Priv.-Doz. Dr. A. Naber
Exercises to the lecture Nano-Optics, WS 2023/24

Exercises (III)
(Discussion is on Friday, 22.12.2023)

Problem 1:

For a wavelength $\lambda_0 = 633\,\mathrm{nm}$ (vacuum) the optical properties of silver can be described very well using a complex index of refraction $\tilde{n} = 0.06 + \mathrm{i}\,4.15$. Herein the imaginary part describes the absorption of light within the material. The reflection and absorption properties of light striking a silver interface at normal incidence shall be calculated.

For this, generalize first the formula for the reflection R of light (normal incidence) at the interface of vacuum to a dielectric material,

$$R = \left(\frac{n-1}{n+1}\right)^2 ,$$

to materials with a complex index of refraction $\tilde{n}$. Determine the penetration depth d of the light intensity I using the electric field $\vec{E}(\vec{r},t) = E_0 \exp[i(\vec{k}\vec{r} - \omega t)]$. The penetration depth is defined as the depth at which the original intensity I_0 has dropped to I_0/e (with Euler's number e).

Problem 2:

In the lecture we derived the dispersion relation $\omega(k_x)$ of surface plasmon polaritons (SPPs) and found two branches in the corresponding graph. The lower branch ($\omega < \omega_{\mathrm{sp}}$) represents the evanescent waves propagating along the surface of the interface between metal and dielectric media. But what is the meaning of the upper branch ($\omega > \omega_{\mathrm{p}}$) in the dispersion relation? Show that the dispersion relation in this case leads to the expression

$$\tan\alpha = \frac{n_1}{n_2}$$

in which α represents the angle at which the wave-vector $\vec{k}_2$ of the propagating plane wave in the dielectric media impinges on the interface. Discuss this result.