Modern Physics

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Compton effect

Wave-particle dualism

- Thermal radiation
- Planck's radiation law
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Compton effect 1

Albert Einstein (1905)

the energy quanta of electromagnetic radiation are “localized in spatial points, move without division and can only be absorbed and generated as a whole”

Arthur Holly Compton (1922)

the energy quanta carry a momentum: $p = \frac{h}{\lambda}$ and $\vec{p} = \hbar\vec{k}$

$$\underline{E = h\nu = \frac{hc}{\lambda} = cp}$$

Compton effect 1 Comment 1

In 1900 Max Planck postulated that the energy of electromagnetic radiation can only be absorbed and emitted in quanta of $\hbar\omega$.

With this assumption, the spectrum of thermal radiation can be perfectly explained.

In 1905, Albert Einstein formulated Planck's assumption in explaining the photo effect more precisely: the energy quanta of electromagnetic radiation are "localized in spatial points, move without division and can only be absorbed and generated as a whole".

When Arthur Compton investigated the scattering of X-rays on graphite in 1922, he was able to show explicitly that the quanta of electromagnetic radiation behave like relativistic particles.

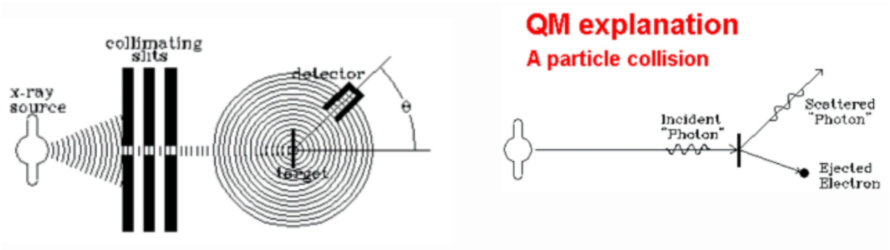
Compton effect 1 Comment 2

In 1927 Arthur Compton was awarded the Nobel Prize for “for his discovery of the effect named after him”.

The relationship between energy and momentum corresponds to the formula that results from the special theory of relativity for a particle without rest mass.

Compton effect 2

Scattering of high-energy photons on quasi-resting electrons



Compton effect 2 Comment

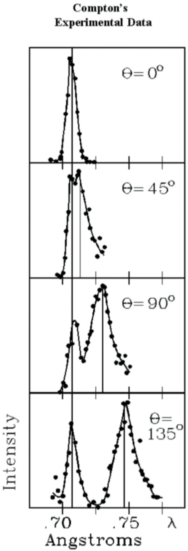
The illustration shows the experimental setup.

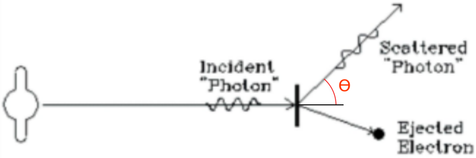
An X-ray beam is generated through several apertures.

The X-ray beam hits a sample and the scattered X-ray beam is measured at the angle θ with respect to the incident beam.

With the detector, the intensity and also the wavelength, i.e. the energy of the scattered X-rays can be measured.

Compton effect 3





Compton effect 3 — Comment 1

The left figure shows experimental results for different scattering angles θ.

The measurements show that there is an elastic component in the spectrum of the scattered X-rays.

In elastic scattering, the wavelength does not change when the scattering angle is changed.

The spectrum also contains an inelastic component, i.e. the wavelength changes when the scattering angle is changed.

The wavelength becomes larger with increasing angle θ and and the energy of the scattered photons is less than the energy of the incident photons.

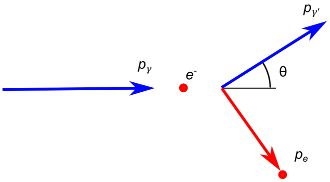
Compton effect 3 Comment 2

The figure on the right illustrates the interpretation of the inelastic scattering.

The incident photon transfers part of its energy and momentum to an electron.

If the binding energy and the kinetic energy of the electron can be neglected compared to the energy of the incident photon, then the effect can be described by the collision of a photon with a quasi-free electron that is at rest.

Compton effect 4



energy conservation

$$\boxed{E_\gamma + m_0 c^2 = E_{\gamma'} + E_e}$$

momentum conservation

$$\boxed{\vec{p}_\gamma = \vec{p}_{\gamma'} + \vec{p}_e}$$

The energy and momentum of the electron can be eliminated with the help of $E_e^2 - c^2 \vec{p}_e^{\,2} = m_0^2 c^4$ and $(\vec{p}_\gamma - \vec{p}_{\gamma'})^2 = \vec{p}_e^{\,2}$

Compton effect 4 Comment 1

Compton showed that the inelastic component can be traced back to the collision of a photon with an electron that is quasi at rest.

Quasi at rest means that both the kinetic energy and the binding energy of the electron can be neglected compared to the energy of the photon.

