Goldstone bosons – the non-linear sigma model

Monika Blanke

Higgs as pseudo-Nambo-Goldstone boson

Hierarchy problem – how to shield Higgs from quadratic divergences?

Goldstone theorem

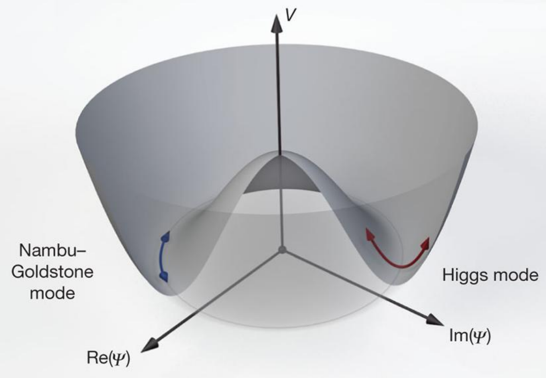
For each continuous symmetry which is spontaneously broken, there exists a massless scalar degree of freedom, the Goldstone boson.

- *local symmetry:* GB eaten by corresponding gauge boson
- *global symmetry:* GB remains as physical degree of freedom

➢ **Conceptual idea**

- implement Higgs field as GB of spontaneously broken global symmetry
- generate Higgs potentiel via small explicit symmetry breaking
- known analog: pions in QCD

A look at the potential



Higgs mode: mass from curvature of potential around minimum
Goldstone mode(s): massless, shift symmetry along flat direction(s)

Spontaneous symmetry breaking

Spontaneous breaking of symmetry group G to its subgroup H

- spontaneously broken symmetry ensures presence of degenerate valley at minimum of potential
- vacuum singles out point along flat direction
- GB fields parametrise motion along valley – correspond to broken directions in symmetry group
  ➢ massless as potential is flat
- *formally:* GBs span the coset G/H
  ➢ massless GB mode for each broken generator

Effective GB Lagrangian

Basic idea

- origin of symmetry breaking is irrelevant for GB dynamics
- GBs parametrically lighter than other degrees of freedem (e. g. Higgs modes) – the latter can be integrated out

➢ **effective theory description in terms of non-linear Goldstone fields**

Ansatz:

- global symmetry G broken to H by VEV Σ_0
- GBs $\Pi^a(x)$ parametrized by NGB matrix

$$U_{\mathsf{NGB}} = e^{i\Pi^a(x)T^a/f}$$

T^a – broken generators of G/H
f – pion decay constant, magnitude determined by Σ_0

Constructing the non-linear sigma model

U_{NGB} acts on VEV Σ_0 to rotate it along broken directions

$$\boxed{\Sigma = U_{\mathsf{NGB}}\left[\Sigma_0\right]}$$

exact form depends on Σ_0's symmetry transformation properties, e. g.

$$\begin{aligned} \Sigma &= U_{\mathsf{NGB}}\Sigma_0 && \text{for } \Sigma_0 \text{ in fundamental} \\ \Sigma &= U^\dagger_{\mathsf{NGB}}\Sigma_0 U_{\mathsf{NGB}} && \text{for } \Sigma_0 \text{ in adjoint} \end{aligned}$$

➢ effective GB Lagrangian built by forning invariants using the Σ field

non-linear sigma model (nlσm)

Example: QCD and the chiral Lagrangian

- **QCD Lagrangian** (full theory, three flavours)

$$\mathcal{L}_{\mathsf{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} + \sum_{i=u,d,s} \bar{q}_i (i \not{D} - m_{qi}) q_i$$

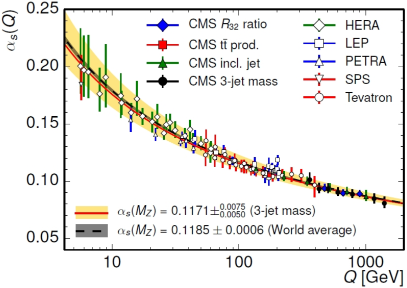
- for $m_q \to 0$: global chiral flavour symmetry $SU(3)_L \times SU(3)_R$
  ➢ should be reflected in spectrum of QCD bound states
- **observation:** bound states only reflect one $SU(3)_V$ (e. .g. meson octet)

