The flavour puzzle

Monika Blanke

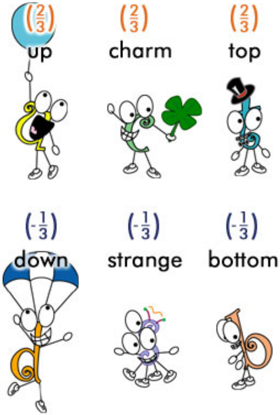
What is flavour?

Three generations of quarks and leptons

- identical gauge quantum numbers
- different masses

➢ **flavour physics** describes interactions that distinguish between flavours

here: only quark flavour physics



Quarks in the SM

Recall: parity violation of electroweak interactions

➢ left-handed quarks are introduced as $SU(2)_L$ doublets

$$Q_j = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

and right-handed quarks as $SU(2)_L$ singlets

$$U_j = u_R, c_R, t_R \qquad D_j = d_R, s_R, b_R$$

Gauge couplings of the quarks

$$\mathcal{L}_{\text{fermion}} = \sum_{j=1}^{3} \bar{Q}_j i \not{D}_Q Q_j + \bar{U}_j i \not{D}_U U_j + \bar{D}_J i \not{D}_D D_j$$

with the covariant derivatives ($Y_Q = 1/6$, $Y_U = 2/3$, $Y_D = -1/3$)

$$\begin{aligned} D_{Q,\mu} &= \partial_\mu + i g_s T^a G^a_\mu + i g \tau^a W^a_\mu + i Y_Q g' B_\mu \\ D_{U,\mu} &= \partial_\mu + i g_s T^a G^a_\mu \qquad\qquad\quad + i Y_U g' B_\mu \\ D_{D,\mu} &= \partial_\mu + i g_s T^a G^a_\mu \qquad\qquad\quad + i Y_D g' B_\mu \end{aligned}$$

➢ **flavour universality**: gauge couplings are equal for all three generations

Yukawa couplings

flavour non-universality introduced by Yukawa couplings between the Higgs field and the quarks:

$$\mathcal{L}_{\mathsf{Yuk}} = \sum_{i,j=1}^{3} \left(-Y_{U,ij}\bar{Q}_{Li}\tilde{H}U_{Rj} - Y_{D,ij}\bar{Q}_{Li}HD_{Rj} + h.c.\right)$$

where i, j are generation indices and $\tilde{H} = \epsilon H^* = (H^{0*}, -H^-)^T$

➢ replacing H by its vacuum expectation value $\langle H \rangle = (0, v)^T$, we obtain the quark mass terms

$$\sum_{i,j=1}^{3} \left(-m_{U,ij}\bar{u}_{Li}u_{Rj} - m_{D,ij}\bar{d}_{Li}d_{Rj} + h.c.\right)$$

with the quark mass matrices given by $m_A = vY_A$ $(A = U, D, E)$.

Diagonalising the fermion mass matrices

- quark mass matrices m_U, m_D, m_L are 3×3 complex matrices in the generation space ("flavour" space) with *a priori* arbitrary entries

- can be diagonalised by bi-unitary field redefinitions

$$u_L = \hat{U}_L u_L^m \qquad u_R = \hat{U}_R u_R^m \qquad d_L = \hat{D}_L d_L^m \qquad d_R = \hat{D}_R d_R^m$$

  with ^{m} denoting quarks in the mass eigenstate basis

- in this basis

$$m_U^{\text{diag}} = \hat{U}_L^\dagger m_U \hat{U}_R \qquad m_D^{\text{diag}} = \hat{D}_L^\dagger m_D \hat{D}_R$$

  are diagonal

➢ **Is the SM Lagrangian invariant under these field redefinitions?**

The CKM matrix

- transformations of the right-handed quarks are indeed unphysical, i. e. they leave the rest of the Lagrangian invariant
- however, u_{Li} and d_{Li} form the $SU(2)_L$ doublets Q_{Li}
  ➢ kinetic term gives rise to the interaction

$$\frac{g}{\sqrt{2}}\bar{u}_{Li}\gamma_\mu W^{\mu+}d_{Li}$$

- transforming to the mass eigenstate basis, we obtain

$$\frac{g}{\sqrt{2}}\bar{u}_{Li}\hat{U}^\dagger_{L,ij}\hat{D}_{L,jk}\gamma_\mu W^{\mu+}d_{Lk}$$

