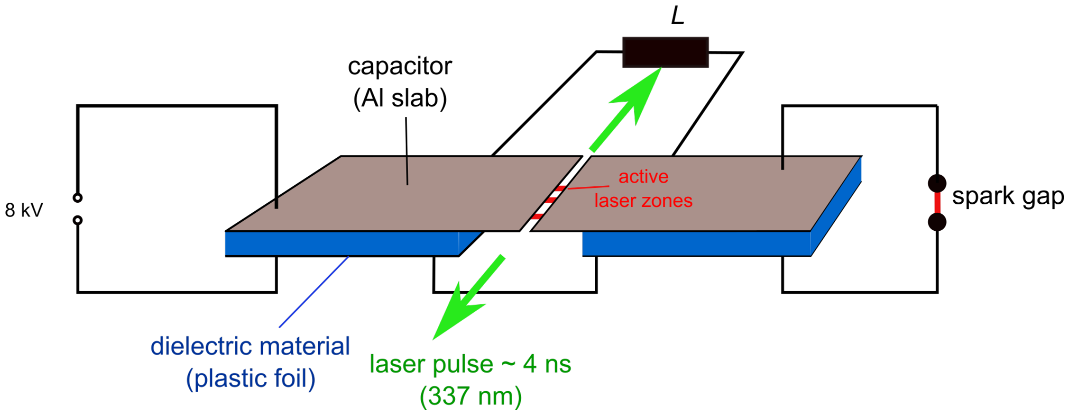
The laser generates pulses in the ultraviolet spectral range with a pulse duration of approximately 4 ns.

The ultra violet light is detected by the photoluminescence of paper that absorbs the laser light.

As with the He-Ne laser, the laser light is generated by a transition between excited states of the N_2 molecule.

In contrast to the He-Ne laser, the lifetime of the lower energy level is long, so that the condition of population inversion is quickly suppressed due to saturation.

Laser 18: N_2 Laser



Laser 18 Comment 1

The sketch shows the basic experimental setup.

Two capacitors are separated by a small gap created by the edges of the Al plates.

The capacitors are connected by an inductance (or impedance).

High voltage is applied to one capacitor.

This creates a high electric field in the gap between the two capacitors.

Ionized air molecules are accelerated, collide and create an avalanche of excited molecules.

Laser 18 Comment 2

The voltage applied to the second capacitor increases slowly due to the inductance.

If the voltage of the second capacitor reaches a critical value, the spark gap ignites and the two capacitors are discharged.

The first capacitor is immediately charged again by the high voltage source, so that a large electric field builds up again in the gap between the two aluminum plates.

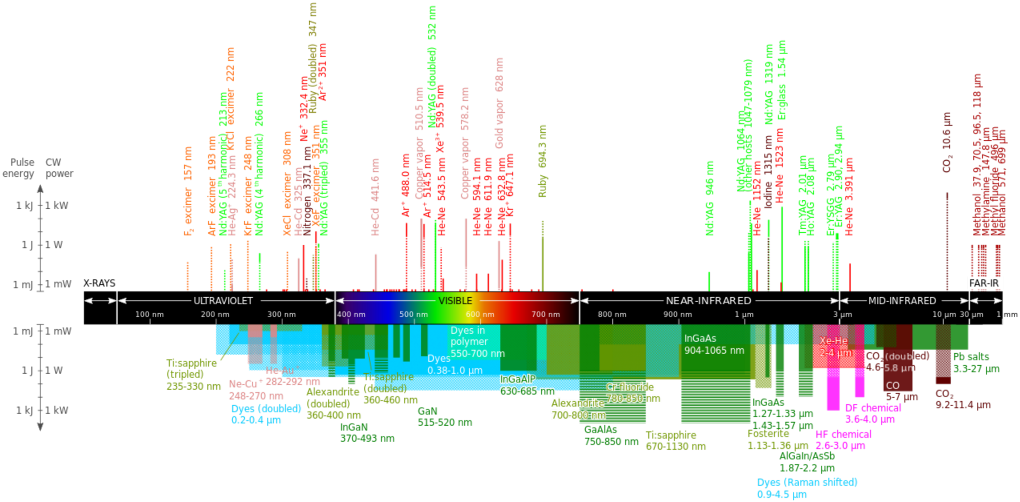
In this way, laser pulses are generated almost permanently.

In contrast to the He-Ne laser, this laser does not need a resonator.

Laser 18 Comment 3

However, only light is amplified that propagates essentially along the gap between the aluminum plates.

Perpendicular to the gap, the photon loss rate is obviously so large that no population inversion can be achieved in these directions.



Wavelengths of commercially available lasers. Laser types with distinct laser lines are shown above the wavelength bar, while below are shown lasers that can emit in a wavelength range. The height of the lines and bars gives an indication of the maximal power/pulse energy commercially available, while the color codifies the type of laser material. Most of the data comes from Weber's book Handbook of laser wavelengths.

