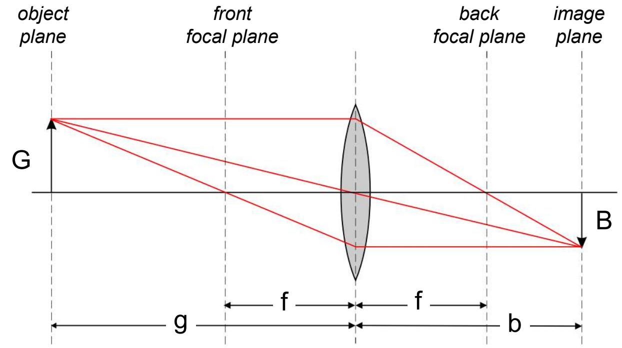
2 Classical optics and microscopy

2.1 Microscopic imaging

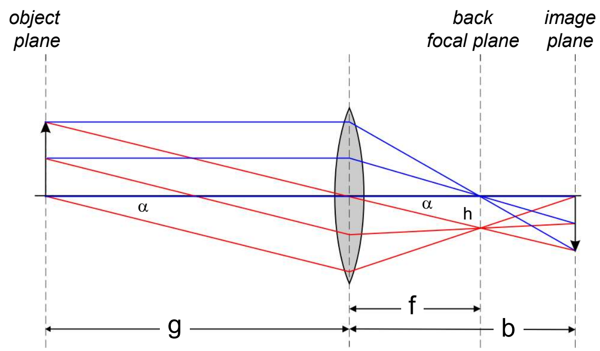
2.1.1 Geometrical optics
2.1.2 Primary aberrations and Abbe's sine condition
2.1.3 Resolving power and criteria
2.1.4 Image formation
2.1.5 Fourier optics

Geometrical optics - paraxial approximation



lens equation (thin lens) $$\frac{1}{f} = \frac{1}{g} + \frac{1}{b}$$

Geometrical optics – paraxial approximation



$$h = f \cdot \tan\alpha \approx f \cdot \alpha$$

Geometrical Optics – Abberations

Paraxial approximation: $\sin\varphi \cong \varphi$

Exact Taylor series: $\sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \ldots$

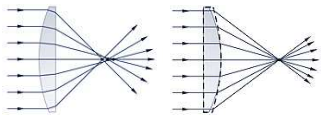
„abberations of third order“

Primary (monochromatic) abberations

- Spherical abberation
- Astigmatism
- Coma
- Field curvature
- Field distortion

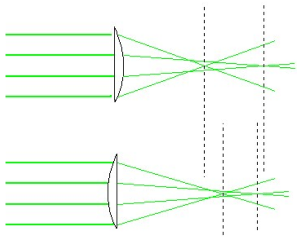
Spherical Abberation

For monochromatic light the abaxial rays are refracted stronger then the paraxial rays.


spherical   Asphericallly corrected

Workaround

- Suppression of abaxial rays using a small aperture close to the lens („aperture stop“ or „stop“).
- Lens shape, e.g., plano-convex lenses for incident collimated pencil of rays.
- Corrected lens systems
- Aspherical lenses



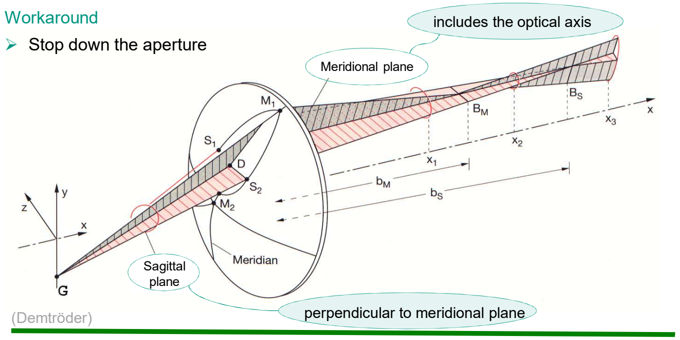
Astigmatism („absence of points“)

Astigmatism of oblique „pencils of rays“

- Occurs for objects remote from the symmetry axis.
- The image of a point produces two orthogonal lines at different positions, inbetween is a blurred area.

Workaround

- Stop down the aperture



(Demtröder)

Coma

- Magnification depends on angle of rays with reference to the optical axis.
- Arises if object is remote from the optical axis.