The illustration outlines this situation.

The law of conservation of energy states that the sum of the energy of the photon and the rest energy of the electron before the collision must be equal to the sum of the energy of the photon and the total energy of the electron after the collision.

Compton effect 4 Comment 2

The law of conservation of momentum states that the momentum of the incident photon must be equal to the sum of the momentum of the photon and that of the electron after the collision.

With the relativistic energy-momentum relationship, the energy and momentum of the electron can be eliminated.

Compton effect 5

$$\boxed{\frac{1}{E_{\gamma'}} - \frac{1}{E_\gamma} = \frac{1}{m_0 c^2}(1 - \cos\theta)}$$

with $E_\gamma = h\nu = hc/\lambda$

$$\frac{\lambda'}{hc} - \frac{\lambda}{hc} = \frac{1}{m_0 c^2}(1 - \cos\theta)$$

Compton formula

$$\boxed{\lambda' - \lambda = \lambda_C(1 - \cos\theta)}$$

Compton wavelength

$$\boxed{\lambda_C = \frac{h}{m_0 c} = 2.43 \cdot 10^{-12}\,\mathrm{m}}$$

Compton effect 5 Comment

A small calculation results in the first formula outlined in red.

The Compton formula results from Planck's law $E = h\nu$.

In the forward direction, the wavelength of the scattered photon coincides with the wavelength of the incident photon.

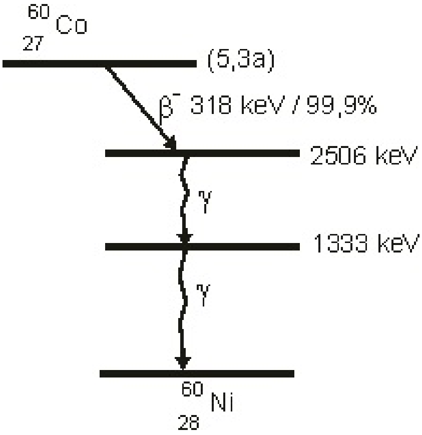
If the direction of observation deviates from the direction of incidence, the wavelength of the inelastically scattered photon increases and reaches the greatest value with backward scattering.

The strength of the effect is determined by the Compton wavelength.

The Compton wavelength is about five orders of magnitude below the wavelength of visible light.

Compton effect: Example 6

radioactive decay of ^{60}Co



Compton effect 6 Comment 1

The Compton effect is usually observed in the gamma decay of nuclei.

The figure shows the decay of cobalt to nickel.

The cobalt nucleus ejects an electron in the β decay and thus increases its nuclear charge by one elementary charge.

Cobalt turns into nickel.

However, the resulting nucleus of nickel is in an excited state and changes to the ground state through two successive gamma decays.

Gamma radiation is electromagnetic radiation.

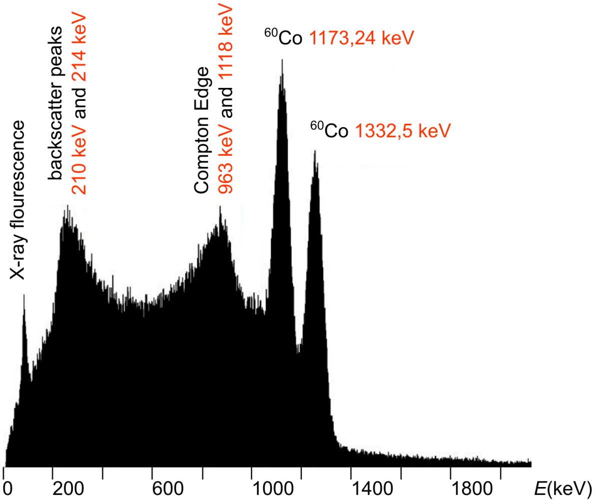
Compton effect 6 Comment

The energy of gamma particles is much greater than the typical energy of X-rays.

The first gamma particle has an energy of 1173 keV.

The second gamma particle has an energy of 1333 keV.

Compton effect: example 7



Compton effect 7 Comment 1

The figure shows the spectrum of gamma radiation when cobalt decays.

Since the nickel nuclei can emit the gamma particles in all directions, when the gamma spectrum is measured, a distribution over all angles θ in the Compton formula results.

The two peaks of the elastic scattering can be clearly seen in the spectrum.

The energy corresponds to the energy of the nuclear transition.

Then there is a broad spectrum that extends to the so-called backscatter peaks. The energy of these peaks is slightly greater than 200 keV.

The sharp peak at even lower energy is due to fluorescence.

The gamma quanta excite the electrons of the cobalt and nickel atoms and the excitation energy is released by the emission of fluorescence photons.

