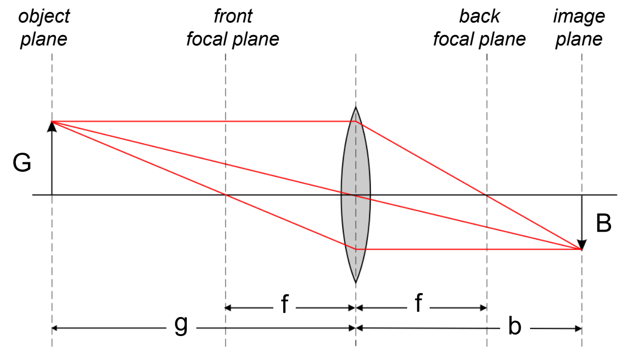
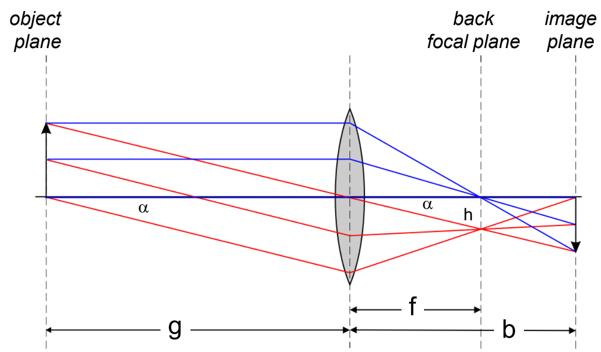
2 Classical optics and microscopy

Geometrical optics - paraxial approximation



lens equation (thin lens) $$\frac{1}{f} = \frac{1}{g} + \frac{1}{b}$$

Geometrical optics – paraxial approximation



$$h = f \cdot \tan\alpha \approx f \cdot \alpha$$

Geometrical Optics – Abberations

Paraxial approximation: $\sin\varphi \cong \varphi$

Exact Taylor series: $\sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \ldots$

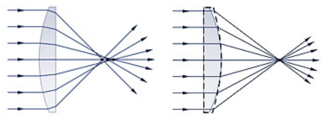
„abberations of third order"

Primary (monochromatic) abberations

- Spherical abberation
- Astigmatism
- Coma
- Field curvature
- Field distortion

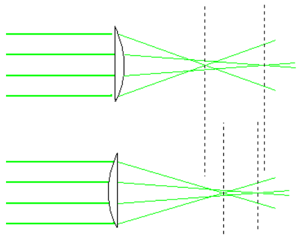
Spherical Abberation

For monochromatic light the abaxial rays are refracted stronger then the paraxial rays.



Workaround

- Suppression of abaxial rays using a small aperture close to the lens („aperture stop“ or „stop“).
- Lens shape, e.g., plano-convex lenses for incident collimated pencil of rays.
- Corrected lens systems
- Aspherical lenses



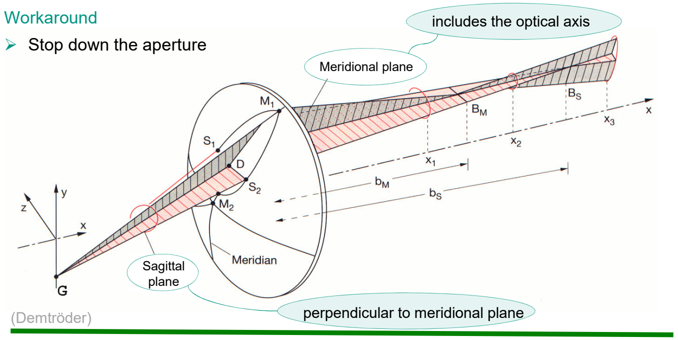
Astigmatism („absence of points“)

Astigmatism of oblique „pencils of rays“

- Occurs for objects remote from the symmetry axis.
- The image of a point produces two orthogonal lines at different positions, inbetween is a blurred area.

Workaround

- Stop down the aperture



(Demtröder)

Coma

- Magnification depends on angle of rays with reference to the optical axis.
- Arises if object is remote from the optical axis.

