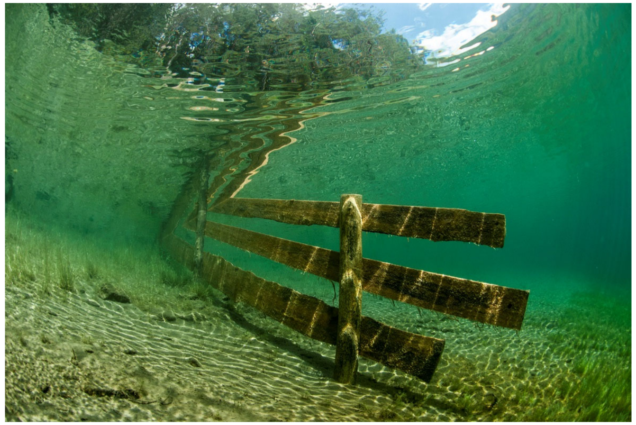
1.2 *Surface waves*

Total Internal Reflection



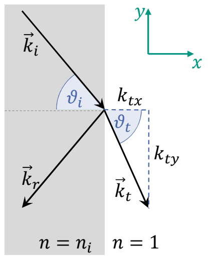
https://www.fotocommunity.de/photo/zaun-mit-totalreflexion-bernd-nies/41041376

Total Internal Reflection

Snell's law for total internal reflection (to air, $n_t = 1$):

$$\frac{\sin\vartheta_i}{\sin\vartheta_t} = \frac{1}{n_i} \quad \Rightarrow \quad \sin\vartheta_t = n_i \sin\vartheta_i \geq 1$$

$$\Rightarrow \quad \cos\vartheta_t = i\underbrace{\sqrt{n_i^2 \sin^2\vartheta_i - 1}}_{\text{real number} \geq 0}$$



Components of the wave vector $\vec{k}_t$:

$$k_{ty} = k_0 \sin\vartheta_t = k_0\, n_i \sin\vartheta_i$$

$$k_{tx} = k_0 \cos\vartheta_t = k_0\, i\sqrt{n_i^2 \sin^2\vartheta_i - 1}$$

Electric field of transmitted light is $\vec{E}_t = \vec{E}_0\, e^{i(\vec{k}_t \vec{r} - \omega t)}$; $|\vec{k}_t| = k_0 = \frac{\omega}{c_0}$

Inserting k_{tx} and k_{ty} in phase of $\vec{E}_t$ then gives

$$\vec{k}_t \cdot \vec{r} = k_{tx}\, x + k_{ty}\, y = i\sqrt{n_i^2 \sin^2\vartheta_i - 1} \cdot k_0 x + n_i \sin\vartheta_i \cdot k_0 y$$

Surface Wave

The electric field $\vec{E}_t$ of the wave in the low-index medium (here air) is an <u>evanescent wave</u>:

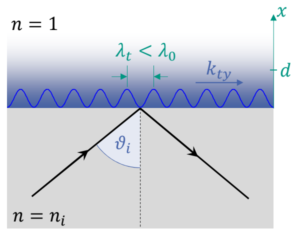
$$\vec{E}_t = \vec{E}_{t0} \cdot e^{i(\vec{k}_t \vec{r} - \omega t)} = \vec{E}_{t0} \cdot e^{-i\omega t} \cdot \underbrace{e^{i k_{ty} y}}_{\substack{\text{propagation} \\ \text{in y-direction}}} \cdot \underbrace{e^{-\frac{x}{d}}}_{\substack{\text{decay} \\ \text{in x-direction}}}$$

The surface wave propagates along the interface with

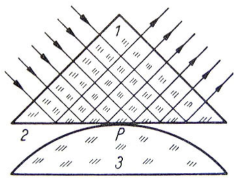
$$k_{ty} = \frac{2\pi}{\lambda_t} \quad ; \quad \lambda_t = \frac{\lambda_0}{n_i \sin\vartheta_i}$$