Compton effect 8

$$\boxed{\frac{1}{E_{\gamma'}} - \frac{1}{E_\gamma} = \frac{1}{m_0 c^2}(1 - \cos\theta)}$$

backscattering $\theta = 180°$

$$\frac{1}{E_{\gamma'}} - \frac{1}{E_\gamma} = \frac{2}{m_0 c^2}$$

for $E_\gamma >> m_0 c^2$ $\quad E_{\gamma'} \approx \frac{m_0 c^2}{2} = 250\,\text{keV}$

Compton effect 8 Comment 1

The energy of the backscatter can be understood with the formula outlined in red.

In the case of backward scattering, the right-hand side of the formula corresponds to the reciprocal value of half the rest energy of the electron, i.e. the reciprocal value of around 250 keV.

If the incident photon has a very high energy, then its contribution in the Compton formula can be neglected and the energy of the scattered photon tends towards half the rest energy of the electron.

If the energy of the incident photon cannot be neglected, then the reciprocal value of the energy of the incident photon is added to the reciprocal value of half the rest energy and the energy of the scattered photon is slightly smaller.

Compton effect 8 Comment 2

With the energy of the two photons of the cobalt decay, the values 210 and 214 keV result for the backscatter peaks.

Another distinctive feature in the Compton spectrum is the Compton edge.

The reason for the Compton edge lies in the high kinetic energy of the electrons, which are created when the photons are backscattered.

These electrons are strongly decelerated on their way through the material of the sample and emit photons, as can be expected according to Maxwell's electrodynamics.

One speaks of bremsstrahlung.

Compton effect 8 Comment 3

In the extreme case, an electron converts its entire kinetic energy into a photon.

These photons, which absorb almost the entire kinetic energy of the electrons, which were created during the backward scattering, form the Compton edge.

The energy results from the energy of the original photon minus the energy of the backscattered photon.

This means that the energy of the Compton edges can be expected at 963 keV and 1173 keV, respectively

Pair production

Wave-particle dualism

- Thermal radiation
- Planck’s radiation law
- Photoelectric effect
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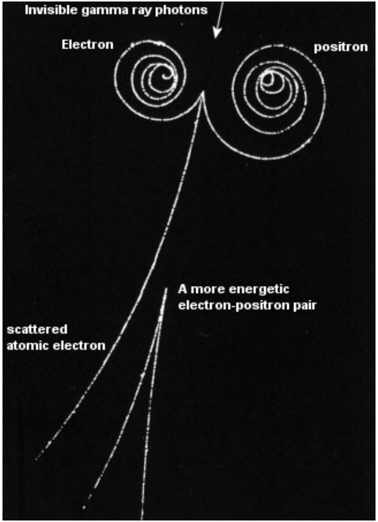
- Matter waves
- Uncertainty relations

Pair production 1



Pair production 1 Comment 1

Arthur Compton's work showed that the quanta of electromagnetic radiation can be treated as particles that obey the laws of conservation of energy and momentum.

Patrick Blackett discovered the conversion of photons into electron-positron pairs while studying cosmic rays in 1933.

The positron is the antiparticle to the electron. Particles and antiparticles are exactly the same in all properties. Only in the case of the charge do the two particles differ in sign.

The illustration shows contrails caused by electrons and positrons in a cloud chamber. A cloud chamber is a chamber that is filled with saturated water vapor.

A magnetic field is applied to the cloud chamber, so that the path of charged particles is curved due to the Lorentz force ($\vec{F}_L = q\vec{v} \times \vec{B}$).

Pair production 1 Comment 2

Only charged particles lead to contrails.

The uncharged photons are not visible.

In the upper part of the figure, a photon decays into an electron-positron pair.

The decay takes place near an atom and part of the energy of the photon is transferred to an electron in the atom, which is knocked out of the electron shell.

A second event can be seen in the lower part of the figure.

During this decay, the entire energy of the photon is transferred to the electron-positron pair.

The curvature of the particle trajectories shows that the kinetic energy of the electron-positron pair is greater than the kinetic energy of the electron-positron pair that was created in the upper area of the image.

Pair production 2

1^{st} **condition** for pair production

$$\boxed{h\nu > 2m_0c^2}$$

$m_0c^2 \approx 500\,\text{keV}$ denotes the rest energy of the electron

Pair production 2 Comment

The 1st condition for pair formation is that the energy of the photon is greater than twice the rest energy of the electron.

Pair production 3

2nd condition for pair production

$$\boxed{\vec{p}_\gamma \neq 0}$$

In the center-of-momentum frame of the electron-positron pair

$$\boxed{\vec{p}_e + \vec{p}_{\bar{e}} = 0}$$

$$\boxed{\textit{A free photon can not decay into an electron positron pair!}}$$

A photon can only decay if there is a partner with whom momentum can be exchanged

Pair production 3 Comment 1

The momentum of a photon $E_\gamma = cp_\gamma$ is never zero.

The momentum of the electron positron pair is zero in the center-of-momentum frame.

Therefore the law of conservation of momentum cannot be fulfilled for the decay of a single photon as long as it moves freely through space.

Free photons cannot decay.

