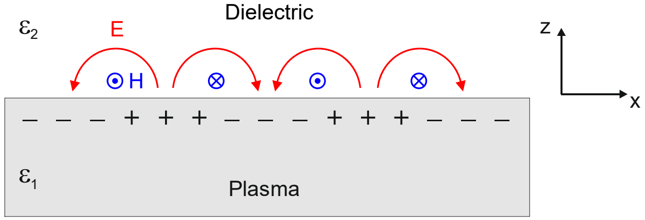
Surface Plasmons

(SP - surface plasmon; SPP - surface plasmon polariton)



z < 0:

$$\vec{E}_1 = (E_{x1}, 0, E_{z1}) \cdot e^{i(k_{x1}x - k_{z1}z - \omega t)}$$

$$\vec{H}_1 = (0, H_{y1}, 0) \cdot e^{i(k_{x1}x - k_{z1}z - \omega t)}$$

z > 0:

$$\vec{E}_2 = (E_{x2}, 0, E_{z2}) \cdot e^{i(k_{x2}x + k_{z2}z - \omega t)}$$

$$\vec{H}_2 = (0, H_{y2}, 0) \cdot e^{i(k_{x2}x + k_{z2}z - \omega t)}$$

SPP – dispersion relation

Boundary conditions:

$$\left.\begin{array}{l} E_{x1} = E_{x2} \quad (= E_x) \\ H_{y1} = H_{y2} \quad (= H_y) \end{array}\right\} \Rightarrow k_{x1} = k_{x2} \quad (= k_x)$$

Maxwell equation:

$$\text{curl}\,\vec{H} = \dot{\vec{D}} = \epsilon_0 \epsilon \dot{\vec{E}} \quad \Rightarrow \quad -\frac{\partial H_y}{\partial z} = \epsilon_0 \epsilon \dot{E}_x$$

$$\text{curl}\,\vec{H} = \begin{pmatrix} \partial_y H_z - \partial_z H_y \\ \partial_z H_x - \partial_x H_z \\ \partial_x H_y - \partial_y H_x \end{pmatrix}$$

$$\left.\begin{array}{ll} \underline{z>0:} & -i\,k_{z2}\,H_y = -i\omega\,\epsilon_2\,\epsilon_0\,E_x \\ \underline{z<0:} & +i\,k_{z1} H_y = -i\omega\,\epsilon_1\,\epsilon_0\,E_x \end{array}\right\} \Rightarrow -\frac{k_{z2}}{k_{z1}} = \frac{\epsilon_2}{\epsilon_1} \quad (1)$$

SPP – dispersion relation

Dispersion relation:

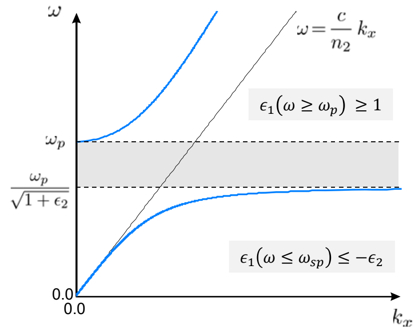
$$\omega = \frac{c}{n} \cdot k \quad \Rightarrow \quad k^2 = \epsilon \cdot \left(\frac{\omega}{c}\right)^2 \quad (2)$$

$$\left.\begin{array}{lll} \underline{z>0:} & k_x^2 + k_{z2}^2 = \epsilon_2 \left(\frac{\omega}{c}\right)^2 \Rightarrow & k_{z2}^2 = \epsilon_2 \left(\frac{\omega}{c}\right)^2 - k_x^2 \\ \underline{z<0:} & k_x^2 + k_{z1}^2 = \epsilon_1 \left(\frac{\omega}{c}\right)^2 \Rightarrow & k_{z1}^2 = \epsilon_1 \left(\frac{\omega}{c}\right)^2 - k_x^2 \end{array}\right\} \overset{(1)}{\Rightarrow} \frac{\epsilon_2^2}{\epsilon_1^2} = \frac{\epsilon_2 \left(\frac{\omega}{c}\right)^2 - k_x^2}{\epsilon_1 \left(\frac{\omega}{c}\right)^2 - k_x^2}$$

Result:

$$k_x(\omega) = \sqrt{\frac{\epsilon_1 \cdot \epsilon_2}{\epsilon_1 + \epsilon_2}} \cdot \frac{\omega}{c}$$

