A brief tour of the Standard Model

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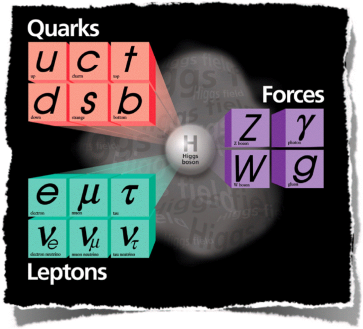
Standard Model (SM) in a nutshell

Basic ingredients

- **matter fields** – three generations of quark and leptons
- **gauge fields** – force carriers of strong, weak and electromagnetic interactions
- **Higgs field** – responsible for electroweak symmetry breaking, particle masses



Gauge symmetry

gauge symmetry group G_{SM} – group of local symmetry transformations under which SM Lagrangian is invariant

$$G_{\text{SM}} = \underbrace{SU(3)_c}_{\text{QCD}} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{electroweak}}$$

Interactions and force carriers

- $SU(3)_c$ – strong interactions (QCD) ➢ gluons
- $SU(2)_L \times U(1)_Y$ – electroweak (EW) interactions ➢ $W^\pm$, Z, photon

Interlude: Lie groups

$\boldsymbol{SU(N)}$ – continuous groups ("Lie groups") of special unitary $N \times N$ matrices, i. e. $U_N^\dagger U_N = \mathbb{1}$, $\det U_N = 1$

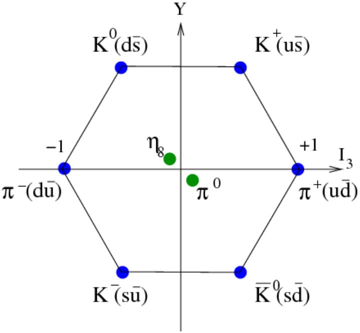
- $N^2 - 1$ generators
- for local symmetry: each generator corresponds to one massless spin-1 particle ➢ $N^2 - 1$ gauge bosons
- non-abelian: elements do not commute ➢ gauge bosons self-interact

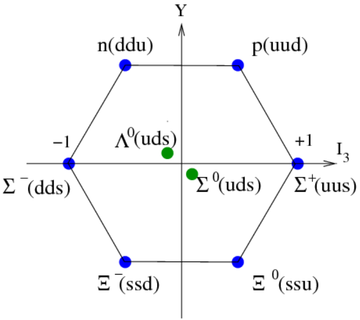
$\boldsymbol{U(1)}$ – group of 1-dim. unitary transformations, i. e. rotations by complex phase $e^{i\phi}$

- abelian, one generator

Basics of QCD

- observed spectra of mesons and baryons led to the postulation of three quark flavours u, d, s which carry colour-charges (red, green, blue)





Quantum chromodynamics (QCD)

Basics of QCD

- observed spectra of mesons and baryons led to the postulation of three quark flavours u, d, s which carry colour-charges (red, green, blue)
- (q_r, q_g, q_b) form a triplet under the gauge symmetry $SU(3)$
- the 8 gauge bosons of $SU(3)$ are the gluons
- mesons and baryons carry no colour, they are $SU(3)$ singlets
  ➢ confinement

The QCD Lagrangian

$$\mathcal{L}_{\mathsf{QCD}} = \bar{q}_i (i \not{D}_{ij} - m \delta_{ij}) q_j - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$

- q_i: quark field of colour $i = r, g, b$ with mass m

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- $D_{\mu,ij}$: covariant derivative

$$D_{\mu,ij} = \partial_\mu \delta_{ij} + i g_s T^a_{ij} G^a_\mu$$

- G^a_μ: gluon field ($a = 1, \ldots, 8$)

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- G^a_μ: gluon field $(a = 1, \ldots, 8)$
- $G^a_{\mu\nu}$: gluon field strength tensor

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\nu + g_s f_{abc} G^b_\mu G^c_\nu$$

Electroweak symmetry

$SU(2) \times U(1)_Y$

- unified theory of weak $(W^{\pm}, Z)$ and electromagnetic (photon) interactions ➢ gauge couplings g, g'

$$-\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

- 3+1 electroweak gauge bosons
  - massless photon ✓

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- 3+1 electroweak gauge bosons
  - massless photon ✓
  - $W^\pm, Z$ with

$$m_W = 80\,\text{GeV} \qquad m_Z = 91\,\text{GeV}$$

➢ **electroweak symmetry is broken, only $U(1)_{\text{em}}$ conserved**

The Higgs field

- introduce a complex scalar field

$$H = \begin{pmatrix} \phi^+ \\ (h + i\phi^0)/\sqrt{2} \end{pmatrix}$$

$SU(2)_L$ doublet with hypercharge $Y_H = 1/2$

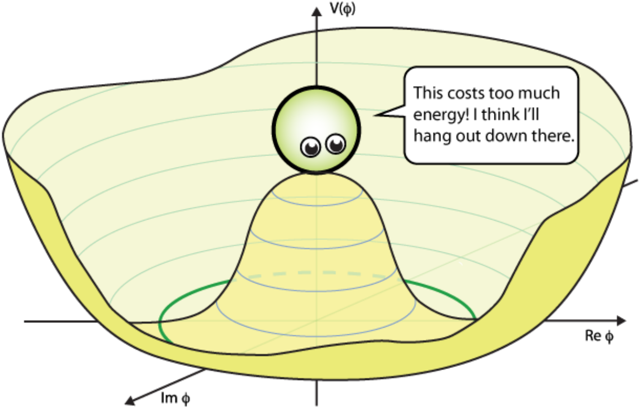
- most general renormalisable potential

$$V(H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

For $\mu^2 < 0$, the potential has a "mexican hat" shape and the vacuum breaks electroweak symmetry:

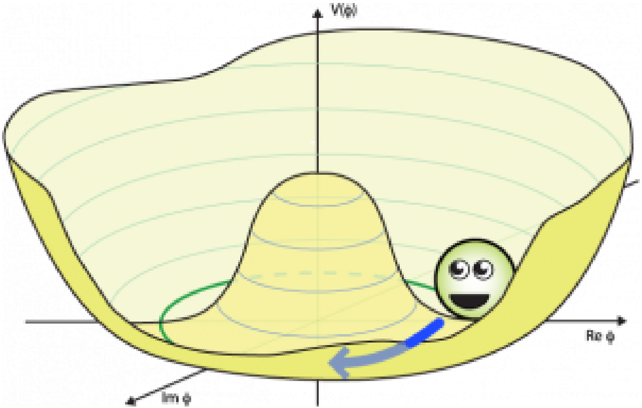
$$\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0$$

Spontaneous symmetry breaking



Thanks to Flip Tanedo for the cartoon!

Spontaneous symmetry breaking



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Masses for $W^{\pm}$ and Z

Inserting $\langle H \rangle$ in couplings between Higgs and $SU(2)_L \times U(1)_Y$ gauge fields gives

$$\boxed{\frac{g^2 v^2}{4} W_\mu^+ W^{\mu -} + \frac{(g^2 + g'^2) v^2}{4} Z_\mu Z^\mu}$$

with $(\tan\theta_W = g'/g)$

$$\begin{aligned} W_\mu^\pm &= (W_\mu^1 \mp i W_\mu^2)/\sqrt{2} \\ Z_\mu &= \cos\theta_W W_3^\mu - \sin\theta_W B_\mu \\ A_\mu &= \sin\theta_W W_3^\mu + \cos\theta_W B_\mu \end{aligned}$$

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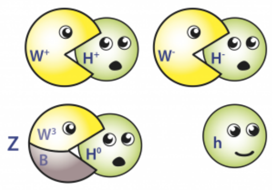
➢ **mass terms for $W^\pm$ and Z, while photon A_μ remains massless**

The Higgs degrees of freedom

- $\phi^\pm$ and ϕ^0 can be removed from theory by suitable gauge transformation
  ➢ absorbed by $W^\pm$ and Z to become their third degree of freedom

  Goldstone bosons "eaten" by the gauge bosons corresponding to the broken generators

- one physical degree of freedom remains – Higgs boson h with $m_h \sim 125\,\mathrm{GeV}$



Quark sector

Left-handed quarks are introduced as $SU(2)_L$ doublets

$$Q_j = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

and right-handed quarks as $SU(2)_L$ singlets

$$U_j = u_R, c_R, t_R \qquad D_j = d_R, s_R, b_R$$

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The correct hypercharges Y can be obtained from the electric charge

$$Q = T_3 + Y$$

where $T_3 = \pm 1/2$ for doublet components and $T_3 = 0$ for singlets.

Lepton sector

Also the left-handed leptons are $SU(2)_L$ doublets

$$L_j = \begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix}$$

and the right-handed charged leptons are $SU(2)_L$ singlets

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Leptons do not carry QCD colour, and $U(1)_Y$ hypercharges are chosen according to $Q = T_3 + Y$.

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Note that the SM does *not* contain right-handed neutrinos!
➢ no neutrino masses within the SM

Yukawa sector and fermion masses

- chiral transformations properties under EW symmetry prohibits Dirac mass terms for quarks and leptons.
- Yukawa couplings are invariant under gauge symmetry

$$\mathcal{L}_{\mathsf{Yuk}} = \sum_{i,j=1}^{3} (-Y_{U,ij}\bar{Q}_{Li}\tilde{H}U_{Rj} - Y_{D,ij}\bar{Q}_{Li}HD_{Rj} - Y_{E,ij}\bar{L}_{Li}HE_{Rj} + h.c.)$$

where i, j are generation indices and $\tilde{H} = \epsilon H^* = (H^{0*}, -H^-)^T$

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where i, j are generation indices and $\tilde{H} = \epsilon H^* = (H^{0*}, -H^-)^T$

- EW symmetry breaking yields fermion mass terms

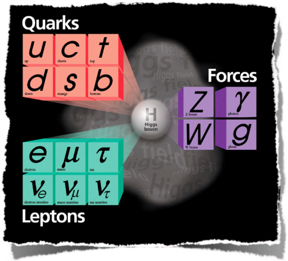
$$\sum_{i,j=1}^{3} (-m_{U,ij}\bar{u}_{Li}u_{Rj} - m_{D,ij}\bar{d}_{Li}d_{Rj} - m_{E,ij}\bar{\ell}_{Li}\ell_{Rj} + h.c.)$$

with fermion masses given by $m_A = vY_A$ $(A = U, D, E)$

Summary

Study goal: brief recap of the Standard Model

- ➢ particle content
- ➢ gauge symmetry
- ➢ Higgs mechanism
- ➢ generation of particle masses



Further reading (if needed)

- M. Mühlleitner, *The Standard Model of Particle Physics*, `https://www.itp.kit.edu/~maggie/icise/standardmodel.pdf`
- your favourite particle physics textbook