The decay of a photon is only possible when other particles (atoms or molecules) are nearby, so that the exchange of energy and momentum becomes possible.

In one of the two electron-positron decays, which are shown in the cloud chamber image, so much momentum is transferred to an atom that an electron is knocked out of the electron shell (upper half of the figure).

Pair production 3 Comment 2

When an electron-positron pair annihilates, two photons are created.

The momentum is conserved during this process.

Radioactive tracers (β^+-decay) are used for functional imaging in medicine.

The gamma radiation emitted during the pair annihilation is detected and forms an image analogous to X-ray tomography.

This is known as positron emission tomography (PET).

Pair production 3 Comment 3

Electrons and positrons will immediately be called true particles.

Photons, on the other hand, are the quantum particles of electromagnetic radiation.

The conversion of photons into electron-positron pairs and vice versa the conversion of electron-positron pairs into photons shows that there is no differentiation between the quantum particles of a wave and apparently true particles.

It turns out that electrons and positrons are also the quantum particles of electron waves.

These waves are sometimes called matter waves.

Matter waves

Wave-particle dualism

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- Planck’s radiation law
- Photoelectric effect
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- Compton effect
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- Matter waves
- Uncertainty relations

Matter waves 1: classical mechanics and wave mechanics

Particle wave dualism for electromagnetic waves

energy and frequency

$$\boxed{E = h\nu = \hbar\omega}$$

momentum and wavelength

$$\boxed{p = \frac{h}{\lambda} \quad \text{and} \quad \vec{p} = \hbar\vec{k}}$$

special relativity

$$\boxed{E = cp} \quad \text{and} \quad \nu = \frac{c}{\lambda}$$

Matter waves 1 Comment 1

During the 18th century, under the influence of Newton, light was considered a particle phenomenon.

In particular, the Newtonian particle image was confirmed by the aberration of the light from the stars.

In the 19th century, it was accepted that light is a wave phenomenon.

However, the aberration of the light from the stars made the theory of aether very complicated and contradictory.

By realizing that light is an electromagnetic wave and by the theory of special relativity, these problems were solved.

Matter waves 1 Comment 2

With the explanation of the photoelectric effect, Albert Einstein showed that the energy of electromagnetic waves is quantized.

Arthur Compton showed that there is also a momentum associated with the energy quanta of electromagnetic radiation, as predicted by the special theory of relativity.

The energy quanta of electromagnetic radiation are particles that we call photons.

In 1924 Louis de Broglie postulated that the wave-particle dualism is a general phenomenon of nature.

With his excellent knowledge of classical mechanics, he was able to show how the properties of waves are related to the properties of the particles.

Matter waves 1

Comment 3

The formulas summarize the wave-particle dualism of electromagnetic waves.

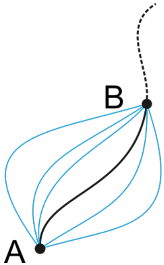
The first formula outlined in red gives Planck’s law for the energy of light quanta and the second formula outlined in red the relationship between the wavelength and the momentum of the particles.

In 1924 de Broglie showed that these formulas, which follows from the special theory of relativity in connection with electromagnetic waves, must generally apply to all types of particles.

The following slides outline the historical background.

Matter waves 2: classical mechanics and wave mechanics

Fermat 1660: Light takes a path where the transit time takes an extreme value.



Matter waves 2 Comment

Pierre de Fermat did not care about waves or particles but realized that light always takes the path that can be traversed within the shortest possible time.

Fermat was the first ever to use an extremal principle to derive a law of nature.

The figure illustrates the extremal principle.

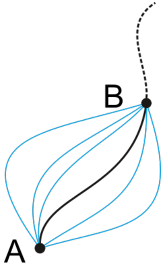
There are several possible paths between point A and B, but in nature only one path is used: the path of the shortest time.

In the 17th and 18th centuries the wave theory of Huygens (1650) and the corpuscle theory of light by Newton (1704) were discussed.

Independent of these ideas, Fermat's theory makes it possible to describe the propagation of light in the context of geometrical optics.

Matter waves 3: classical mechanics and wave mechanics

Lagrange 1788: Newtonian mechanics can be traced back to an extremal principle.



The action is extremal for the movement of a mass *m* between point A and B.

Matter waves 3

Comment

Joseph-Louis Lagrange rediscovered Fermat’s extremum principle and showed that Newton’s mechanics can also be derived from an extremal principle.

In mechanics, however, it is not the time, but the action that becomes extremal for the path of a particle between point A and B.

The action S is a new physical quantity discovered by Lagrange.

Matter waves 4: classical mechanics and wave mechanics

In 1834 showed Carl Jacobi that the theory of classical mechanics and geometrical optics are identical theories.

1924 Louis de Broglie wondered:

Since geometrical optics is the limiting case of wave optics

can it be that classical mechanics is the limiting case of a wave mechanics ?

Matter waves 4

Comment

The mathematician Carl Jacobi showed in 1834 based on the work of Fermat and Lagrange that the theory of geometric optics and classical mechanics are identical theories.

