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Exercises to the lecture Nano-Optics, WS 2023/24

Exercises (I)
(Discussion is on Friday, 1.12.2023)

Problem 1:

Derive the continuity relations for the normal and tangential components of the fields $\vec{E}$ und $\vec{D}$ at the plane interface between two isotropic dielectric materials with different dielectric permittivities ϵ_1 and ϵ_2.

Problem 2:

For normal incidence the reflection coefficient r of light at the interface between two media is normally given by

$$r = \frac{E_r}{E_i} = \frac{n_i - n_t}{n_i + n_t}$$

with incident electric field E_i, reflected electric field E_r, and indices of refraction n_i and n_t. For its derivation, it is assumed that the permeability of both media is $\mu \simeq 1$ so that $n = \sqrt{\epsilon\mu} \simeq \sqrt{\epsilon}$, which is an excellent approximation for the vast majority of materials.

We drop this common assumption and study here the reflection and transmission properties of a plane electromagnetic wave in vacuum impinging in normal direction on an unusual material with permeability $\mu_t = -1$ and permittivity $\epsilon_t = -1$. At first, derive the reflection coefficient r as a function of the impedances Z_i and Z_t using the continuity relations ($Z = E/H = \sqrt{\mu_0\mu/\epsilon_0\epsilon}$). Then calculate r for the given numbers of μ_t and ϵ_t. Finally, determine the directions of the wave vector $\vec{k}_t$ and the poynting vector $\vec{S}_t = \vec{E}_t \times \vec{H}_t$ of the transmitted wave.