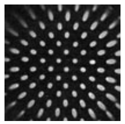
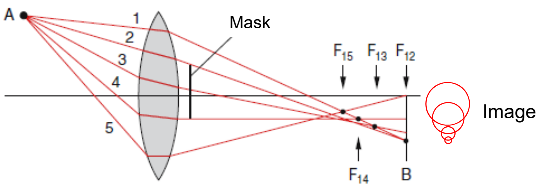


Image of a holey plate with coma

Workaround

- Stop down aperture
- Special lens system → *Aplanat*



(Demtröder)

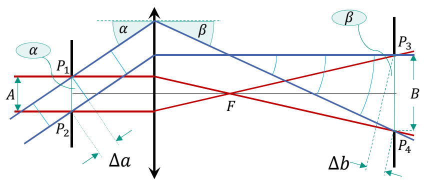
Wide angle rays - Abbe‘s Sine Condition

In case of ideal point imaging all contributing rays must have the same optical path (distance of wavefronts). Thus we get

$$\overline{P_1P_4} = \overline{P_2P_3} \quad \Rightarrow \quad \overline{P_1P_4'} = \overline{P_2P_3'} \quad \Rightarrow \quad \Delta a = \Delta b$$

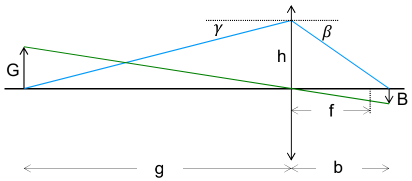
$$\left.\begin{aligned}\frac{\Delta a}{A} &= \sin\alpha \\ \frac{\Delta b}{B} &= \sin\beta\end{aligned}\right\} \Rightarrow \quad \frac{|B|}{|A|} = \frac{\sin\alpha}{\sin\beta}$$

Sine condition



Abbe's Sine Condition

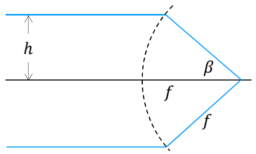
$$\frac{G}{B} = \frac{\sin\beta}{\sin\gamma}$$



The sine condition can be simplified in case of a very large object distance $g \to \infty$:

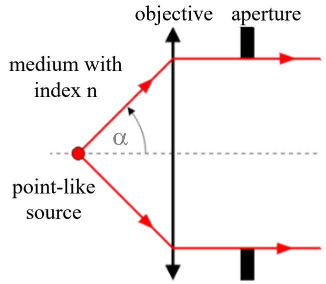
$$\frac{h}{g} \cong \sin\gamma \quad ; \quad \frac{G}{B} = \frac{g}{f} \cong \frac{h}{f\sin\gamma}$$

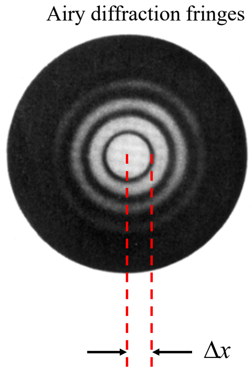
$$\frac{\sin\beta}{\sin\gamma} = \frac{G}{B} = \frac{h}{f\sin\gamma} \Rightarrow \boxed{\frac{h}{f} = \sin\beta}$$



(intuitive scheme)

Resolving power

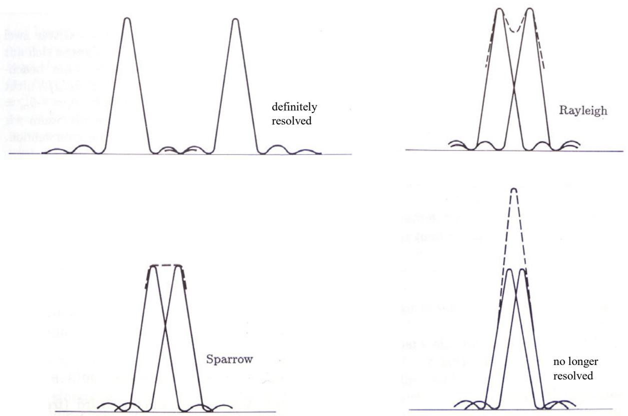




$$\Delta x = 0.61\frac{\lambda}{NA} \quad \text{mit} \quad NA = n\sin\alpha$$

Born & Wolf, Principles of Optics (Pergamon Press 1993)

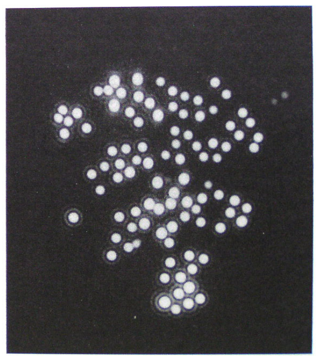
Resolution criteria



E. Hecht, Optik

Illumination

Incoherent illumination



Coherent illumination

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Image formation according to Abbe