90 years later, Louis de Broglie had the seminal idea.

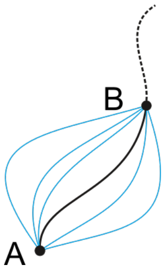
Geometric optics is clearly the limit case of wave optics when the wavelength is small compared to the dimensions of the optical devices.

Louis de Broglie wondered whether classical mechanics can be the limiting case of a wave mechanics that is valid in the microscopic world.

E.g. the quantized energies of atoms can be easily understood if the microscopic world is determined by waves.

The quantized energies of the atoms are then the result of standing waves.

Matter waves 5: classical mechanics and wave mechanics



in classical mechanics is the differential of the action is

$$\boxed{dS = \vec{p}d\vec{r} - Hdt}$$

the Hamilton function H corresponds to the energy $E = E_{kin} + E_{pot}$

but: The speed of the particle has to be replaced by the momentum $\vec{v} = \vec{p}/m$

$$\boxed{H = \frac{\vec{p}^{\,2}}{2m} + E_{pot}}$$

Matter waves 5 Comment 1

De Broglie’s theory begins with the action of mechanics.

The action is extreme for the path between point A and B.

The action S on the way between A and B changes if the end point B is moved in time dt and space $d\vec{r}$.

The first formula outlined in red shows the change in the action S.

The differential of the action is given by the momentum multiplied with $d\vec{r}$ minus the Hamilton function multiplied with dt.

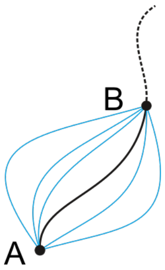
Matter waves 5 — Comment 2

The Hamilton function corresponds to the energy of the particle, e.g. the sum of the kinetic and potential energy.

The energy becomes the Hamilton function when the velocity is expressed in terms of momentum.

The second equation outlined in red gives the Hamilton function of a particle with kinetic and potential energy.

Matter waves 6: classical mechanics and wave mechanics



necessary time that the light needs for the path $d\ell$

$$dt = \frac{d\ell}{c}$$

and with $c = \omega/k$

$$dt = \frac{1}{\omega} k d\ell$$

and

$$\boxed{dt \propto \vec{k} d\vec{r}}$$

Matter waves 6

Comment

The underlined equation gives the additional time needed for light when endpoint B is shifted by $d\ell$.

If one replaces the speed of light by the quotient of angular velocity ω and the wave number k, one finds that dt is proportional to the product of wave number and the shift $d\ell$.

The equation outlined in red results when the wave vector $\vec{k}$ and the spatial displacement vector $d\vec{r}$ are used.

The product of wave vector and $d\vec{r}$ is part of the phase of a wave.

Matter waves 7: classical mechanics and wave mechanics

differential of the phase

$$d\varphi = \vec{k}d\vec{r} - \omega dt$$

differential of the action

$$dS = \vec{p}d\vec{r} - Hdt$$

de Broglie:

The phase of a matter wave is proportional to the action of the particle

Matter waves 7 Comment

The first equation gives the differential of the phase of a plane wave.

The second equation gives the differential of the action of a particle.

de Broglie concluded that the action of the particle is proportional to the phase of the corresponding matter wave.

Matter waves 8: classical mechanics and wave mechanics

With Planck's law $H = E = \hbar\omega$ one gets for the phase of a matter wave

$$\boxed{\varphi = \frac{S}{\hbar}}$$

and

$$\boxed{\vec{p} = \hbar\vec{k}}$$

de Broglie wavelength

$$\boxed{\lambda = \frac{h}{p}}$$

Matter waves 8 Comment

Using the results for the wave-particle duality of electromagnetic waves, de Broglie suggests that the phase of a matter wave is equal to the action of the particle divided by Planck's constant $\hbar$.

Since the frequency of an electromagnetic wave does not change on its way, de Broglie assumes that his result is correct at least if the energy of the particle is conserved.

The Hamilton function corresponds to the energy when the energy is conserved.

This also results in the relationship between the momentum of the particles and the wave number vector.

The wavelength of a matter wave is also known as the de Broglie wavelength.

Matter waves 9

de Broglie (1924): Wave particle duality is a common property of nature

de Broglie wavelength

$$\boxed{\lambda = \frac{h}{p}}$$

First test (Davisson and Germer) in 1927: Diffraction of electron waves on nickel

Matter waves 9 Comment

de Broglie concludes that wave particle duality is a general property of nature.

The de Broglie wavelength was first tested experimentally in 1927 by Davisson and Germer.

Davisson and Germer studied the scattering of electrons on nickel crystals.

They observed that the diffraction of electrons is similar to the diffraction of X-rays.

Louis Victor Pierre Raymond, 7th Duc de Broglie was awarded the Nobel Prize in 1929 “for his discovery of the wave nature of electrons”.

