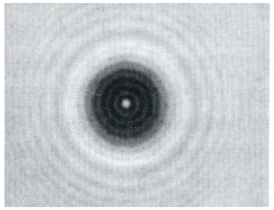
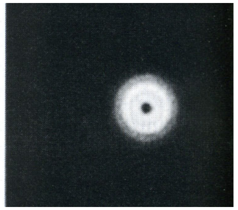
Diffraction at a Circular Aperture: Babinet Principle

Circular Disc



Circular Aperture



The diffraction pattern at an aperture and a complementary disc are equal in intensity if the undiffracted light is subtracted.

Demtröder, Experimentalphysik 2

Generalized Babinet Principle

for an infinitely thin, ideally conducting film

The following relations hold for the diffracted fields (near- and far-fields) of an aperture A and its complementary disc K:

$$E_A \rightarrow -H_K$$
$$H_A \rightarrow E_K$$

Near-fields at a circular nano-aperture: approximate solution of Bouwkamp

Oblate-spheroidal coordinates (with radius a of the aperture):

$$z = auv, \quad x = a\sqrt{(1-u^2)(1+v^2)}\cos\varphi, \quad y = a\sqrt{(1-u^2)(1+v^2)}\sin\varphi,$$

where $0 \le u \le 1$, $-\infty \le v \le \infty$, $0 \le \theta \le 2\pi$. The surfaces $v = 0$ and $u = 0$ correspond to the aperture and the screen, respectively.

Defining equation (spheroid)

$$\frac{x^2+y^2}{1+v^2} + \frac{z^2}{v^2} = a^2$$

Solution for $a \ll \lambda$ (valid for distances to the aperture $r \ll a$):

$$E_x/E_0 = ikz - \frac{2}{\pi} ikau \left[1 + v \arctan v + \frac{1}{3}\frac{1}{u^2+v^2} + \frac{x^2-y^2}{3a^2(u^2+v^2)(1+v^2)^2}\right]$$

$$E_y/E_0 = -\frac{4ikxyu}{3\pi a(u^2+v^2)(1+v^2)^2},$$

$$E_z/E_0 = -\frac{4ikxv}{3\pi(u^2+v^2)(1+v^2)},$$

$$H_x/H_0 = -\frac{4xyv}{\pi a^2(u^2+v^2)(1+v^2)^2},$$

$$H_y/H_0 = 1 - \frac{2}{\pi}\left[\arctan v + \frac{v}{u^2+v^2} + \frac{v(x^2-y^2)}{\pi a^2(u^2+v^2)(1+v^2)^2}\right]$$

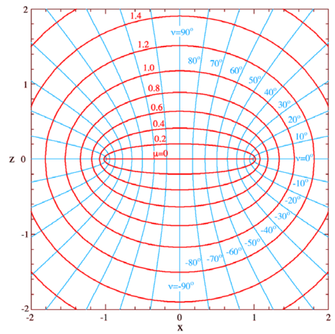
$$H_z/H_0 = -\frac{4ayu}{\pi a^2(u^2+v^2)(1+v^2)},$$

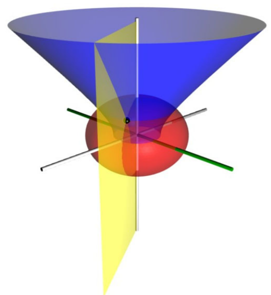
Novotny & Hecht, Nanooptics, p. 190-191

Oblate-spheroidal coordinates

Coordinate transformation:

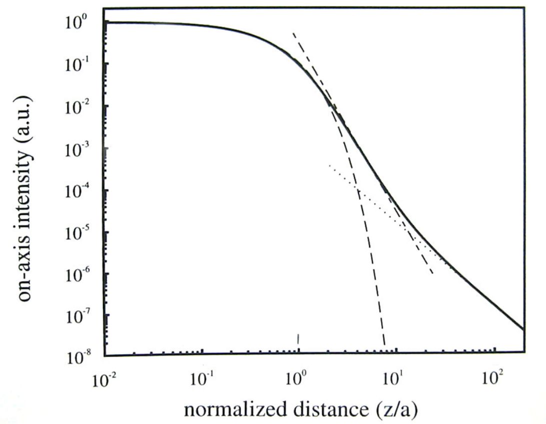
$$x = a \cosh\mu \cos\nu \cos\phi \qquad v = \sinh\mu$$
$$y = a \cosh\mu \cos\nu \sin\phi \qquad u = \sin\nu$$
$$z = a \sinh\mu \sin\nu$$





Wikipedia

On-axis intensity behind a circular aperture in an ideal conductor



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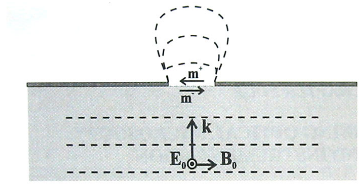
M.H.P.Moers (1995)

Transmission through Circular Apertures



Near-field Intensity along the symmetry axis

$$I(0,0,z) \propto \left(\frac{4kD}{3\pi}\right)^2 \exp\left(-\frac{3\pi z}{2D}\right)$$ with aperture diameter D