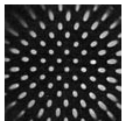
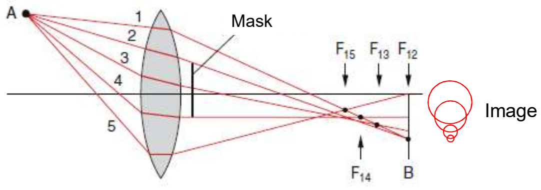


Image of a holey plate with coma

Workaround

- Stop down aperture
- Special lens system → *Aplanat*



(Demtröder)

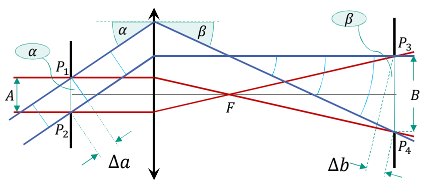
Wide angle rays - Abbe‘s Sine Condition

In case of ideal point imaging all contributing rays must have the same optical path (distance of wavefronts). Thus we get

$$P_1P_4 = P_2P_3 \Rightarrow \overline{P_1P_4} = \overline{P_2P_3} \Rightarrow \Delta a = \Delta b$$

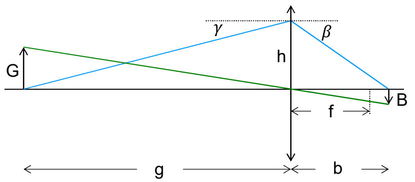
$$\left.\begin{aligned} \frac{\Delta a}{A} &= \sin\alpha \\ \frac{\Delta b}{B} &= \sin\beta \end{aligned}\right\} \Rightarrow \frac{|B|}{|A|} = \frac{\sin\alpha}{\sin\beta}$$

Sine condition



Abbe's Sine Condition

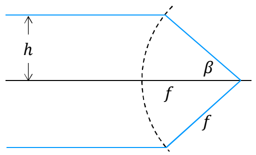
$$\frac{G}{B} = \frac{\sin\beta}{\sin\gamma}$$



The sine condition can be simplified in case of a very large object distance $g \to \infty$:

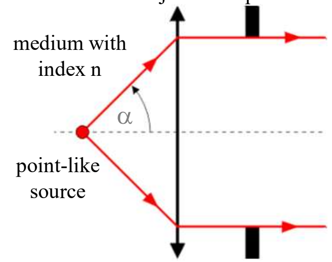
$$\frac{h}{g} \cong \sin\gamma \quad ; \quad \frac{G}{B} = \frac{g}{f} \cong \frac{h}{f\sin\gamma}$$

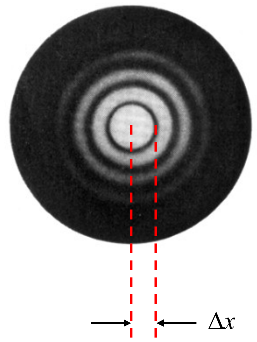
$$\frac{\sin\beta}{\sin\gamma} = \frac{G}{B} = \frac{h}{f\sin\gamma} \Rightarrow \boxed{\frac{h}{f} = \sin\beta}$$



(intuitive scheme)

Resolving power

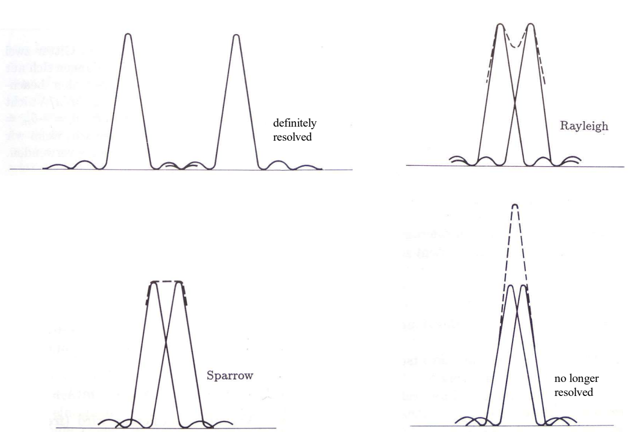




$$\Delta x = 0.61\frac{\lambda}{NA} \quad \text{mit} \quad NA = n\sin\alpha$$

Born & Wolf, Principles of Optics (Pergamon Press 1993)

