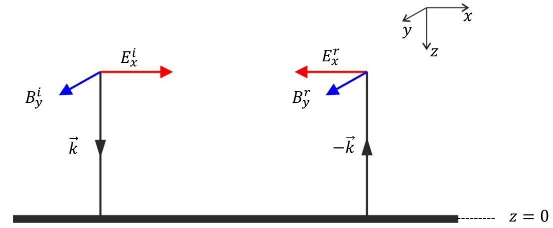
Reflection at an ideally conducting metal

$$\vec{k} = \begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix}$$

$$\vec{B} = \frac{1}{\omega}\,\vec{k} \times \vec{E}$$



Assumption: Only $\vec{E}$ interacts with metal (electrons). Then the fields are given by

$$\vec{E}^i = \begin{pmatrix} E_x^i \\ 0 \\ 0 \end{pmatrix} ; \quad E_x^i = E_0 e^{i(kz-\omega t)} \qquad \vec{E}^r = \begin{pmatrix} E_x^r \\ 0 \\ 0 \end{pmatrix} \quad E_x^r = -E_0 e^{i(-kz-\omega t+\phi)}$$

$$\vec{B}^i = \begin{pmatrix} 0 \\ B_y^i \\ 0 \end{pmatrix} ; \quad B_y^i = B_0 e^{i(kz-\omega t)} \qquad \vec{B}^r = \begin{pmatrix} 0 \\ B_y^r \\ 0 \end{pmatrix} \quad B_y^r = +B_0 e^{i(-kz-\omega t+\phi)}$$

Reflection at an ideally conducting metal

Due to the ideal conduction, $\vec{E} = \vec{E}^i + \vec{E}^r$ must vanish at the surface ($z = 0$).

Thus we get $\phi = 0$ and

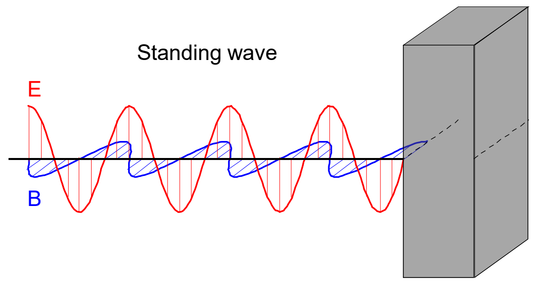
$$E_x = E_x^i + E_x^r = i \cdot 2E_0 \sin(kz)\, e^{-i\omega t}$$

$$B_y = B_y^i + B_y^r = 2B_0 \cos(kz)\, e^{-i\omega t}$$

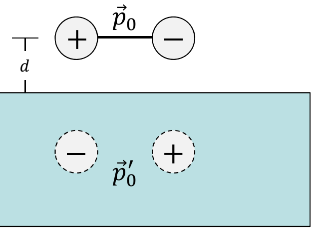
The current density $\vec{j}$ in the slab is created by the incident electric field E_x. Then the inductance prevents the current to increase infinitely. The current j_x produces a magnetic field H_y^r which in turn creates the reflecting electric field. Thus

$$\vec{j} = \mathrm{curl}\, \vec{H}^r \quad \Rightarrow \quad j_x = -\frac{\partial H_y^r}{\partial z} = i\, k\, H_0\, e^{-i\omega t}$$

Reflexion at a Metallic Surface



Molecule in Front of a Metal Surface: Fluorescence Quenching



Molecular dipole

Mirror charges

- Damping through ohmic losses (current in metal)
- Damping through the excitation of SPPs
- Weaker or even forbidden dipole emission (depending on distance)
  → change of spontaneous emission rate!

Emission Enhancement: Purcell-Effect

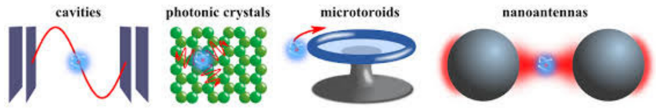
By increasing the density of final states ρ for the photons emitted from a molecule their spontaneous life time τ will decrease.

Example

Free space: $\rho_f = \frac{2}{\pi}\frac{n^3\omega^2}{c^3}$

Cavity: $\rho_c = \frac{2\pi}{V}\frac{Q}{\omega}$

$$\Rightarrow \quad \frac{\rho_c}{\rho_f} = \frac{\pi^2}{V}\frac{c^3 Q}{\omega^3}$$



Radiation of a Black Body at Temperature T

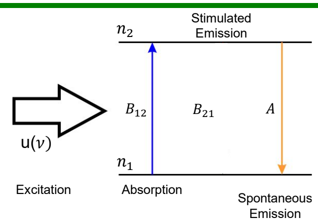
Knowing the density of states ρ_f the spectral energy density $u(\nu,T)$ of blackbody radiation can be immediately derived:

$$\rho_f(\nu) = \frac{8\pi\,\nu^2}{c^3}$$ density of propagation modes in vacuum

$$\langle n\rangle = \frac{1}{e^{h\nu/kT}-1}$$ averaged excitational state $\langle n\rangle$ of a quantum oscillator at temperature T

$$\Rightarrow \quad u(\nu,T) = \rho_f(\nu)\cdot\langle n\rangle\, h\nu \;=\; \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

Stimulated Emission



$B_{12} = B_{21} = B$

for non-degenerate states

$$\dot{n}_2 = B\,u(\nu)\,n_1 - B\,u(\nu)\,n_2 - A\,n_2$$

Example: $n_1(0) = 0,\; n_2(0) = 1 \;\Rightarrow\; n_2(t) \sim e^{-(B\,u+A)\,t}$!

Steady state solution: $\dot{n}_2 = 0 \;\Rightarrow\; \frac{n_2}{n_1} = \frac{B\,u(\nu)}{B\,u(\nu)+A}$

Assume $u(\nu,T)$ is a blackbody radiation with temperature T and the molecule is in balance with the radiation.

$$\frac{n_2}{n_1} = e^{-h\nu/kT} \quad\Rightarrow\quad u(\nu,T) = \frac{A}{B}\frac{1}{e^{h\nu/kT}-1} \overset{!}{=} \frac{8\pi\,\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

The coefficient *A* (spontaneous emission) depends on the density of states!