Matter waves 10: Davisson Germer Experiment

The momentum of the electrons:

When electrons are accelerated by a voltage U, the kinetic energy is

$$E_{kin} = eU$$

the total energy of the electron is

$$E = m_0 c^2 + E_{kin}$$

with $E^2 - c^2 p^2 = m_0^2 c^4$ and $E^2 = m_0^2 c^4 + 2 m_0 c^2 E_{kin} + E_{kin}^2$ is the momentum

$$\boxed{c^2 p^2 = E_{kin}^2 + 2 m_0 c^2 E_{kin}}$$

small kinetic energy approximation $E_{kin} << m_0 c^2$ (i.e. U is very much smaller than 500 kV)

$$\boxed{p = \sqrt{2 m_0 E_{kin}}}$$

Matter waves 10 Comment

The first underlined equation gives the kinetic energy of electrons that are accelerated by the voltage *U*.

For the total energy, the rest energy must be added.

The momentum can be calculated with the relativistic energy-momentum relation.

This results in the formula outlined in red.

The square of the kinetic energy can be neglected if the kinetic energy is much smaller than the rest energy.

In this case the equation on the last line results.

Matter waves 11: Davisson Germer Experiment

de Broglie wavelength of electrons ($m_0c^2 = 500$ keV)

$$\boxed{\begin{aligned}\lambda =& \frac{h}{p} = \frac{h}{\sqrt{2m_0eU}} = \frac{hc}{\sqrt{2\,m_0c^2eU}}\\ =& \frac{4.14\cdot 10^{-15}\,\mathrm{eVs}\,3\cdot 10^{8}\,\mathrm{ms}^{-1}}{\sqrt{2\cdot 500\cdot 10^{3}\,\mathrm{eV}\cdot eU}}\\ =& 12.42\cdot 10^{-10}\,\frac{\sqrt{\mathrm{V}}\,\mathrm{m}}{\sqrt{U}}\end{aligned}}$$

Comparison with X-rays:

e.g. the K_α-line of molybdenum: $\lambda = 0.71\cdot 10^{-10}$ m

the corresponding acceleration voltage for electrons is

$$U = 305\,\mathrm{V}$$

Matter waves 11

Comment

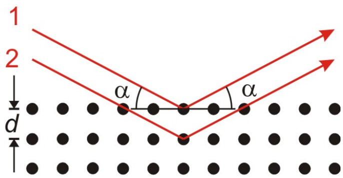
The small calculation in the box gives the de Broglie wavelength for electrons as a function of the acceleration voltage *U*.

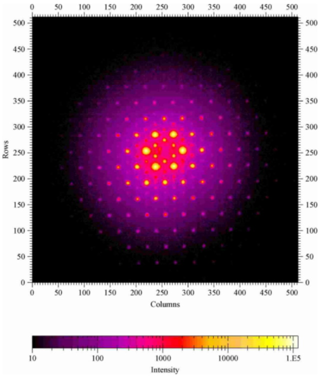
A moderate voltage of 305 V is required if the de Broglie wavelength of the electrons is to be equal to the wavelength of the X-rays of the K_α line of molybdenum.

Matter waves 12: Davisson Germer Experiment

Bragg formula

$$n\lambda = 2d \sin \alpha_n$$



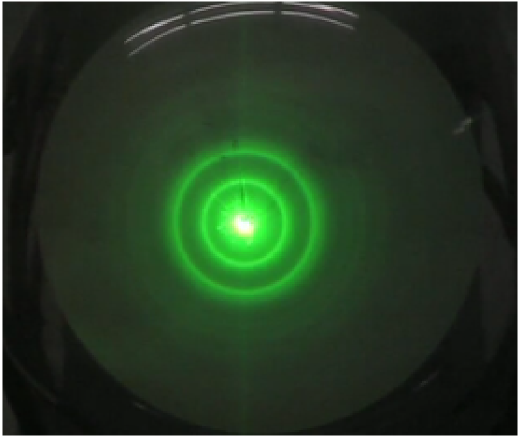


Matter waves 12

Comment

The angles of the diffraction maxima can also be calculated for electron waves using the Bragg formula for X-rays.

Matter waves 13: Davisson Germer Experiment



(elektronenbeugung.m4v)

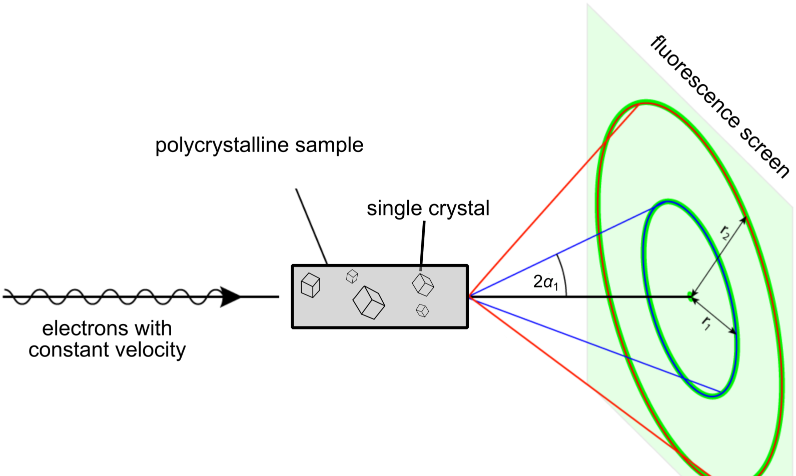
Matter waves 13

Comment

The video shows the diffraction of an electron beam on a polycrystalline sample of graphite.