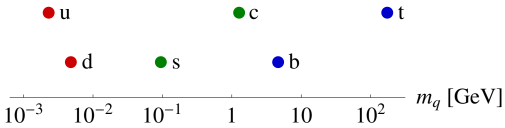
➢ The combination $\hat{V}_{\text{CKM}} = \hat{U}^\dagger_L\hat{D}_L$ is physical and is called the CKM matrix. It leads to flavour violating charged current interactions.

Present status

CKM matrix is found to be close to the unit matrix

$$|V_{\mathsf{CKM}}| \sim \begin{pmatrix} \mathbf{0.974} & 0.225 & 0.0036 \\ 0.225 & \mathbf{0.973} & 0.041 \\ 0.0089 & 0.041 & \mathbf{0.999} \end{pmatrix}$$

also quark masses exhibit strong hierarchy



Where does this hierarchical structure come from?

➢ **flavour hierarchy problem**

Flavour changing neutral currents

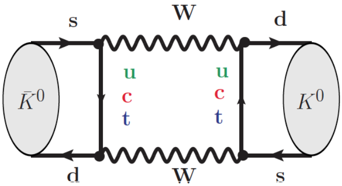
What about flavour changing neutral currents (FCNCs)?

- absent at the tree level, because

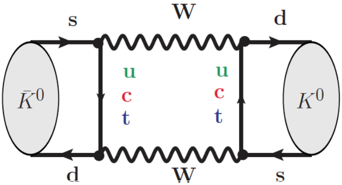
$$g_Z \bar{d}_{Li} \hat{D}^\dagger_{L,ij} \hat{D}_{L,jk} \gamma_\mu Z^\mu d_{Lk} \equiv g_Z \bar{d}_{Li} \gamma_\mu Z^\mu d_{Li}$$

- generated however by loop diagrams with $W^\pm$ boson exchanges

Example: neutral K meson mixing



GIM mechanism



$$\mathcal{M} \propto \sum_{i,j=u,c,t} V_{id}^* V_{is} V_{jd}^* V_{js} F(x_i, x_j)$$

$F(x_i, x_j)$: loop function that depends on mass square ratios $x_i = m_i^2/M_W^2$

CKM unitarity: $\displaystyle\sum_{i=u,c,t} V_{id}^* V_{is} = 0$

➢ **GIM mechanism** Glashow, Iliopoulos, Maiani (1970)
FCNCs are suppressed by quark mass differences $x_i - x_j$

SM suppression of FCNCs

Flavour changing neutral currents in the SM strongly suppressed by

- loop suppression $\propto g^2/(16\pi^2)$
- CKM hierarchy

$$\underbrace{V_{ts}^* V_{td}}_{K\ \text{system}} \sim 5 \cdot 10^{-4} \ll \underbrace{V_{tb}^* V_{td}}_{B_d\ \text{system}} \sim 10^{-2} < \underbrace{V_{tb}^* V_{ts}}_{B_s\ \text{system}} \sim 4 \cdot 10^{-2}$$

  ➢ K decays in general most sensitive to BSM physics

- GIM mechanism
- chirality of weak interactions: purely left-handed
  ➢ no strong RGE enhancements

Any of these could be absent in the presence of new physics!
➢ **strong sensitivity!**

New physics in the effective theory language

- SM is renormalisable
  ➢ all terms in the Lagrangian have mass dimension ≤ 4

- parametrise NP in a model-independent way by including higher dimensional effective operators consistent with SM symmetries

  ➢ effective theory

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \sum_i \frac{C_i}{\Lambda} \mathcal{O}_i^{\mathsf{dim\ 5}} + \sum_i \frac{C_i}{\Lambda^2} \mathcal{O}_i^{\mathsf{dim\ 6}} + \cdots$$

  - dimension 5: Weinberg operator ➢ neutrino masses
  - dimension 6: e. g. four fermion interactions mediating FCNCs

