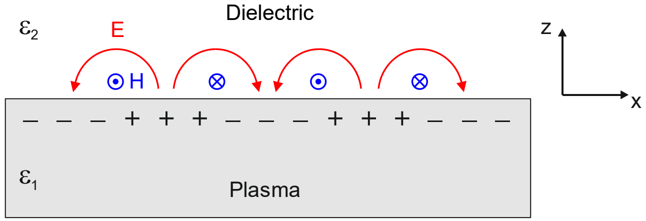
Surface Plasmons

(SP - surface plasmon; SPP - surface plasmon polariton)



z < 0:

$$\vec{E}_1 = \left(E_{x1}, 0, E_{z1}\right)\cdot e^{i(k_{x1}x - k_{z1}z - \omega t)}$$
$$\vec{H}_1 = \left(0, H_{y1}, 0\right)\cdot e^{i(k_{x1}x - k_{z1}z - \omega t)}$$

z > 0:

$$\vec{E}_2 = \left(E_{x2}, 0, E_{z2}\right)\cdot e^{i(k_{x2}x + k_{z2}z - \omega t)}$$
$$\vec{H}_2 = \left(0, H_{y2}, 0\right)\cdot e^{i(k_{x2}x + k_{z2}z - \omega t)}$$

SPP – dispersion relation

Boundary conditions:

$$\left.\begin{aligned} E_{x1} &= E_{x2} \quad (= E_x) \\ H_{y1} &= H_{y2} \quad (= H_y) \end{aligned}\right\} \Rightarrow \; k_{x1} = k_{x2} \quad (= k_x)$$

Maxwell equation:

$$\text{curl}\,\vec{H} = \dot{\vec{D}} = \epsilon_0 \epsilon \dot{\vec{E}} \quad \Rightarrow \quad -\frac{\partial H_y}{\partial z} = \epsilon_0 \epsilon \dot{E}_x$$

$$\text{curl}\,\vec{H} = \begin{pmatrix} \partial_y H_z - \partial_z H_y \\ \partial_z H_x - \partial_x H_z \\ \partial_x H_y - \partial_y H_x \end{pmatrix}$$

$$\left.\begin{aligned} &\underline{z>0:} \quad -i\,k_{z2}\,H_y = -i\omega\,\epsilon_2\,\epsilon_0\,E_x \\ &\underline{z<0:} \quad +i\,k_{z1} H_y = -i\omega\,\epsilon_1\,\epsilon_0\,E_x \end{aligned}\right\} \Rightarrow \quad -\frac{k_{z2}}{k_{z1}} = \frac{\epsilon_2}{\epsilon_1} \qquad (1)$$

SPP – dispersion relation

Dispersion relation:

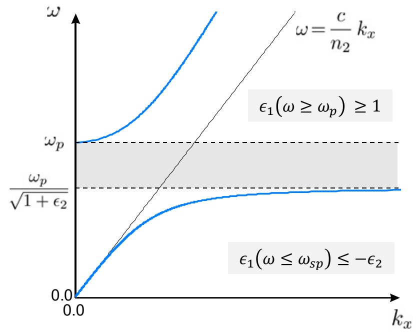
$$\omega = \frac{c}{n}\cdot k \qquad \Rightarrow \qquad k^2 = \epsilon \cdot \left(\frac{\omega}{c}\right)^2 \qquad (2)$$

$$\left.\begin{aligned} &\underline{z>0:} \quad k_x^2 + k_{z2}^2 = \epsilon_2 \left(\frac{\omega}{c}\right)^2 \Rightarrow k_{z2}^2 = \epsilon_2 \left(\frac{\omega}{c}\right)^2 - k_x^2 \\ &\underline{z<0:} \quad k_x^2 + k_{z1}^2 = \epsilon_1 \left(\frac{\omega}{c}\right)^2 \Rightarrow k_{z1}^2 = \epsilon_1 \left(\frac{\omega}{c}\right)^2 - k_x^2 \end{aligned}\right\} \overset{(1)}{\Rightarrow} \quad \frac{\epsilon_2^2}{\epsilon_1^2} = \frac{\epsilon_2 \left(\frac{\omega}{c}\right)^2 - k_x^2}{\epsilon_1 \left(\frac{\omega}{c}\right)^2 - k_x^2}$$