Matter waves 14: Davisson Germer Experiment

diffraction on a polycrystalline sample

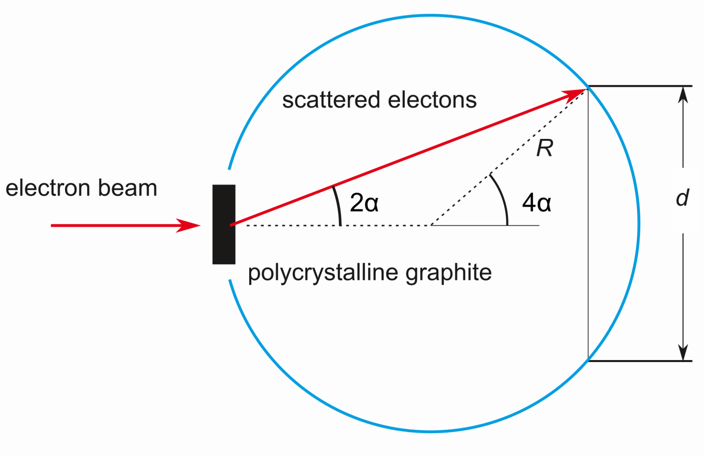


Matter waves 14 Comment

If a polycrystalline sample is used, the diffraction peaks of the single crystals merge into diffraction rings due to the random orientation of the single crystals within the sample.

This is known as powder diffraction.

Materiewellen 15: Davisson Germer Experiment



Matter waves 15 Comment

The illustration shows a sketch of the experiment.

The electron beam is formed in an electron tube that ends in a sphere.

The sample is placed at the entrance of the sphere.

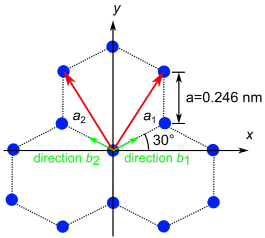
The diffracted electrons are stopped in a fluorescence layer on the inner side of the sphere.

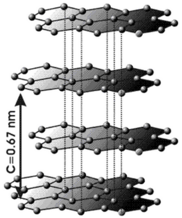
With the known radius of the sphere, the diffraction angle α can be easily calculated from the diameter of the diffraction rings.

The video shows that the radius of the rings increases as the accelerating voltage is reduced.

With Bragg's law, this result is evident since the wavelength increases as the voltage is decreased.

Matter waves 16: Davisson Germer Experiment





$$\vec{a}_1 = a\left(\frac{\sqrt{3}}{2}\vec{x} + \frac{3}{2}\vec{y}\right)$$

$$\vec{a}_2 = a\left(-\frac{\sqrt{3}}{2}\vec{x} + \frac{3}{2}\vec{y}\right)$$

$$\vec{a}_3 = c\vec{z}$$

$$\vec{b}_1 = \frac{2\pi}{a}\left(\frac{1}{\sqrt{3}}\vec{x} + \frac{1}{3}\vec{y}\right)$$

$$\vec{b}_2 = \frac{2\pi}{a}\left(-\frac{1}{\sqrt{3}}\vec{x} + \frac{1}{3}\vec{y}\right)$$

$$\vec{b}_3 = \vec{z} = \frac{2\pi}{c}\vec{z}$$

Matter waves 16 Comment

The two figures show the crystal structure of graphite.

The red vectors $\vec{a}_1$ and $\vec{a}_2$ in the hexagonal plane give the lattice vectors which, together with the vector $\vec{a}_3$ along the c-axis, span the primitive unit cell .

It is simple to calculate with the vectors $\vec{a}_{i=1,2,3}$ the basis vectors $\vec{b}_{i=1,2,3}$ of the reciprocal lattice.

$$\vec{b}_1 = \frac{2\pi}{V_{\text{Cell}}}(\vec{a}_2 \times \vec{a}_3)$$
$$\vec{b}_2 = \frac{2\pi}{V_{\text{Cell}}}(\vec{a}_3 \times \vec{a}_1)$$
$$\vec{b}_3 = \frac{2\pi}{V_{\text{Cell}}}(\vec{a}_1 \times \vec{a}_2)$$

The direction of the vectors $\vec{b}_{1,2}$ is indicated by the green arrows in the sketch of the honeycomb lattice.