Laser 19 Comment

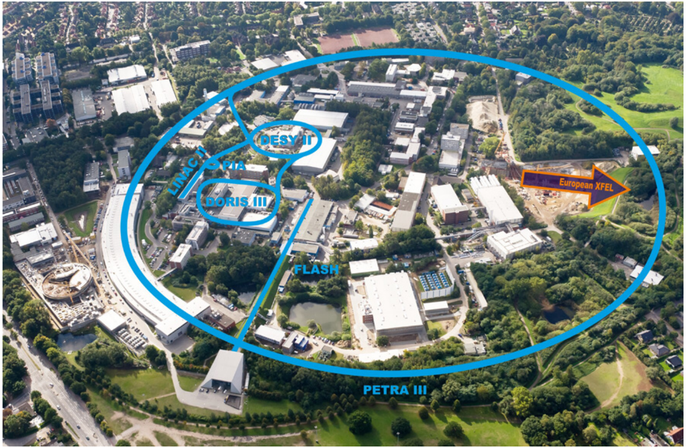
The figure gives an overview of the lasers based on stimulated emission of radiation.

The wavelength ranges from infrared to ultraviolet.

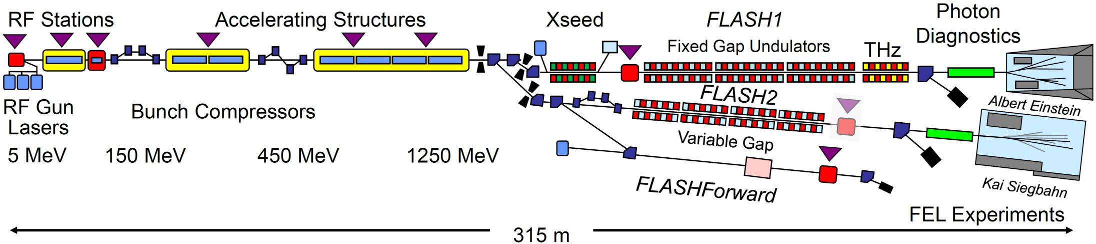
The shortest wave length is 157 nm.

Lasers with shorter wave length are no longer based on the stimulated emission of radiation.

Laser 20: FEL (Free Electron Laser)

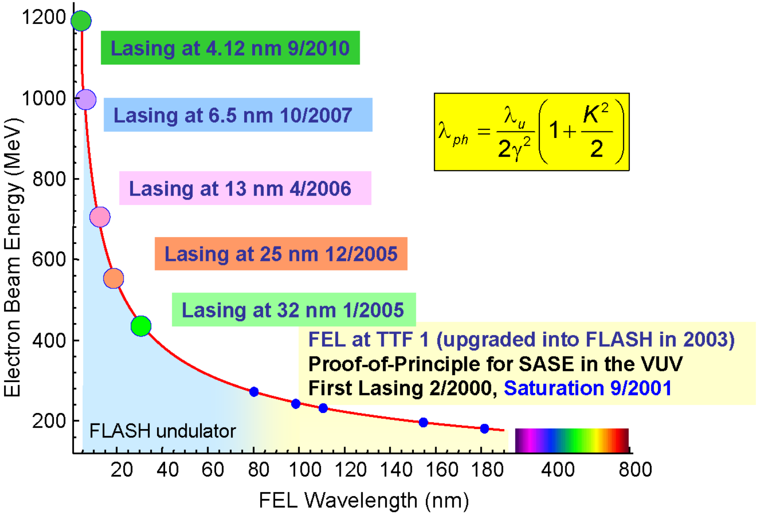


Laser 21: FEL (Free Electron Laser)



Schematic layout of FLASH (not to scale). Beam direction is from left to right, the total length is 315 m. (Image: DESY/Siegfried Schreiber)

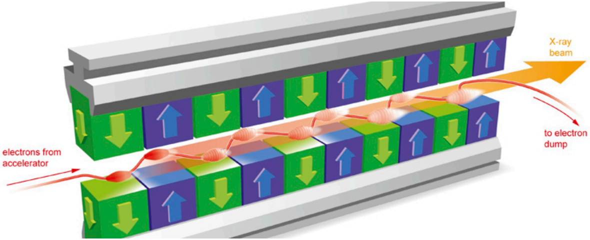
Laser 22: FEL



$$\lambda_{ph} = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

Laser 23: XFEL (X-Ray Free-Electron Laser, $\lambda \approx 0.05$ to 4.7 nm)

Laser 24: XFEL



In the undulator, the electron bunches are guided by structures with periodically arranged magnets (green and blue). This brings the electrons on a tight slalom course. In doing so, they emit X-ray light, which continues to intensify.

