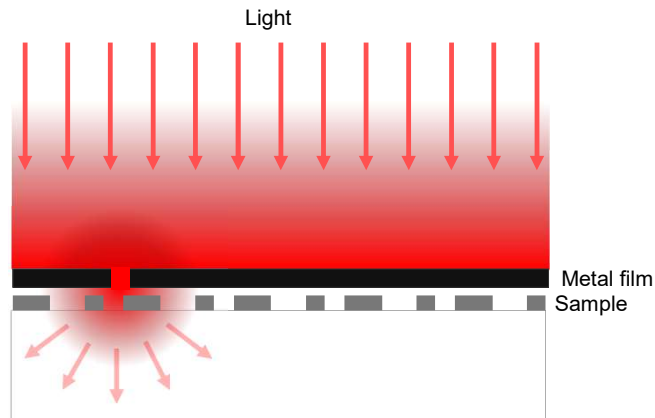


E. Synge: Microscopy with holes



E. H. Synge (1928)

XXXVIII. *A Suggested Method for extending Microscopic Resolution into the Ultra-Microscopic Region.* By E. H. SYNGE*.



1890 - 1957

... It remains to consider the most obvious technical difficulties. These seem to be four in number :—

- (1) The source of illumination, which must be of very great intensity.
- (2) The making of small adjustments of order 10^{-7} cm., which would be required in the vertical inter-adjustments of the section and the opaque plate, and the making of regular increments of motion of 10^{-6} cm. in the plane of the section, the conditions in both cases being such that friction cannot be entirely eliminated.
- (3) The planing of the biological section so that it presents a surface which does not diverge from a plane by more than a fraction of 10^{-6} cm.
- (4) The construction, in an opaque plate or film, of a hole whose diameter is of the order 10^{-6} cm.

Scanning Near-Field Optical Microscopy (SNOM): A Brief Historical Overview

1928 E. H. Synge: Microscopic Imaging by scanning a submicroscopic light source across a sample

1956 J. A. O'Keefe: „Resolving Power of Visible Light“

1956 A. V. Baez: Experiments with sound waves

1972 Ash & Nicholls: Micro-wave experiment with sub-wavelength sized apertures at a resolution of $\lambda/60$.

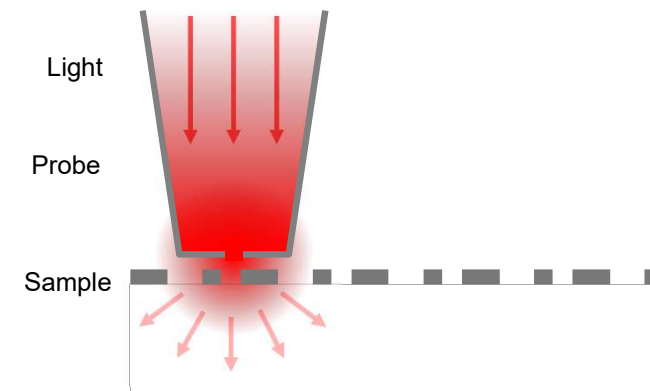
1984 D. Pohl, W. Denk & M. Lanz: Demonstration of a resolution of $\lambda/20$ using visible light.

