1 Theoretical aspects

1.1 Basics

Maxwell's Equations

In vacuum, with sources ρ and j, the Microscopic Maxwell Equations say

$$\begin{aligned} \operatorname{div}\vec{D} &= \rho \\ \operatorname{curl}\vec{E} &= -\frac{\partial}{\partial t}\vec{B} \\ \operatorname{div}\vec{B} &= 0 \\ \operatorname{curl}\vec{H} &= \vec{j}+\frac{\partial}{\partial t}\vec{D} \end{aligned}$$

$\vec{E}$	electric field	[E]	=	$\frac{\mathrm{V}}{\mathrm{m}}$
$\vec{D}=\varepsilon_0\vec{E}$	electric flux density	[D]	=	$\frac{\mathrm{C}}{\mathrm{m}^2}$
$\vec{H}$	magnetic field	[H]	=	$\frac{\mathrm{A}}{\mathrm{m}}$
$\vec{B}=\mu_0\vec{H}$	magnetic flux density	[B]	=	T

In isotropic, linear media following relations hold:

$$\begin{aligned} \vec{D} &= \varepsilon_0\vec{E}+\vec{P} &= \varepsilon_0\varepsilon\vec{E} \\ \vec{B} &= \mu_0(\vec{H}+\vec{M}) &= \mu_0\mu\vec{H} \end{aligned}$$

$\vec{P}=\varepsilon_0\chi_e\vec{E}$	polarisation	[P]	=	$\frac{\mathrm{C}}{\mathrm{m}^2}$
$\vec{M}=\chi_m\vec{H}$	magnetisation	[M]	=	$\frac{\mathrm{A}}{\mathrm{m}}$

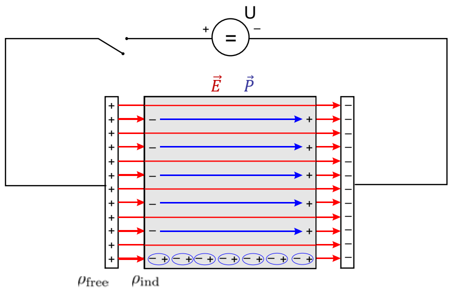
An equivalent set of equations as for the microscopic ME is valid, however, now only for fields averaged over a small but finite volume (Macroscopic Maxwell Equations).

Contributions to ρ and $\vec{j}$ are divided in „free“ and „bound“ or „external“ and „internal“:

$$\begin{aligned} \rho &= \rho_{\mathrm{free}}+\rho_{\mathrm{bound}} \\ \vec{j} &= \vec{j}_{\mathrm{ext}}+\cancel{\vec{j}_{\mathrm{int}}} \end{aligned}$$

(almost) never significant in optics

Polarisation (dielectric material)



The entire information of the optical properties of a material is contained in the polarisation $\vec{P}$. However, usually other quantities are used which are closely related to the polarisation: $\varepsilon, \chi, n, \alpha$, ...

$$\left.\begin{aligned} \operatorname{div}\varepsilon_0\vec{E} &= \rho_{\mathrm{free}}+\rho_{\mathrm{ind}} \\ \operatorname{div}\vec{P} &= -\rho_{\mathrm{ind}} \end{aligned}\right\} \quad \operatorname{div}\underbrace{(\varepsilon_0\vec{E}+\vec{P})}_{\vec{D}} = \rho_{\mathrm{free}} \quad ; \quad \vec{E} = \frac{1}{\varepsilon_0}\left(\vec{D}-\vec{P}\right)$$

Averaged field!

mostly omitted!

Important Quantities and Relations

Lorentz force: $\vec{F} = q\cdot(\vec{E}+\vec{v}\times\vec{B})$

Continuity equation: $\operatorname{div}\vec{j}+\dot{\rho} = 0$

Energy density: $w = \frac{1}{2}\left(\vec{D}\cdot\vec{E}+\vec{B}\cdot\vec{H}\right)$

Energy flux density: $\vec{S} = \vec{E}\times\vec{H}$ with Intensity $I = \langle S\rangle_t$

Ohm's law: $\vec{j} = \sigma(\vec{r})\,\vec{E}$ with conductivity σ

Energy Density and Poynting-Vector

$$w = \frac{1}{2}\left(\vec{D}\cdot\vec{E} + \vec{B}\cdot\vec{H}\right) \quad ; \quad [\mathrm{w}] = \frac{\mathrm{J}}{\mathrm{m}^3}$$

Temporal change of w $\hat{=}$ inflow or outflow + transformation:

$$\begin{aligned}
\frac{\partial w}{\partial t} &= \frac{1}{2}\left(\dot{\vec{D}}\cdot\vec{E} + \vec{D}\cdot\dot{\vec{E}} + \dot{\vec{B}}\cdot\vec{H} + \vec{B}\cdot\dot{\vec{H}}\right) \\
&\overset{\text{in vacuum}}{=} \varepsilon_0\vec{E}\cdot\dot{\vec{E}} + \mu_0\vec{H}\cdot\dot{\vec{H}} && \text{(2. and 4. ME)} \\
&= \vec{E}\left(\vec{\nabla}\times\vec{H} - \vec{j}\right) - \vec{H}\left(\vec{\nabla}\times\vec{E}\right) && \mathrm{div}\,(\vec{a}\times\vec{b}) = -\vec{a}(\vec{\nabla}\times\vec{b}) + \vec{b}(\vec{\nabla}\times\vec{a}) \\
&= -\mathrm{div}\left(\vec{E}\times\vec{H}\right) - \vec{j}\cdot\vec{E}
\end{aligned}$$

thus

$$\boxed{\frac{\partial w}{\partial t} = \underbrace{-\mathrm{div}\,\vec{S}}_{\text{radiation}} \underbrace{-\,\vec{j}\cdot\vec{E}}_{\text{transformation to heat (Ohm's law)}} \quad ; \quad [\mathrm{S}] = \frac{\mathrm{W}}{\mathrm{m}^2}}$$

Wave Equations

Maxwell's equations ($\rho = 0;\ \vec{j} = 0$)

$$\begin{aligned}
\mathrm{curl}\,\vec{E} &= -\frac{\partial}{\partial t}\vec{B} && \mathrm{curl} \\
\mathrm{curl}\,\vec{H} &= \frac{\partial}{\partial t}\vec{D} && \mu_0\mu\frac{\partial}{\partial t} && (\, j_{\mathrm{int}} = 0 \ \rightarrow \mu = 1 \,)
\end{aligned}$$

$$\underbrace{\mathrm{curl}\,\mathrm{curl}}_{\cancel{\mathrm{grad}\,\mathrm{div}}\, - \underbrace{\mathrm{div}\,\mathrm{grad}}_{\Delta}}\vec{E} + \mu_0\frac{\partial^2}{\partial t^2}\vec{D} = 0 \qquad \vec{D} = \varepsilon_0\varepsilon\,\vec{E}$$

$$\boxed{\Delta\vec{E} - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\vec{E} = 0}$$

analogue:

$$\boxed{\Delta\vec{H} - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\vec{H} = 0}$$

with

$$\boxed{\begin{aligned}
c_0 &:= \frac{1}{\sqrt{\mu_0\varepsilon_0}} && ; \ \text{Speed of light in vacuum} \\
c &:= \frac{c_0}{n} && ; \ \text{Speed of light in matter} \\
n &:= \sqrt{\varepsilon\mu} && ; \ \text{Index of refraction}
\end{aligned}}$$

$$(\, \mu = 1 \rightarrow n = \sqrt{\varepsilon} \,)$$