Matter waves 17: Davisson Germer Experiment

distance between the lattice planes

$$\boxed{K = \frac{2\pi}{d}} \quad \rightarrow \quad d = \frac{2\pi}{K}$$

$$|\vec{b}_3| = \frac{2\pi}{c} \quad \rightarrow \quad d = \frac{2\pi}{\frac{2\pi}{c}} = c = 0.67\,\mathrm{nm}$$

$$|\vec{b}_1| = |\vec{b}_2| = |\vec{b}_1 + \vec{b}_2| = \frac{2\pi}{a}\frac{2}{3} \quad \rightarrow \quad d = \frac{2\pi}{\frac{2\pi}{a}\frac{2}{3}} = a\frac{3}{2} = 0.369\,\mathrm{nm}$$

$$|\vec{b}_1 - \vec{b}_2| = \frac{2\pi}{a}\frac{2}{\sqrt{3}} \quad \rightarrow \quad d = \frac{2\pi}{\frac{2\pi}{a}\frac{2}{\sqrt{3}}} = a\frac{\sqrt{3}}{2} = 0.213\,\mathrm{nm}$$

Matter waves 17 Comment

The length of the reciprocal lattice vectors is inversely proportional to the distance between the Bragg planes.

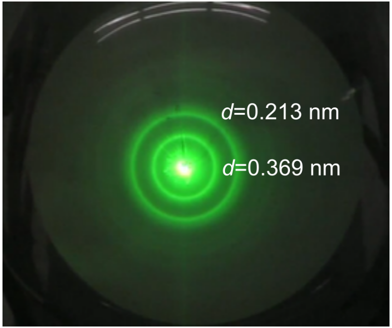
Conversely, the distance between the Bragg planes can be calculated from the length of the reciprocal lattice vectors.

If one takes the vector $\vec{b}_3$ along the z-axis, the lattice parameter c results for the distance between the Bragg planes, which is not surprising.

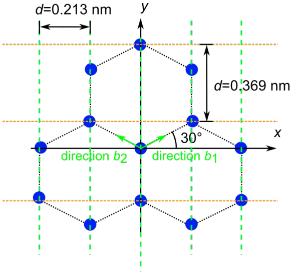
For the vectors $\vec{b}_1$, $\vec{b}_2$ and $\vec{b}_1 + \vec{b}_2$, the distance between the Bragg planes is 0.369 nm.

If one takes the difference $\vec{b}_1 - \vec{b}_2$, then the smaller distance between the Bragg planes of 0.213 nm results.

Matter waves 18: Davisson Germer Experiment



(remember Bragg's law
$n\lambda = 2d \sin \alpha_n$)



brown: lattice planes for $\vec{b}_1 + \vec{b}_2$ (d=0.369 nm)

green: lattice planes for $\vec{b}_1 - \vec{b}_2$ (d=0.213 nm)

Matter waves 18 Comment 1

The comparison with the Bragg formula shows that the diffraction angle decreases with increasing distance between the Bragg planes.

The evaluation of the experiment shows that the outer diffraction ring corresponds to the Bragg plane distance 0.213 nm and the inner ring corresponds to the Bragg plane distance 0.369 nm.

The right figure shows the orientation of some Bragg planes.

The vectors of the reciprocal lattice are always perpendicular to the Bragg planes.

The vector from the sum of $\vec{b}_1$ and $\vec{b}_2$ is oriented parallel to the y-axis and the brown-drawn Bragg planes result.

There is a second set of these Bragg planes that are shifted by 0.123 nm.

Matter waves 18 Comment 2

This second set of Bragg planes arises from the fact that there are two carbon atoms in the primitive cell of the honeycomb lattice.

The vector $\vec{b}_1 - \vec{b}_2$ is parallel to the x-axis and is perpendicular to the Bragg planes drawn in green.

The Bragg planes perpendicular to the c-axis have larger distances and result in the non-resolvable intensity around the central bright point of the non-diffracted electrons.

This demonstration experiment shows that electrons and other particles, similar to photons, can be used for diffraction experiments.

Each type of particle has its own special properties, which make it ideal for special applications. E.g. neutrons have a magnetic moment and are therefore particularly suitable for the exploration of magnetic structures.

Revision

Summary in questions 1

1. What is the Compton Effect?
2. Write down the law of conservation of energy and momentum for the Compton effect.
3. Give the relationship between the energy of the incident photon and the scattered photon.
4. Give the relationship between the wavelength of the incident photon and the scattered photon.
5. Give the order of magnitude of the Compton wavelength.
6. Sketch the Compton spectrum of a nuclear γ decay.
7. What process leads to the Compton edge?

Summary in questions 2

8. Give the energy of the Compton edge.
9. What is pair production?
10. Give the minimum energy of a photon for pair production.
11. Explain why a free photon is a stable particle.
12. Give the formula for the de Broglie wavelength.
13. Calculate the momentum of an electron that is accelerated with the voltage U.
14. What does the collapse of the wave function mean?