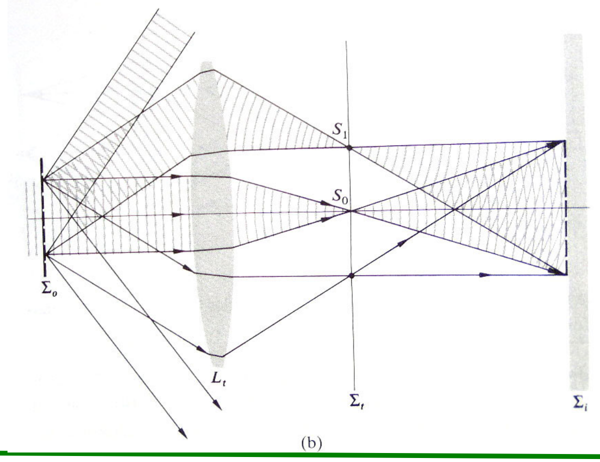

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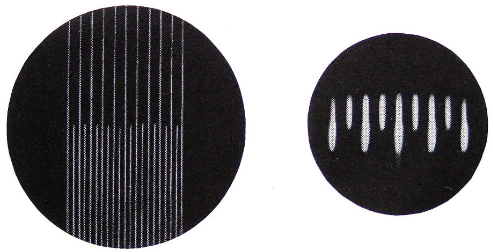
Image formation according to Abbe

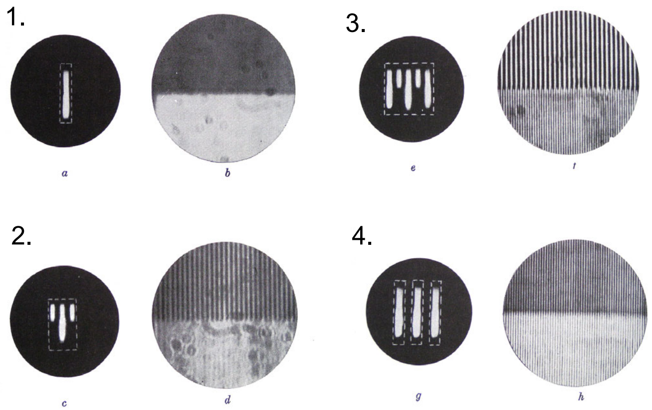


Abbe's diffraction plate

diffraction image (focal plane)

Bergmann-Schäfer, Optik

Image formation according to Abbe



Bergmann-Schäfer, Optik

Diffraction at double slit

Formula for 1. maxima (normal light incidence):

$$\lambda = d \sin\varphi \quad \Rightarrow \quad d = \frac{\lambda}{\sin\varphi}$$

Exciting the slits with a phase difference of π (oblique light incidence) results in 0. and 1. maxima for:

$$\frac{\lambda}{2} = d \sin\varphi \quad \Rightarrow \quad d = \frac{\lambda}{2 \sin\varphi}$$

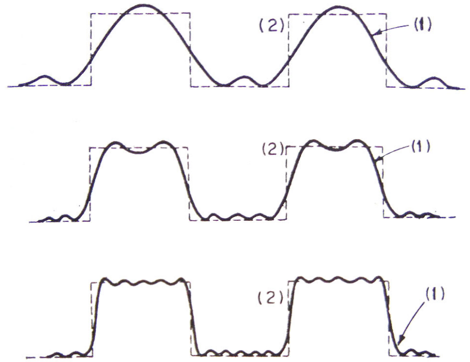
Diffraction at double slit

Formula fo

Exciting the dence) results
in 0. and 1.

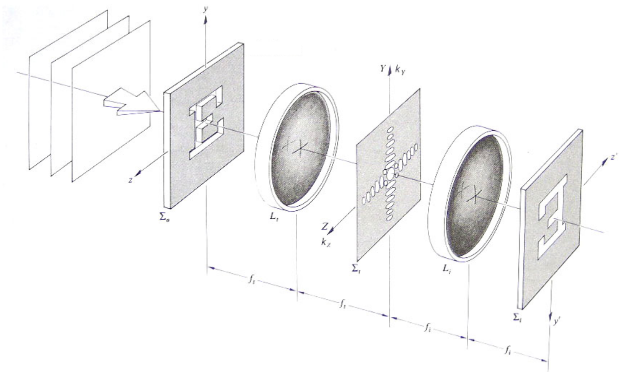
$$\frac{\lambda}{2} = d \sin\varphi \quad \Rightarrow \quad d = \frac{\lambda}{2\sin\varphi}$$

