Karlsruher Institut für Technologie

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Computerpraktikum zur Vorlesung
Moderne Methoden der Datenanalyse - Blatt 4

Exercise 4: Combination of Correlated Measurements

A common problem in science is the combination of several measurements to one single result, e.g., the average value. Not only the uncertainties of the individual measurements have to be taken into account, but also the correlations between them. A wrong treatment of correlations or common systematic effects can lead to biased results.

- **Exercise 4.1: Combination of W mass measurements** **voluntary**

  At the LEP accelerator at CERN the mass of the W boson m_W was measured in two different channels:

  $$\begin{aligned} e^+e^- &\rightarrow W^+W^- \rightarrow q_1\, q_2\, q_3\, q_4 \\ e^+e^- &\rightarrow W^+W^- \rightarrow \ell\, \nu\, q_1\, q_2 \end{aligned}$$

  The experimental signature in the detector for the first channel with four quarks q are four reconstructed jets. The second channel is identified by a lepton ℓ (electron or muon) and two jets. The neutrino is not detected. The measured W masses and the uncertainties are:

  $$\begin{aligned} \text{4 jets channel:} \quad m_W &= (80457 \pm 30 \pm 11 \pm 47 \pm 17 \pm 17)\ \text{MeV} \\ \text{lepton + 2 jets channel:} \quad m_W &= (80448 \pm 33 \pm 12 \pm \ \ 0 \pm 19 \pm 17)\ \text{MeV} \end{aligned}$$

  To facilitate the interpretation of the results, different uncertainties are given, originating from different sources: The first two uncertainties are the statistical and systematic experimental uncertainties, which are uncorrelated. The third uncertainty originates from the theory applied for the analysis and is only present in the first channel. The fourth uncertainty comes from a common theoretical model applied for both channels, and thus is 100 % correlated. Also the last uncertainty is 100 % correlated between both measurements, since it represents the uncertainty on the LEP accelerator beam energy.

  - Construct a covariance matrix of the two W mass measurements taking into account all uncertainties and their correlations. Use this covariance matrix to define a χ^2 expression containing the average W mass $\bar{m}_W$ as a free parameter. Determine $\bar{m}_W$ and its uncertainty by minimizing the χ^2 expression e.g. with the help of the `iminuit` python package.

    For this exercise, you have to write your own χ^2-function to be minimized. Take a look at the previous exercise sheet to learn how this can be done.

- Because the minimization of the χ^2 expression in exercise 4.1 is a linear problem it can be solved analytically. Determine $\bar{m}_W$ and its error analytically and compare them to the result from above.
- Estimate the contributions from statistical, systematic, theoretical, and accelerator based uncertainties to the error of the combined W mass measurement. Use the quadratic difference between the total error and the error calculated with a covariance matrix where one component is removed.

- **Exercise 4.2: Normalisation uncertainty** **obligatory**

  Two measurements $y_1 = 8.0$ and $y_2 = 8.5$ of the same physical quantity with an uncorrelated relative statistical error of 2 % and a common normalisation error of 10 % should be combined.

  - Construct a covariance matrix and a χ^2 expression and determine its minimum with `iminuit` or analytically.

    Is the result reasonable? What could be the cause for the unexpected value? Make a plot of the covariance ellipse in the $y_1' y_2'$ plane defined by

    $$\Delta y^T V^{-1} \Delta y = c^2, \quad \Delta y = \begin{pmatrix} y_1 - y_1' \\ y_2 - y_2' \end{pmatrix}$$

    for $c = 1$ and $c = 2$ together with the line $y_1' = y_2'$. V is the covariance matrix. To draw the ellipse a `TGraph` object can be used. The points on the ellipse can be calculated as a function of the angle ϕ if Δy is expressed by ϕ and the radius r. You can use the function given in the Jupyter notebook to draw the ellipse. Pay attention to where the bisector intersects with the ellipse.

  - Use an additional normalisation parameter N for the treatment of the common normalisation uncertainty instead of taking it into account in the covariance matrix of y_1 and y_2. Add a term to the χ^2 expression for the normalisation with an expected value of 1 and an error of 10 %. The normalisation factor N can be applied either to the measured values y_i or to the fit parameter $\bar{y}$. Try out both ways (using `iminuit` for the χ^2 minimisation) and compare the results. Which one is the more meaningful result and why?

    Determine $\bar{y}$ from the correct χ^2 expression in an analytical way. How does the normalisation error affect the averaged value and its error?

    Construct a covariance matrix of y_1 and y_2 containing the normalisation uncertainty of 10 % relative to the average value $\bar{y}$. Solve the corresponding χ^2 minimisation with `iminuit` and plot the covariance ellipse.