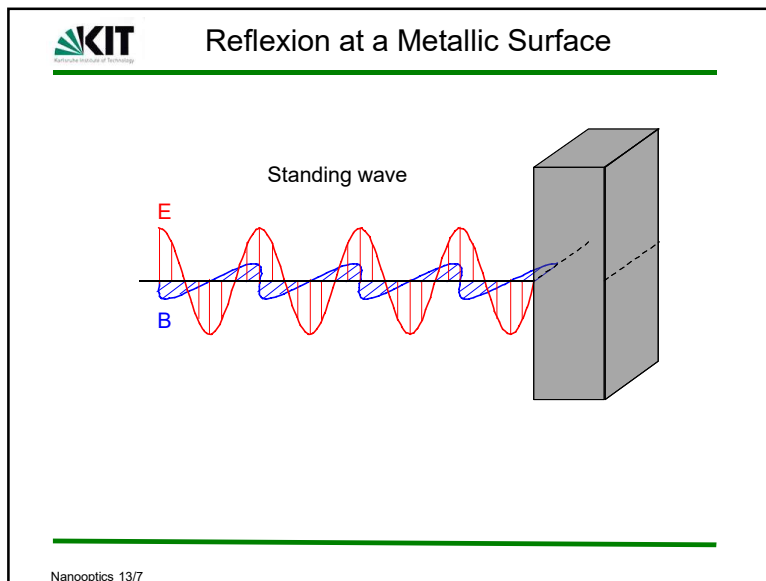
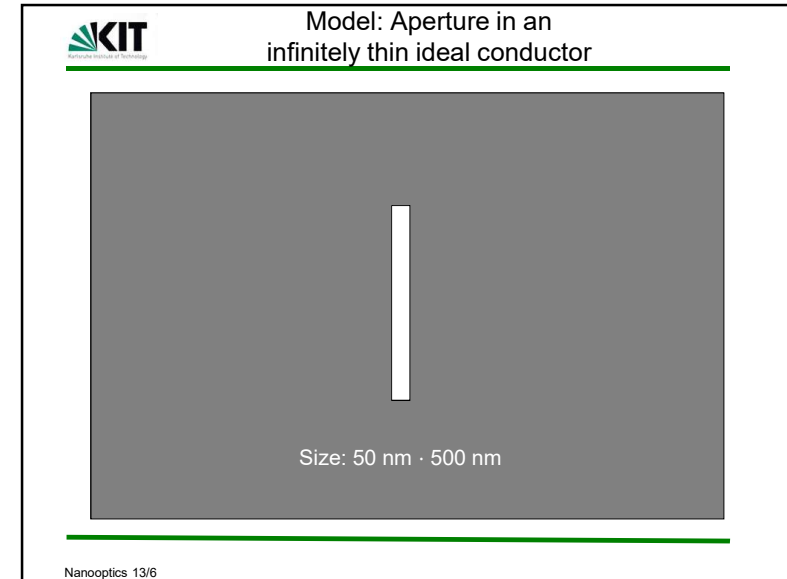
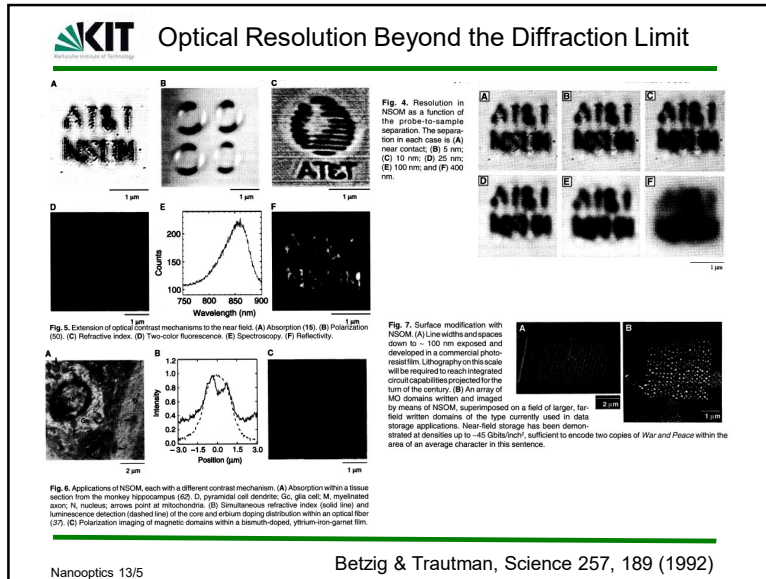
...

1991 E. Betzig et al.: „Breaking the Diffraction Barrier – Optical Microscopy on a Nanometric Scale“

...

Near-Field Imaging with a Tapered Probe





KIT Karlsruhe Institute of Technology

A slit in a metal screen as scatterer for the electromagnetic field

$\vec{E}$ perpendicular to long side of the slit

In first order the fields inside the slit are equal to the fields at the metal surface

- At the edges the tangential components of E and H must be continuous.
- The incoming E-field is cancelled by the opposite field created by the charges at the slit's rim.
- The additional H-field is created by the displacement current $\vec{D}$.

$\vec{E}_{total}(z=0) \approx 0$ $\text{curl } \vec{H} = \vec{j} + \dot{\vec{D}}$

High transmission ($\vec{S} = \vec{E} \times \vec{H}$) !

Zakharian et al. (2004)

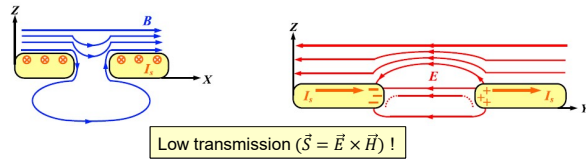
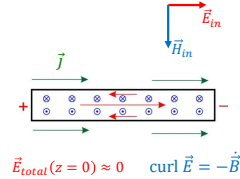
Nanooptics 13/8

A slit in a metal screen as scatterer for the electromagnetic field

$\vec{E}$ parallel to long side of the slit

In first order the fields inside the slit are equal to the fields at the metal surface

- At the edges the tangential component of E and the normal component of B must be continuous.
- The incoming E -field is cancelled (at the edges) by induction caused by the magnetic field penetrating the slit.



Optical Near-Fields at a Circular Aperture



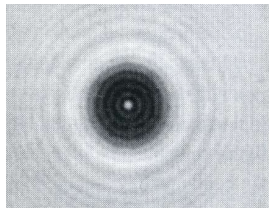
Hans Bethe
(1906 – 2005)



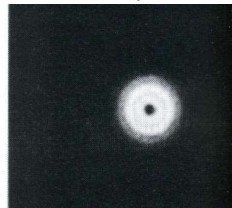
Christoffel Jacob Bouwkamp
(1915 – 2003)

Diffraction at a Circular Aperture: Babinet Principle

Circular Disc



Circular Aperture



The diffraction pattern at an aperture and a complementary disc are equal in intensity if the undiffracted light is subtracted.

