Emission Enhancement: Purcell-Effect

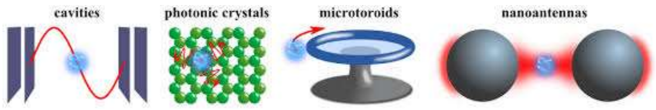
By increasing the density of final states ρ for the photons emitted from a molecule their spontaneous life time τ will decrease.

Example

Free space: $\rho_f = \frac{2}{\pi}\frac{n^3\omega^2}{c^3}$

Cavity: $\rho_c = \frac{2\pi}{V}\frac{Q}{\omega}$

$$\Rightarrow \quad \frac{\rho_c}{\rho_f} = \frac{\pi^2}{V}\frac{c^3 Q}{\omega^3}$$



Radiation of a Black Body at Temperature T

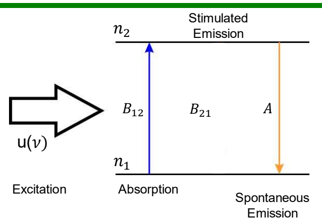
Knowing the density of states ρ_f the spectral energy density $u(\nu,T)$ of blackbody radiation can be immediately derived:

$\rho_f(\nu) = \frac{8\pi\,\nu^2}{c^3}$ — density of propagation modes in vacuum

$\langle n\rangle = \frac{1}{e^{h\nu/kT}-1}$ — averaged excitational state $\langle n\rangle$ of a quantum oscillator at temperature T

$$\Rightarrow \quad u(\nu,T) = \rho_f(\nu)\cdot\langle n\rangle\, h\nu \;=\; \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

Stimulated Emission



$B_{12} = B_{21} = B$

for non-degenerate states

$$\dot{n}_2 = B\,u(\nu)\,n_1 - B\,u(\nu)\,n_2 - A\,n_2$$

Example: $n_1(0) = 0,\ n_2(0) = 1 \quad \Rightarrow \quad n_2(t) \sim e^{-(B\,u+A)\,t}$!

Steady state solution: $\dot{n}_2 = 0 \quad \Rightarrow \quad \frac{n_2}{n_1} = \frac{B\,u(\nu)}{B\,u(\nu)+A}$

Assume $u(\nu,T)$ is a blackbody radiation with temperature T and the molecule is in balance with the radiation.

$$\frac{n_2}{n_1} = e^{-h\nu/kT} \quad \Rightarrow \quad u(\nu,T) = \frac{A}{B}\frac{1}{e^{h\nu/kT}-1} \overset{!}{=} \frac{8\pi\,\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

The coefficient *A* (spontaneous emission) depends on the density of states!