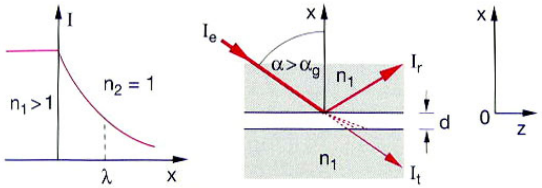
and exponential decay in x-direction given by

$$d = \frac{\lambda_0}{2\pi\sqrt{n_i^2 \sin^2\vartheta_i - 1}}$$



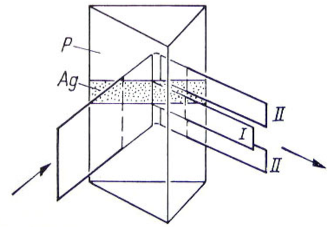
Optical Tunneling

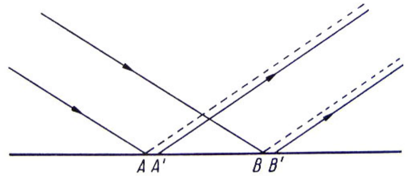




Bergmann-Schäfer, Optik

Goos-Hänchen-Shift





Bergmann-Schäfer, Optik

Light Propagation in Metals

Model : Electron gas
Parameter : Charge density $\rho = f \cdot (-e)$
with number density f

$$E = E_x = E_0 \cdot e^{ikz} \cdot e^{-i\omega t}$$

$$k = n(\omega)\frac{2\pi}{\lambda_0} \quad \text{with} \quad n(\omega) = \sqrt{\varepsilon(\omega)}$$

ε > 0 : Propagating wave
ε < 0 : Damped or evanescent wave

Dielectric Function of a Free-Electron-Gas

Interaction of an electron with the electric field E in the electron gas:

$$m\,\ddot{x}(t) = -e\,E(t) \quad ; \quad E(t) = E_0\,\exp(-i\omega t)$$

Ansatz: $x(t) = x_0\,\exp(-i\omega t)$

$$-\omega^2 m\,x(t) = -e\,E(t) \qquad \boxed{x(t) = \frac{e}{m\omega^2}\,E(t)}$$ (phase shift of 180°!)

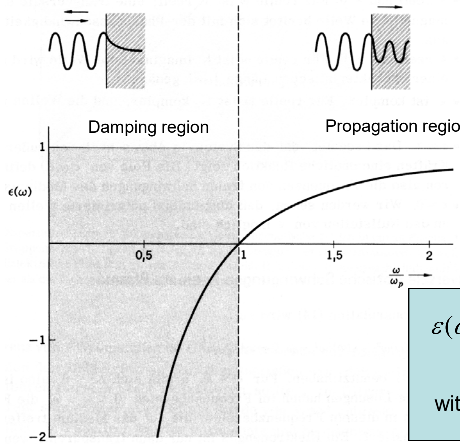
With „dipole moment“ $p = -e\,x$ and number density f of the electrons, it follows

$$P(t) = f \cdot p(t) = -\frac{f\,e^2}{m\omega^2}\,E(t) \overset{!}{=} \varepsilon_0\,\chi_e(\omega)\,E(t)$$

$$\Rightarrow \quad \varepsilon(\omega) = 1 + \chi_e(\omega) = 1 - \frac{f\,e^2}{\varepsilon_0 m\omega^2}$$

or with $\omega_p := \sqrt{\dfrac{f\,e^2}{\varepsilon_0\,m}}$ $$\boxed{\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}}$$

Plasma Optics



$$\boxed{\varepsilon(\omega) = 1 - \frac{\omega_P^2}{\omega^2}}$$

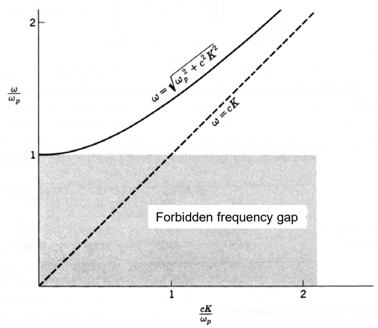
with $\omega_P = \sqrt{\dfrac{f\,e^2}{\varepsilon_0\,m}}$