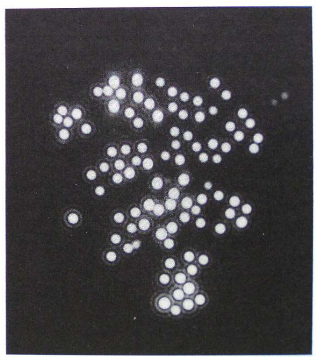
Resolution criteria



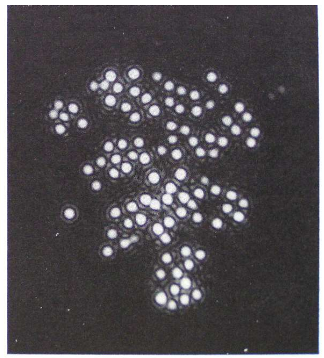
E. Hecht, Optik

Illumination

Incoherent illumination

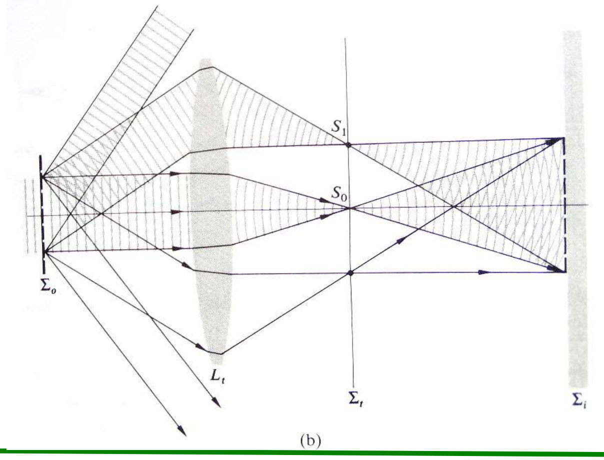


Coherent illumination



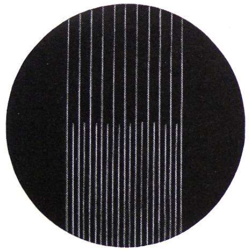
E. Hecht, Optik

Image formation according to Abbe

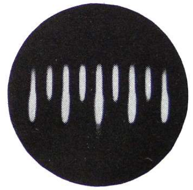


E. Hecht, Optik

Image formation according to Abbe



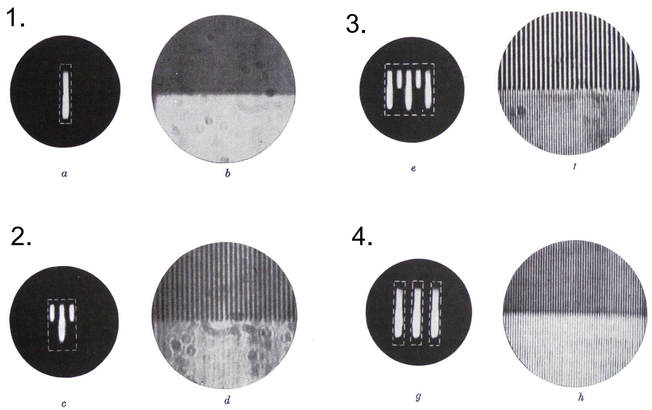
Abbe's diffraction plate



diffraction image (focal plane)

Bergmann-Schäfer, Optik

Image formation according to Abbe



Bergmann-Schäfer, Optik

Diffraction at double slit

Formula for 1. maxima (normal light incidence):

$$\lambda = d \sin\varphi \quad \Rightarrow \quad d = \frac{\lambda}{\sin\varphi}$$

Exciting the slits with a phase difference of π (oblique light incidence) results in 0. and 1. maxima for:

$$\frac{\lambda}{2} = d \sin\varphi \quad \Rightarrow \quad d = \frac{\lambda}{2\sin\varphi}$$

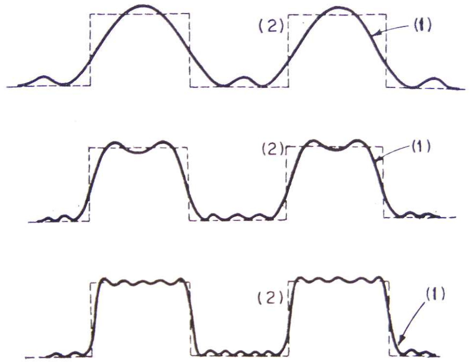
Diffraction at double slit

Formula fo

Exciting the ... dence) results
in 0. and 1. ...