SPPs at a Dielectric-Metal Interface



$$k_x(\omega) = \sqrt{\frac{\epsilon_1 \cdot \epsilon_2}{\epsilon_1 + \epsilon_2}} \cdot \frac{\omega}{c}$$

Dielectric-Metal Interface:

$$\epsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\epsilon_2 \geq 1; \quad n_2 = \sqrt{\epsilon_2}$$

SPP – decay length

If k_{zi} is the z-component of the wavevector in media i with permittivity ϵ_i, then:

$$k = \frac{\omega}{n} \quad \Longrightarrow \quad k_i^2 = \epsilon_i \cdot \left(\frac{\omega}{c}\right)^2 = k_x^2 + k_{zi}^2 \quad \Longrightarrow \quad k_{zi} = \frac{\epsilon_i}{\sqrt{\epsilon_1 + \epsilon_2}} \cdot \frac{\omega}{c}$$

Media 2 is a dielectric and media 1 is a metal. Then for the upper branch of the dispersion relation, we get:

$$\omega > \omega_p \quad \Longrightarrow \quad \epsilon_1(\omega) > 0 \quad \Longrightarrow \quad k_{z1} \text{ and } k_{z2} \text{ are real (propagating waves)}$$

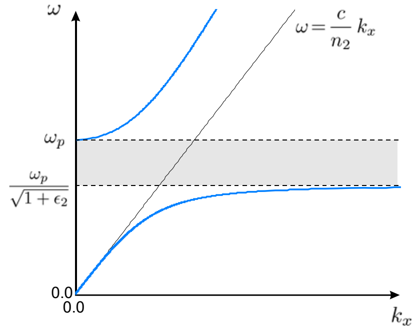
For the lower branch of the dispersion relation, we have:

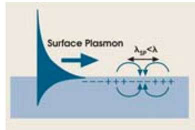
$$\omega < \frac{\omega_p}{\sqrt{1+\epsilon_2}} \quad \Longrightarrow \quad \epsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega^2} < -\epsilon_2 \quad \Longrightarrow \quad k_{z1} \text{ and } k_{z2} \text{ are imaginary!}$$

The decay lengths of the evanescent wave are: $d_i = \frac{1}{|k_{zi}|} = \frac{\lambda}{2\pi}\sqrt{\left|\frac{\epsilon_1 + \epsilon_2}{\epsilon_i^2}\right|}$

Silver @ 600 nm: $d_1 = 24$ nm $\quad d_2 = 390$ nm

Dispersion relation of SPPs

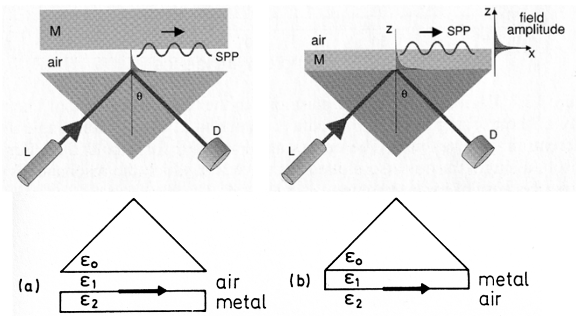




$$k_x(\omega) = \sqrt{\frac{\epsilon_1 \cdot \epsilon_2}{\epsilon_1 + \epsilon_2}} \cdot \frac{\omega}{c} \quad ; \quad k_{zi} = \frac{\epsilon_i}{\sqrt{\epsilon_1 + \epsilon_2}} \cdot \frac{\omega}{c}$$

Excitation of SPPs by ATR

(ATR = attenuated total reflection)

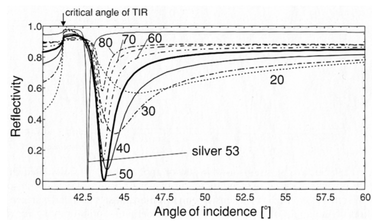


Otto-configuration

Kretschmann-configuration

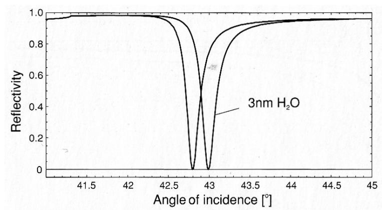
H. Raether, Surface Plasmons; Novotny & Hecht, Nano-Optics

SPP Excitation of Gold and Silver films



SPP excitation of gold and silver in Kretschmann-configuration.

(film thicknesses in nm)



Shift of SPP resonance induced by a 3 nm layer of water on a 53 nm silver film (calculation).

Novotny & Hecht, Nano-Optics