Cross section for radiation to the far-field

$$\sigma = \frac{64}{27\pi} k^4 D^6$$

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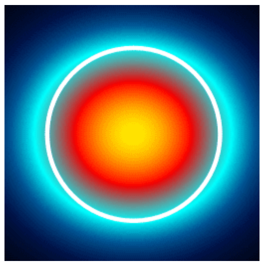
Bouwkamp (1950); M.H.P.Moers (1995)

Field-Map at an Aperture



diameter D=50 nm distance 50 nm

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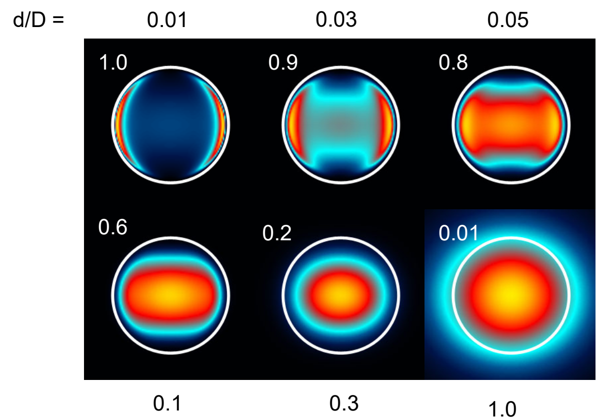
Bouwkamp (1950)

Field-Map at a Circular Aperture



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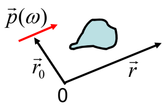
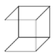
Field Susceptibility Method (Green's dyadic)



$$\vec{E}(\vec{r},\omega) = \underline{S}(\vec{r},\vec{r}_0,\omega)\cdot\vec{p}(\omega)$$

Field susceptibility (or Green's dyadic function or Field Propagator)

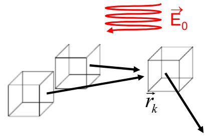
Incident field polarizes the cells
⇒ a dipole is induced



Polarization Polarizability

$$\vec{p}(r_k,\omega) = \chi(r_k,\omega)\cdot\vec{E}(\vec{r}_k,\omega)$$

Each induced dipole creates a field that interacts with each of the other cells



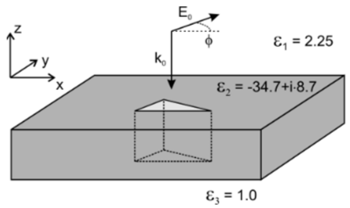
Induced field

$$\vec{E}(\vec{r},\omega) = \underline{S}(\vec{r},\vec{r}_k,\omega)\cdot\vec{p}(\vec{r}_k,\omega)$$

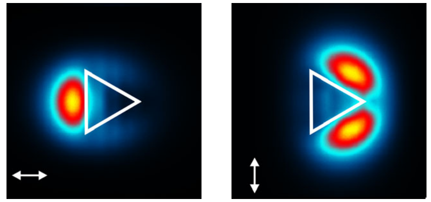
Lippman-Schwinger equation:

$$\vec{E}(\vec{r},\omega) = \vec{E}_0(\vec{r},\omega) + a^3\sum_{\text{all cells}}\underline{S}(\vec{r},\vec{r}_k,\omega)\cdot\chi\cdot\vec{E}(\vec{r}_k,w)$$

Numerical Simulation

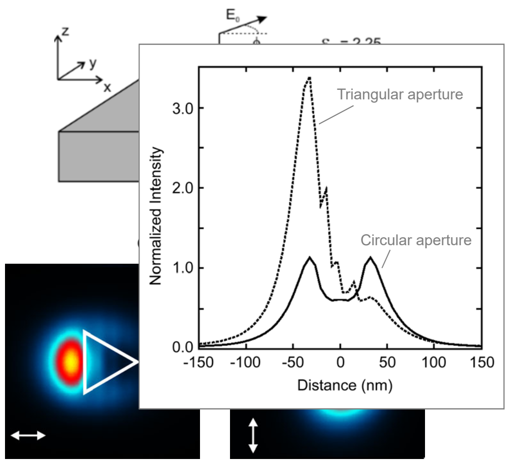


Calculated field intensity



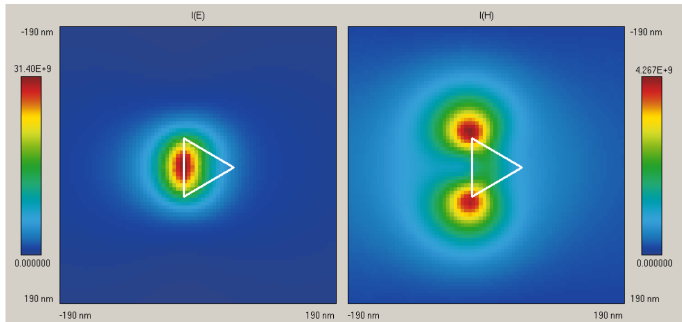
G. Colas des Francs et al. (2005)

Numerical Simulation



G. Colas des Francs et al. (2005)

Numerical Calculations: Electric and Magnetic Fields at a Triangular Aperture



Aperture side length: 80 nm
Gold thickness : 300 nm
Distance: 20 nm

Software: COMSOL

T. Ergin