1.2 *Surface waves*

Light Propagation in Metals

Model : Electron gas

Parameter : Charge density $\rho = f \cdot (-e)$

with number density f

$$E = E_x = E_0 \cdot e^{ikz} \cdot e^{-i\omega t}$$

$$k = n(\omega)\frac{2\pi}{\lambda_0} \quad \text{with} \quad n(\omega) = \sqrt{\varepsilon(\omega)}$$

ε > 0 : Propagating wave
ε < 0 : Damped or evanescent wave

Dielectric Function of a Free-Electron-Gas

Interaction of an electron with the electric field *E* in the electron gas:

$$m\,\ddot{x}(t) = -e\,E(t) \quad ; \quad E(t) = E_0 \exp(-i\omega t)$$

Ansatz: $x(t) = x_0 \exp(-i\omega t)$

$$-\omega^2 m\, x(t) = -e\,E(t) \qquad x(t) = \frac{e}{m\omega^2} E(t)$$ (phase shift of 180°!)

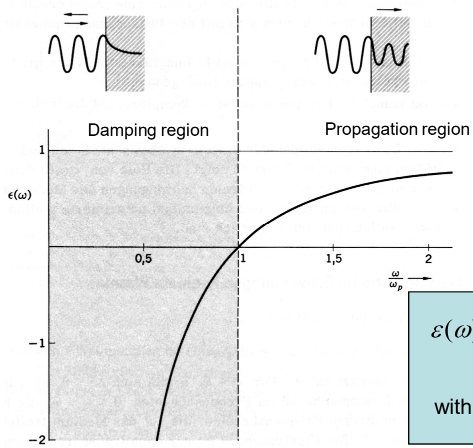
With „dipole moment" $p = -e\,x$ and number density f of the electrons, it follows

$$P(t) = f \cdot p(t) = -\frac{f\,e^2}{m\omega^2} E(t) \overset{!}{=} \varepsilon_0\,\chi_e(\omega)\,E(t)$$

$$\Rightarrow \quad \varepsilon(\omega) = 1 + \chi_e(\omega) = 1 - \frac{f\,e^2}{\varepsilon_0 m\omega^2}$$

or with $\omega_p := \sqrt{\frac{f\,e^2}{\varepsilon_0\,m}}$ $\qquad \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$

Plasma Optics



$$\varepsilon(\omega) = 1 - \frac{\omega_P^2}{\omega^2}$$

with $\omega_P = \sqrt{\frac{f\,e^2}{\varepsilon_0\,m}}$

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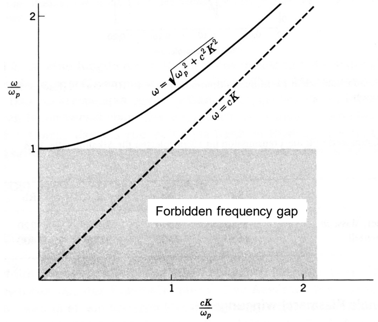
Propagation of Light in Metal (Plasma)

Dispersion relation of electromagnetic waves in the electron gas:

$$\omega = \frac{c_0}{n} k \quad \Rightarrow \quad \varepsilon(\omega)\,\omega^2 = c_0^2 k^2 \quad \Rightarrow \quad \boxed{\omega = \sqrt{\omega_p^2 + c_0^2 k^2}}$$



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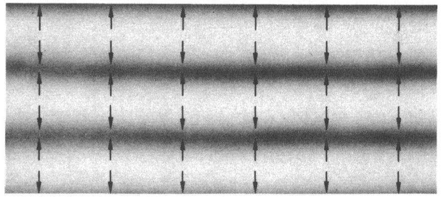
Volume plasmon

Longitudinal resonances of the plasma:



Charge density oscillation. The arrows indicate the displacement of charge.

Resonance: $D = \varepsilon_0 E + P = \varepsilon_0\, \varepsilon(\omega_{res})\, E = 0$

$$\varepsilon(\omega_{res}) = 0 \quad \Rightarrow \quad \omega_{res} = \omega_P$$

C. Kittel, solid state physics