Generalized Babinet Principle

for an infinitely thin, ideally conducting film

The following relations hold for the diffracted fields (near- and far-fields) of an aperture A and its complementary disc K :

$$E_A \rightarrow -H_K$$

$$H_A \rightarrow E_K$$



Near-fields at a circular nano-aperture: approximate solution of Bouwkamp

Oblate-spheroidal coordinates (with radius a of the aperture):

$$z = auv, \quad x = a\sqrt{(1-u^2)(1+v^2)}\cos\varphi, \quad y = a\sqrt{(1-u^2)(1+v^2)}\sin\varphi,$$

where $0 \leq u \leq 1$, $-\infty \leq v \leq \infty$, $0 \leq \varphi \leq 2\pi$. The surfaces $v = 0$ and $u = 0$ correspond to the aperture and the screen, respectively.

Defining equation (spheroid)

$$\frac{x^2 + y^2}{1 + v^2} + \frac{z^2}{u^2} = a^2$$

Solution for $a \ll \lambda$ (valid for distances to the aperture $r \ll a$):

$$E_x/E_0 = ikz - \frac{2}{\pi}ika u \left[1 + v \arctan v + \frac{1}{3} \frac{1}{u^2 + v^2} + \frac{x^2 - y^2}{3a^2(u^2 + v^2)(1 + v^2)^2} \right]$$

$$E_y/E_0 = -\frac{4ikxyu}{3\pi a(u^2 + v^2)(1 + v^2)^2},$$

$$E_z/E_0 = -\frac{4ikxv}{3\pi a(u^2 + v^2)(1 + v^2)^2},$$

$$H_x/H_0 = -\frac{4xyv}{\pi a^2(u^2 + v^2)(1 + v^2)^2},$$

$$H_y/H_0 = 1 - \frac{2}{\pi} \left[\arctan v + \frac{v}{u^2 + v^2} + \frac{v(x^2 - y^2)}{\pi a^2(u^2 + v^2)(1 + v^2)^2} \right]$$

$$H_z/H_0 = -\frac{4ay u}{\pi a^2(u^2 + v^2)(1 + v^2)^2},$$

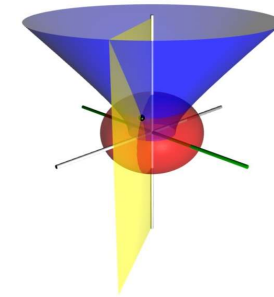
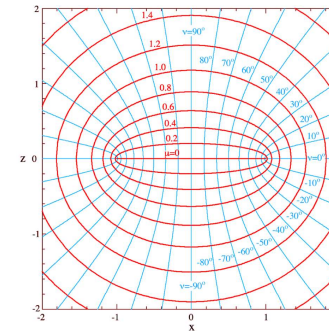
Nanooptics 13/13

Novotny & Hecht, Nanooptics, p. 190-191



Oblate-spheroidal coordinates

Coordinate transformation: $x = a \cosh \mu \cos v \cos \phi$ $v = \sinh \mu$
 $y = a \cosh \mu \cos v \sin \phi$ $u = \sin v$
 $z = a \sinh \mu \sin v$

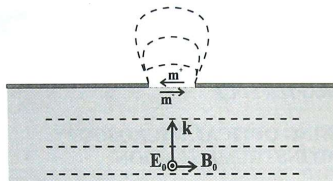


Nanooptics 13/14

Wikipedia



Transmission through Circular Apertures



Near-field Intensity along the symmetry axis

$$I(0,0,z) \propto \left(\frac{4kD}{3\pi} \right)^2 \exp\left(-\frac{3\pi z}{2D} \right) \quad \text{with aperture diameter } D$$

Cross section for radiation to the far-field

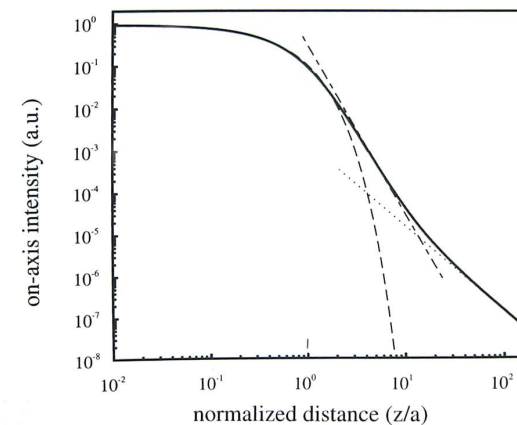
$$\sigma = \frac{64}{27\pi} k^4 D^6$$

Nanooptics 13/15

Bouwkamp (1950); M.H.P.Moers (1995)



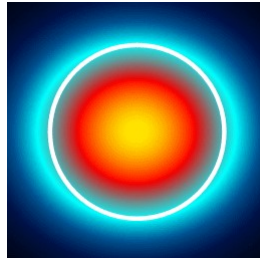
On-axis intensity behind a circular aperture in an ideal conductor



Nanooptics 13/16

M.H.P.Moers (1995)

Field-Map at an Aperture



diameter $D=50$ nm

distance 50 nm

Field-Map at a Circular Aperture