Image formation according to Abbe



H. Beyer, Interferenzmikroskopie

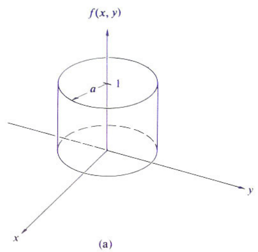
Fourier optics



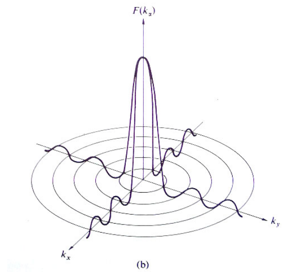
E. Hecht, Optik

Example: Circular Aperture

cylinder function



Fourier transform

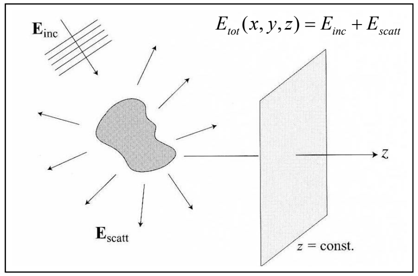


$$f(x,y) = \begin{cases} 1 & \sqrt{x^2+y^2} \le a \\ 0 & \sqrt{x^2+y^2} > a \end{cases}$$

$$F(k_x,k_y) = \iint_{-\infty}^{+\infty} f(x,y)\, e^{i(k_x x + k_y y)} dx\, dy$$

E. Hecht, Optik

Angular Spectrum Representation of Optical Fields



Consider the field *E(x,y,z)* in a plane *z = const* transverse to an arbitrary axis z.

The 2D-Fourier transform Ê of E is then given by:

$$\hat{E}(k_x,k_y;z) = \frac{1}{4\pi^2}\iint E(x,y,z)\exp(-\mathrm{i}[k_x x + k_y y])\,\mathrm{d}x\,\mathrm{d}y$$

so that

$$E(x,y,z) = \iint \hat{E}(k_x,k_y;z)\exp(\mathrm{i}[k_x x + k_y y])\,\mathrm{d}k_x\mathrm{d}k_y$$

Novotny & Hecht, Nanooptics (2006)

Angular Spectrum Representation of Optical Fields

We assume that the medium in the plane is homogeneous, isotropic and source-free. Then

$$(\nabla^2 + k^2)\,E(x,y,z) = 0; \quad k = n(\omega/c) \quad \text{and} \quad n = \sqrt{\mu\varepsilon}$$ (Helmholtz equation)

Inserting the Fourier representation of *E(x,y,z)* into the Helmholtz equation we find

$$\hat{E}(k_x,k_y;z) = \hat{E}(k_x,k_y;0)\exp[\mathrm{i}k_z z]; \quad k_z = \sqrt{k^2 - k_x^2 - k_y^2}$$

$$E(x,y,z) = \iint \hat{E}(k_x,k_y;0)\exp(\mathrm{i}[k_x x + k_y y + k_z z])\,\mathrm{d}k_x\mathrm{d}k_y$$

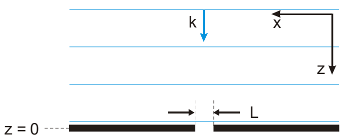
For the case of a purely dielectric medium, the angular spectrum is a superposition of only two characteristic solutions: plane waves and evanescent waves.

Plane waves: $\exp(\mathrm{i}[k_x x + k_y y])\exp(\pm\mathrm{i}|k_z|z); \quad k_x^2 + k_y^2 \le k^2,$

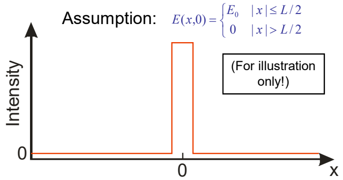
Evanescent waves: $\exp(\mathrm{i}[k_x x + k_y y])\exp(-|k_z|z); \quad k_x^2 + k_y^2 > k^2.$

Novotny & Hecht, Nanooptics (2006)

Example: Line Source (Slit)



Assumption: $E(x,0) = \begin{cases} E_0 & |x| \le L/2 \\ 0 & |x| > L/2 \end{cases}$



$$\hat{E}(k_x; z=0) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} \mathrm{E}(x,0)\exp(-\mathrm{i}k_x x)\,\mathrm{d}x = \frac{E_0}{4\pi^2}\int_{-L/2}^{L/2}\exp(-\mathrm{i}k_x x)\,\mathrm{d}x$$

$$= \frac{E_0 L}{4\pi^2}\frac{\sin(k_x L/2)}{k_x L/2}$$

$$E(x,z) = \int_{-\infty}^{\infty}\hat{E}(k_x; z=0)\exp(\mathrm{i}[k_x x + k_z z])\,\mathrm{d}k_x \qquad k_z = \sqrt{k^2 - k_x^2} = \sqrt{(\omega/c)^2 - k_x^2}$$

Propagating waves: $|k_z| \le \frac{\omega}{c}$; Evanescent waves: $|k_z| > \frac{\omega}{c}$

Vigoureux et al., AO (1992)