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Exercises to the lecture Nano-Optics, WS 2022/23

Exercises (VII)
(Discussion is on Friday, 20.1.2023)

Problem 1:

A tiny circular aperture of radius a in an infinitely thin, ideally conducting metal film located in the xy-plane at $z=0$ is irradiated in normal direction by a plane, linearly x-polarized electromagnetic wave with spatial frequency $k=2\pi/\lambda$. The center of the aperture is located at $x=y=z=0$ and its radius a is small compared to the wavelength λ of the wave, $a \ll \lambda$. Then behind the aperture the electric field E_x on the z-axis for $x=y=0$ is given for $z \ll a$ by the expression

$$\frac{E_x}{E_0} = -i\,\frac{2ka}{\pi} \cdot \left(1 - \frac{\pi}{2}\frac{z}{a} + \frac{z}{a}\,\arctan\frac{z}{a} + \frac{1}{3}\,\frac{1}{1+\left(\frac{z}{a}\right)^2}\right) .$$

Calculate the Taylor series of the function $E_x(\frac{z}{a})$ up to the second power of $\frac{z}{a}$ and show that the result is similar (but not equal) to the corresponding Taylor expansion of an exponential function. Show that the intensity of the field $I(z)$ can therefore be approximately represented by

$$\frac{I(z)}{I_0} = \left(\frac{8ka}{3\pi}\right)^2 \cdot \exp\left(-\frac{3\pi}{4}\frac{z}{a}\right) .$$

Finally plot the intensity of the exact expression as well as the approximate solution for $ka = 0.3$ and $0 \le \frac{z}{a} \le 2$ into the same graph and discuss the differences.