

### Exercises (IV)

(Discussion is on Friday, 12.1.2024)

#### **Problem 1:**

We discussed the solution of Surface-Plasmon-Polaritons (SPPs) in the case of a free electron gas as a simple model for a metal. Another model often used in (nano-) optics is an ideally conducting medium (ICM), i.e., a metal with vanishing electrical resistance and thus infinite conductivity,  $\sigma \rightarrow \infty$ . Find first a relation of the conductivity  $\sigma$  to the permittivity  $\epsilon$  at optical frequencies and then try to find a solution of SPPs at the air-ICM interface using the dispersion relation derived in the lecture.

*Hint:* Use Maxwell's equation  $\text{curl } \vec{H} = \vec{j} + \dot{\vec{D}}$  and the harmonic Ansatz  $\vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r} - \omega t)}$  to identify  $\sigma/\epsilon_0\omega$  as the imaginary part of the complex permittivity  $\epsilon$  of the metal.

#### **Problem 2:**

A plane electromagnetic wave of angular frequency  $\omega$  in vacuum is reflected back at normal incidence from the interface to a metal with given conductivity  $\sigma$ . Determine the reflection coefficient  $R$  and the penetration depth  $d$  of the wave into the metal. Assume that  $\sigma$  is a real quantity and that  $\sigma/\omega \gg \epsilon_0\epsilon'$  in which  $\epsilon'$  is the real part of the permittivity  $\epsilon$ .