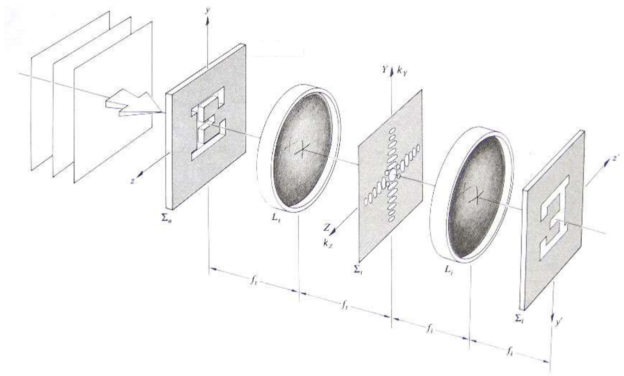
$$\frac{\lambda}{2} = d \sin\varphi \quad \Rightarrow \quad d = \frac{\lambda}{2 \sin\varphi}$$

Image formation according to Abbe



H. Beyer, Interferenzmikroskopie

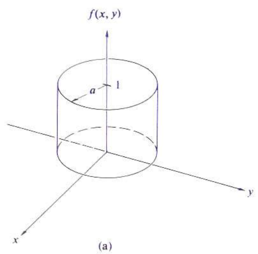
Fourier optics



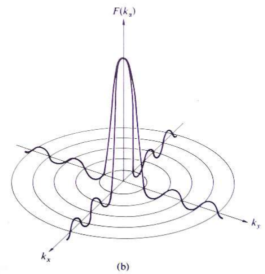
E. Hecht, Optik

Example: Circular Aperture

cylinder function



Fourier transform

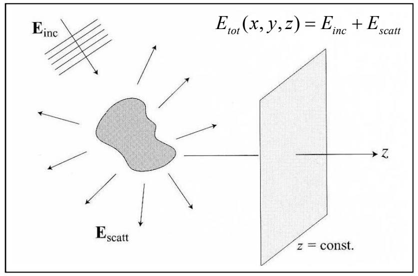


$$f(x,y) = \begin{cases} 1 & \sqrt{x^2+y^2} \le a \\ 0 & \sqrt{x^2+y^2} > a \end{cases}$$

$$F(k_x,k_y) = \int\int_{-\infty}^{+\infty} f(x,y)\, e^{i(k_x x + k_y y)} dx\, dy$$

E. Hecht, Optik

Angular Spectrum Representation of Optical Fields



Consider the field *E(x,y,z)* in a plane *z* = *const* transverse to an arbitrary axis z.

The 2D-Fourier transform Ê of E is then given by:

$$\hat{E}(k_x,k_y;z) = \frac{1}{4\pi^2}\iint E(x,y,z)\exp(-\mathrm{i}[k_x x + k_y y])\,\mathrm{d}x\,\mathrm{d}y$$

so that

$$E(x,y,z) = \iint \hat{E}(k_x,k_y;z)\,\exp(\mathrm{i}[k_x x + k_y y])\,\mathrm{d}k_x\mathrm{d}k_y$$

Novotny & Hecht, Nanooptics (2006)

Angular Spectrum Representation of Optical Fields

We assume that the medium in the plane is homogeneous, isotropic and source-free. Then

$$(\nabla^2 + k^2)\,E(x,y,z) = 0; \quad k = n(\omega/c) \quad \text{and} \quad n = \sqrt{\mu\varepsilon} \qquad \text{(Helmholtz equation)}$$

Inserting the Fourier representation of *E(x,y,z)* into the Helmholtz equation we find

$$\hat{E}(k_x,k_y;z) = \hat{E}(k_x,k_y;0)\,\exp[\mathrm{i}\,k_z z]; \quad k_z = \sqrt{k^2 - k_x^2 - k_y^2}$$

$$E(x,y,z) = \iint \hat{E}(k_x,k_y;0)\,\exp(\mathrm{i}[k_x x + k_y y + k_z z])\,\mathrm{d}k_x\mathrm{d}k_y$$

For the case of a purely dielectric medium, the angular spectrum is a superposition of only two characteristic solutions: plane waves and evanescent waves.

Plane waves: $\exp(\mathrm{i}[k_x x + k_y y])\,\exp(\pm\mathrm{i}|k_z|z); \quad k_x^2 + k_y^2 \le k^2,$

Evanescent waves: $\exp(\mathrm{i}[k_x x + k_y y])\,\exp(-|k_z|z); \quad k_x^2 + k_y^2 > k^2.$

Novotny & Hecht, Nanooptics (2006)