Result:

$$k_x(\omega) = \sqrt{\frac{\epsilon_1 \cdot \epsilon_2}{\epsilon_1 + \epsilon_2}} \cdot \frac{\omega}{c}$$

SPPs at a Dielectric-Metal Interface



$$k_x(\omega) = \sqrt{\frac{\epsilon_1 \cdot \epsilon_2}{\epsilon_1 + \epsilon_2}} \cdot \frac{\omega}{c}$$

Dielectric-Metal Interface:

$$\epsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\epsilon_2 \geq 1; \quad n_2 = \sqrt{\epsilon_2}$$

SPP – decay length

If k_{zi} is the z-component of the wavevector in media i with permittivity ϵ_i, then:

$$k = \frac{\omega}{n} \quad \Longrightarrow \quad k_i^2 = \epsilon_i \cdot \left(\frac{\omega}{c}\right)^2 = k_x^2 + k_{zi}^2 \quad \Longrightarrow \quad k_{zi} = \frac{\epsilon_i}{\sqrt{\epsilon_1 + \epsilon_2}} \cdot \frac{\omega}{c}$$

Media 2 is a dielectric and media 1 is a metal. Then for the upper branch of the dispersion relation, we get:

$$\omega > \omega_p \quad \Longrightarrow \quad \epsilon_1(\omega) > 0 \quad \Longrightarrow \quad k_{z1} \text{ and } k_{z2} \text{ are real (propagating waves)}$$

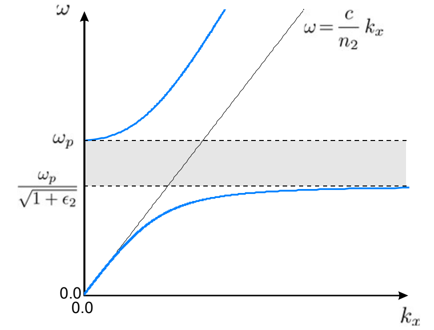
For the lower branch of the dispersion relation, we have:

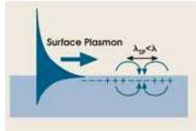
$$\omega < \frac{\omega_p}{\sqrt{1+\epsilon_2}} \quad \Longrightarrow \quad \epsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega^2} < -\epsilon_2 \quad \Longrightarrow \quad k_{z1} \text{ and } k_{z2} \text{ are imaginary!}$$

The decay lengths of the evanescent wave are: $d_i = \frac{1}{|k_{zi}|} = \frac{\lambda}{2\pi}\sqrt{\left|\frac{\epsilon_1 + \epsilon_2}{\epsilon_i^2}\right|}$

Silver @ 600 nm: $d_1 = 24\text{ nm}$ $\quad d_2 = 390\text{ nm}$

Dispersion relation of SPPs

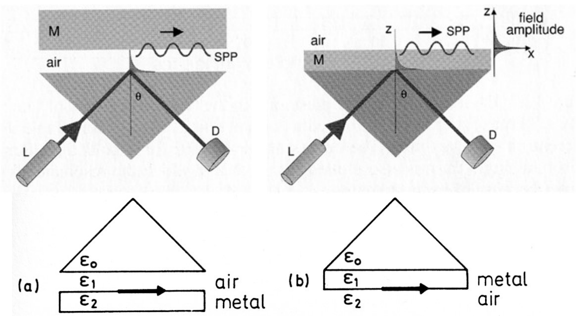




$$k_x(\omega) = \sqrt{\frac{\epsilon_1 \cdot \epsilon_2}{\epsilon_1 + \epsilon_2}} \cdot \frac{\omega}{c} \quad ; \quad k_{zi} = \frac{\epsilon_i}{\sqrt{\epsilon_1 + \epsilon_2}} \cdot \frac{\omega}{c}$$