Amplification by constructive interference: $n\lambda = \mathrm{L_u}\sqrt{1-\frac{v^2}{c^2}}\sqrt{\frac{1-v/c}{1+v/c}} = \mathrm{L_u}(1-v/c)$

$$\nu_n = \frac{c}{\lambda_n} = \frac{n\,c}{\mathrm{L_u}}\frac{(1+v/c)}{(1-v/c)(1+v/c)} \approx \frac{n\,c}{\mathrm{L_u}}2\gamma^2 = \frac{2n\,c}{\mathrm{L_u}}\left(\frac{E}{m_0c^2}\right)^2$$

Laser 24: XFEL Comment 1

The electron beam is forced into a zigzag path by an alternating magnetic field.

If the lateral acceleration is not too great, then electromagnetic radiation is emitted mainly in the direction of the electron beam.

The radiation is amplified by constructive interference when the wavelength is a multiple of the characteristic undulator length L_u. L_u denotes the period of the magnetic field modulation.

Since the electrons move almost at the speed of light, the length contraction must be taken into account in the coordinate system of an electron.

In addition, since the electrons are moving in the direction of the experiment, the Doppler effect must be taken into account and the formula underlined in red results.

Laser 24: XFEL Comment 2

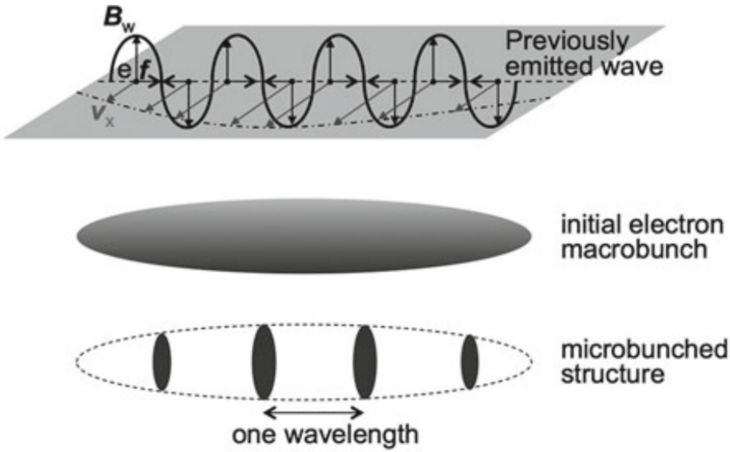
E.g. assuming an undulator period of $L_u = 10\,\text{cm}$ an electron energy of 15 GeV can be estimated for a wavelength of $0.05\,\text{nm}$.

This light is not coherent a priori.

If the magnetic field of the electromagnetic wave created by the accelerated electrons is strong enough, an additional effect leads to coherent radiation.

This effect is called SASE (self-amplified spontaneous emission).

Laser 25: SASE (Self-Amplified Spontaneous Emission)



Microbunching mechanism. Top the Lorentz force due to the interaction between the transverse B-field of previously emitted waves and the transverse velocity of electrons oscillating in an undulator pushes the electrons towards every other node in the wave. In this way, an initially unstructured macrobunch (middle) develops a structure (bottom) consisting of microbunches with the period of one wavelength.

Laser 25: XFEL Comment

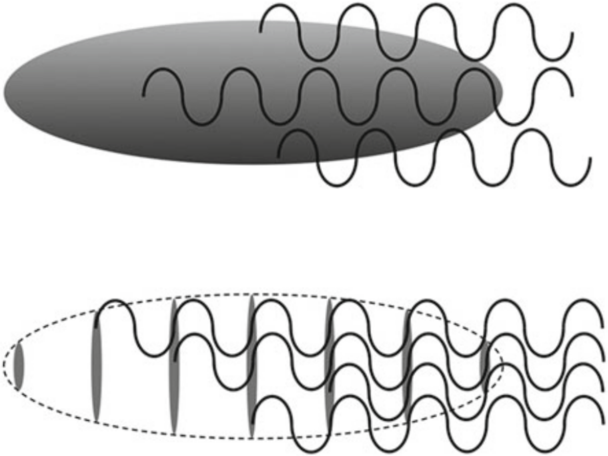
The upper picture shows an electron beam with the transverse velocity v_x, which is imposed on the beam by the magnetic field of the undulator.

The solid line shows the magnetic field of an electromagnetic wave that is radiated by the accelerated electrons.

Due to the Lorentz force, the magnetic field of the wave drives the electrons to a zero crossing of the magnetic field strength in the manner outlined.

The initial bunch of electrons breaks up into a micro-bunched structure as shown in the lower picture.

Laser 26: XFEL



from: Synchrotron Radiation Settimio Mobilio, Federico Boscherini, Carlo Meneghini (Editors),Springer-Verlag Berlin Heidelberg 2015

Laser 26: XFEL Comment

The upper figure shows the incoherent emission of radiation of the initial electron bunch.

The lower figure shows the coherent emission of radiation by the micro-bunched electrons.

Revision

Summary in questions

1. What does the acronym LASER mean?
2. State the basic condition for light amplification by stimulated emission of radiation.
3. Explain how the ruby laser works.
4. Explain the four-level scheme for generating laser light.
5. Explain the role of the helium atoms in the He-Ne laser.
6. Why is the light of some He-Ne lasers polarized?