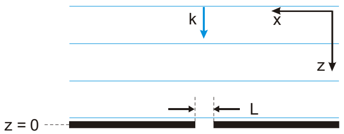
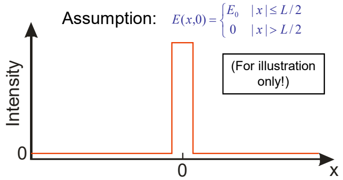
Example: Line Source (Slit)



Assumption: $E(x,0)=\begin{cases}E_0 & |x|\le L/2\\ 0 & |x|>L/2\end{cases}$



$$\hat{E}(k_x; z=0)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\mathrm{E}(x,0)\exp(-\mathrm{i}k_x x)\,\mathrm{d}x=\frac{E_0}{4\pi^2}\int_{-L/2}^{L/2}\exp(-\mathrm{i}k_x x)\,\mathrm{d}x$$
$$=\frac{E_0 L}{4\pi^2}\frac{\sin(k_x L/2)}{k_x L/2}$$

$$E(x,z)=\int_{-\infty}^{\infty}\hat{E}(k_x; z=0)\exp(\mathrm{i}[k_x x+k_z z])\,\mathrm{d}k_x \qquad k_z=\sqrt{k^2-k_x^2}=\sqrt{(\omega/c)^2-k_x^2}$$

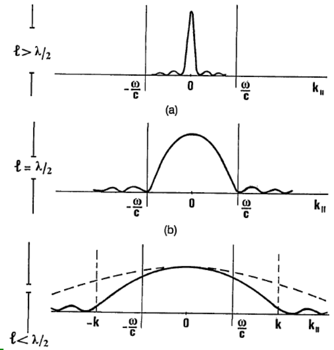
Propagating waves: $|k_z|\le\frac{\omega}{c}$; Evanescent waves: $|k_z|>\frac{\omega}{c}$

Vigoureux et al., AO (1992)

Imaging of a Line Source – Fourier Optics

Classical microscopy: $|k_x|\in[0, k_{max}=\omega\sin\theta/c]$

L is width of a finite line source



$$\Delta x=\frac{\pi c}{\omega\sin\theta}=\frac{\lambda}{2\sin\theta}.$$

(Abbe diffraction limit)

Vigoureux et al., AO (1992)

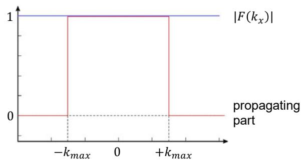
Imaging of a Line Source – Fourier Optics

Point-like light source: $f_s(x)=\delta(x-x_0)$

Spatial frequency spectrum of Delta-function has no limits, $k_x\in[-\infty,\infty]$:

$$F(k_x)=\int_{-\infty}^{\infty}f_s(x)\,e^{ik_x x}\,dx=e^{ik_x x_0}$$

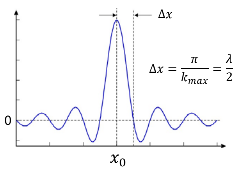
$$|F(k_x)|=1$$



Evanescent waves do not contribute to the image, $k_x=\frac{\omega}{c}\sin\varphi$:

$$k_{max}=\frac{\omega}{c}=\frac{2\pi}{\lambda}$$

$$f_i(x)=\int_{-k_{max}}^{+k_{max}}F(k_x)e^{-ik_x x}\,dk$$
$$=2k_{max}\frac{\sin[k_{max}(x-x_0)]}{k_{max}(x-x_0)}$$



Vigoureux et al., AO (1992)

A Perfect Lens based on an Optical Metamaterial

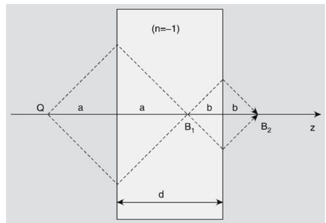
Special solution of Maxwell's equations:

$n=\pm\sqrt{\varepsilon\mu}$ $\quad \varepsilon,\mu<0 \Rightarrow n<0$

Interface Air-Metamaterial, $n=-1$:

$$\frac{\sin\alpha}{\sin\beta}=n=-1$$

A slab of MM with negative n acts as a lens (see figure).



$k=|\vec{k}|=n\frac{\omega}{c_0}=-\frac{\omega}{c_0}$ $\qquad$ Evanescent waves: $k_z=-\sqrt{\frac{\omega^2}{c_0^2}-(k_x^2+k_y^2)}$

According to the negative k, the waves emerge from the right side of the medium and are *enhanced* in amplitude by the transmission process!

T = $e^{ik_z d}=e^{+|k_z|d}$

Since $d=a+b$, the damping of the evanescent waves in air is compensated so that they can fully contribute to the image formation without loss of information.

Pendry, Phys. Rev. Lett. (2000)