Excitation of SPPs by ATR

(ATR = attenuated total reflection)

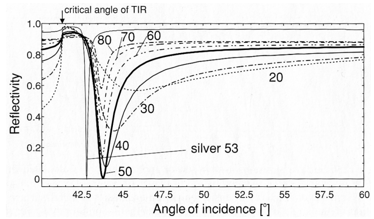


Otto-configuration

Kretschmann-configuration

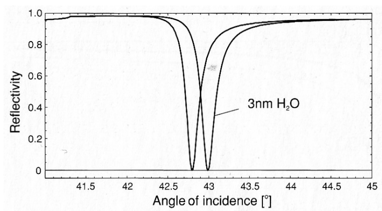
H. Raether, Surface Plasmons; Novotny & Hecht, Nano-Optics

SPP Excitation of Gold and Silver films



SPP excitation of gold and silver in Kretschmann-configuration.

(film thicknesses in nm)



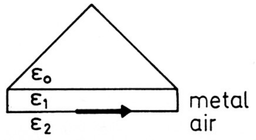
Shift of SPP resonance induced by a 3 nm layer of water on a 53 nm silver film (calculation).

Novotny & Hecht, Nano-Optics

Field amplification

$$T^{el}_{\max} = \frac{\left|E\langle 2/1\rangle\right|^2}{\left|E\langle 0/1\rangle\right|^2} = \frac{1}{\varepsilon_2}\,\frac{2\left|\varepsilon_1'\right|^2}{\varepsilon_1''}\,\frac{\sqrt{\left|\varepsilon_1'\right|(\varepsilon_0-1)-\varepsilon_0}}{1+\left|\varepsilon_1'\right|}$$

with $\varepsilon_1 = \varepsilon_1' + i\,\varepsilon_1''$



Examples:

Ag (600 nm) : T=200
Au (600 nm) : T=30
Al (600 nm) : T=40

H. Raether, Surface Plasmons

1 Theoretical aspects

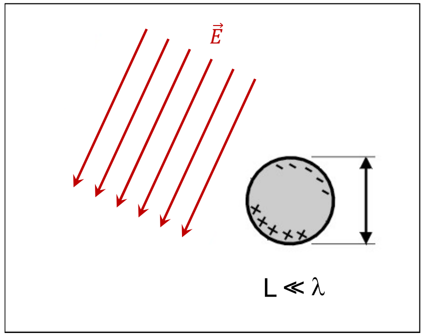
⋮

1.3 Light scattering and emission by particles

1.3.1 Localized Plasmons – Mie scattering
1.3.1 Far- and near-field of a radiating (point-) dipole
1.3.2 Fluorescence of molecules

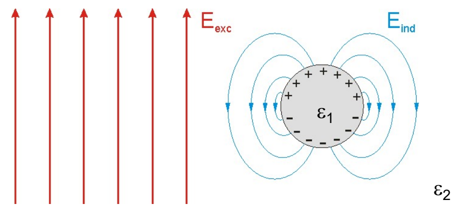
Localized plasmons

Metallic nanoparticle



Localized Plasmon

Induced electric dipole in a spherical metal particle with $\varepsilon_1(\omega) = \varepsilon_1' + \mathrm{i}\,\varepsilon_1''$



$$\vec{p}_{ind} = \alpha\cdot\vec{E}_{exc}\,,\qquad \alpha = 4\pi\varepsilon_0\cdot R^3\cdot\frac{\varepsilon_1(\omega)-\varepsilon_2}{\varepsilon_1(\omega)+2\,\varepsilon_2}$$

Resonance at $\varepsilon_1' \cong -2\varepsilon_2$ with

$$T_{el}^{\max} = \left|\frac{E_{ind}}{E_{exc}}\right|^2 \cong \left|\frac{3\varepsilon_2}{\varepsilon_1''}\right|^2$$ (= 480 for Ag in vacuum)