Priv.-Doz. Dr. A. Naber
Exercises to the lecture Nano-Optics, WS 2022/23

Exercises (VI)
(Discussion is on Friday, 13.1.2023)

Problem 1:

Using an optical microscope (lens L, focal distance f), a point-like light source P in object plane O is imaged to a point P' in image plane I. Simultaneously a similar light source Q with distance s to P is imaged to Q' with distance s' to P' (see drawing). The vacuum emission wavelength of P and Q is λ, object plane O and lens L are embedded in a material with index of refraction n, and focal plane F and image plane I are embedded in a material with index of refraction n'. Show that in the case of diffraction limited resolution (Rayleigh criterion) for incoherent light emission of P and Q their distance $s_{\min}$ is given by

$$s_{\min} = 0.61 \, \frac{\lambda_0}{n \, \sin\varphi} \ ,$$

with the maximum angle φ at which light from an object can pass the aperture of radius a'. In deriving this relation you can assume that $f \ll d'$, $s' \ll d'$, and that the lens fulfills Abbe's sine condition for a best possible imaging process.

Note: The first minimum in the Airy diffraction pattern of the aperture with radius a' is given by the angle $\vartheta = 0.61 \, \lambda_0 / n' a'$.