Effective Hamiltonian for $\Delta F = 2$ transitions

$$\mathcal{H}_{\text{eff}}^{\Delta F=2} = \frac{1}{\Lambda^2} \sum_{i=1}^{5} C_i \mathcal{O}_i + \sum_{i+1}^{3} \tilde{C}_i \tilde{\mathcal{O}}_i$$

with the four fermion operators for $B_{d,s} - \bar{B}_{d,s}$ mixing ($q = d, s$)

$$\mathcal{O}_1 = (\bar{q}^\alpha \gamma_\mu P_L b^\alpha)(\bar{q}^\beta \gamma^\mu P_L b^\beta) \quad \text{(SM operator)}$$

$$\mathcal{O}_2 = (\bar{q}^\alpha P_L b^\alpha)(\bar{q}^\beta P_L b^\beta) \qquad \mathcal{O}_3 = (\bar{q}^\alpha P_L b^\beta)(\bar{q}^\beta P_L b^\alpha)$$

$$\mathcal{O}_4 = (\bar{q}^\alpha P_L b^\alpha)(\bar{q}^\beta P_R b^\beta) \qquad \mathcal{O}_5 = (\bar{q}^\alpha P_L b^\beta)(\bar{q}^\beta P_R b^\alpha)$$

$$\tilde{\mathcal{O}}_1 = (\bar{q}^\alpha \gamma_\mu P_R b^\alpha)(\bar{q}^\beta \gamma^\mu P_R b^\beta)$$

$$\tilde{\mathcal{O}}_2 = (\bar{q}^\alpha P_R b^\alpha)(\bar{q}^\beta P_R b^\beta) \qquad \tilde{\mathcal{O}}_3 = (\bar{q}^\alpha P_R b^\beta)(\bar{q}^\beta P_R b^\alpha)$$

and analogous expressions for $K^0 - \bar{K}^0$ mixing

α, β: colour indices

Constraints on the NP scale

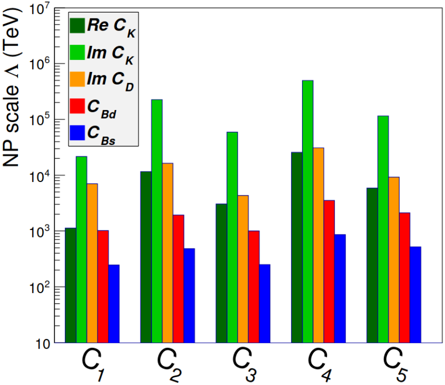
generic NP flavour structure:

$$|C_i| \sim \mathcal{O}(1) \quad \arg C \sim \mathcal{O}(1)$$

- strong constraints on $\Lambda > 100$ TeV
- CP violation in $K^0 - \bar{K}^0$ mixing most constraining

  ➢ scale of scalar LR operator C_4 restricted to $\Lambda \gtrsim 10^5$ TeV

UTFIT (2017)



The flavour of the TeV scale

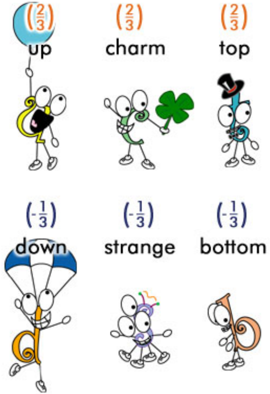
- electroweak naturalness requires NP at the TeV scale
  - ➢ apparenyly incompatible with FCNC constraints

NP flavour problem

- Can NP conserve flavour?
  - ➢ no! - flavour already violated in the SM

- Can NP conserve flavour approximately?
  - ➢ yes!
  - invoke approximate flavour symmetry
  - ...or a dynamical explanation of flavour hierarchies

Summary



Study goal: flavour puzzle

- ➢ Yukawa sector and CKM matrix
- ➢ flavour hierarchies in the SM
- ➢ FCNCs and GIM mechanism
- ➢ flavour probes of NP

Reading assignment

- chapters 1 and 3.1, 3.2 of M. Blanke, *Introduction to Flavour Physics and CP Violation*, `arXiv:1704.03753`