C. Kittel, solid state physics

Propagation of Light in Metal (Plasma)

Dispersion relation of electromagnetic waves in the electron gas:

$$\omega = \frac{c_0}{n}\,k \quad \Rightarrow \quad \varepsilon(\omega)\,\omega^2 = c_0^2\,k^2 \quad \Rightarrow \quad \boxed{\omega = \sqrt{\omega_p^2 + c_0^2\,k^2}}$$



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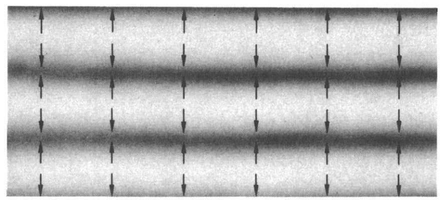
Volume plasmon

Longitudinal resonances of the plasma:



Charge density oscillation. The arrows indicate the displacement of charge.

Resonance: $D = \varepsilon_0\,E + P = \varepsilon_0\,\varepsilon(\omega_{res})\,E = 0$

$$\varepsilon(\omega_{res}) = 0 \quad \Rightarrow \quad \omega_{res} = \omega_P$$

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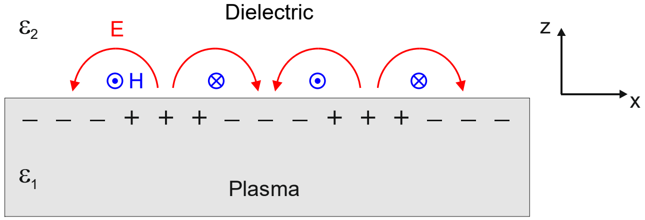
Surface Plasmons

(SP - surface plasmon; SPP - surface plasmon polariton)



z < 0:

$$\vec{E}_1 = (E_{x1}, 0, E_{z1}) \cdot e^{i(k_{x1}x - k_{z1}z - \omega t)}$$
$$\vec{H}_1 = (0, H_{y1}, 0) \cdot e^{i(k_{x1}x - k_{z1}z - \omega t)}$$

z > 0:

$$\vec{E}_2 = (E_{x2}, 0, E_{z2}) \cdot e^{i(k_{x2}x + k_{z2}z - \omega t)}$$
$$\vec{H}_2 = (0, H_{y2}, 0) \cdot e^{i(k_{x2}x + k_{z2}z - \omega t)}$$

SPP – dispersion relation

Boundary conditions:

$$\left.\begin{aligned} E_{x1} &= E_{x2} \quad (= E_x) \\ H_{y1} &= H_{y2} \quad (= H_y) \end{aligned}\right\} \Rightarrow \quad k_{x1} = k_{x2} \quad (= k_x)$$

Maxwell equation:

$$\text{curl}\,\vec{H} = \dot{\vec{D}} = \epsilon_0 \epsilon \dot{\vec{E}} \quad \Rightarrow \quad -\frac{\partial H_y}{\partial z} = \epsilon_0 \epsilon \dot{E}_x$$

$$\text{curl}\,\vec{H} = \begin{pmatrix} \partial_y H_z - \partial_z H_y \\ \partial_z H_x - \partial_x H_z \\ \partial_x H_y - \partial_y H_x \end{pmatrix}$$

$$\left.\begin{aligned} &\underline{z>0:} \quad -i\,k_{z2}\,H_y = -i\omega\,\epsilon_2\,\epsilon_0\,E_x \\ &\underline{z<0:} \quad +i\,k_{z1} H_y = -i\omega\,\epsilon_1\,\epsilon_0\,E_x \end{aligned}\right\} \Rightarrow \quad -\frac{k_{z2}}{k_{z1}} = \frac{\epsilon_2}{\epsilon_1} \qquad (1)$$