➢ QCD dynamics leads to spontaneous breaking

$$SU(3)_L \times SU(3)_R \to SU(3)_V$$

The quark condensate

- RGE running of QCD coupling constant
  - ➢ asymptotic freedom
  - ➢ strong coupling at low energy scales, confinement

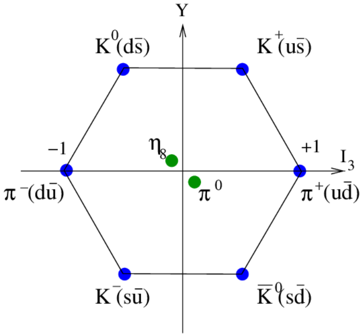


- formation of quark condensate at scale Λ_{QCD}

$$\langle \bar{q}q \rangle = \langle \bar{q}_{Li} q_{Rj} + h.c. \rangle \propto \delta_{ij} \Lambda^3_{\text{QCD}}$$

- $\langle \bar{q}q \rangle$ condensate spontaneously breaks $SU(3)_L \times SU(3)_R \to SU(3)_V$
- 8 GBs, corresponding to broken $SU(3)_A$ generators, forming pseudoscalar octet

Pseudoscalar meson octet



- $\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta_8$ form octet of pseudoscalar mesons
- GBs of spontaneously broken global $SU(3)_A$ symmetry
- finite quark masses constitute small explicit symmetry breaking
  ➢ pseudoscalar mesons acquire mass
- significantly lighter than other QCD bound states (η_1, excited mesons, baryons. . .)
  ➢ protected by global symmetry

Constructing the low-energy effective Lagrangian

- $\langle \bar{q} q \rangle$ is bi-fundamental under $SU(3)_L \times SU(3)_R$
  ➢ assume bi-fundamental VEV

$$\Sigma_0 = f \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

- symmetry transformation $\Sigma_0 \to U_L \Sigma_0 U_R^\dagger$
- Σ_0 invariant under $SU(3)_V$ $(U_L = U_R)$, but breaks $SU(3)_A$ $(U_L = U_R^\dagger)$
- define non-linearly realised pion field using U_{NGB}

$$\boxed{\Sigma(x) = e^{i\Pi^a T^a / f} \Sigma_0 e^{i\Pi^a T^a / f}}$$

Transformation properties

- **unbroken symmetry $SU(3)_V$**

$$\Sigma(x) \to U_V \Sigma(x) U_V^\dagger$$

➢ linearising in pion fields: $\Pi^a T^a \to U_V \Pi^a T^a U_V^\dagger$
i. e. pion fields transform in the adjoint of $SU(3)_V$ (c f. meson octet)

- **broken symmetry $SU(3)_A$**

$$\Sigma(x) \to U_A \Sigma(x) U_A = f e^{2i\Pi^{a\prime} T^a / f}$$

with $\Pi^{a\prime} T^a = \Pi^a T^a + f c^a T^a + \mathcal{O}\left((\Pi^a)^2\right)$

➢ shift symmetry – proof that pions are massless

➢ pions transform non-linearly

Leading order Lagrangian

Guiding principle:

- write down all terms in $\Sigma(x)$ that are allowed by the full $SU(3)_L \times SU(3)_R$ symmetry
- organise them by number of derivatives (powers of momenta)

- zero-derivative term $\mathrm{Tr}\,\Sigma^\dagger\Sigma \propto \mathrm{Tr}\,\mathbb{1}$
- first non-trivial term contains two derivatives

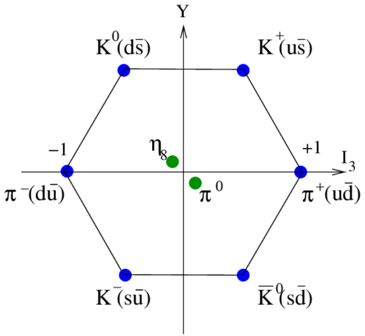
$$\frac{1}{4}\,\mathrm{Tr}\left[(\partial_\mu\Sigma)^\dagger\partial^\mu\Sigma\right]$$

➢ pion kinetic term $\mathrm{Tr}[\partial_\mu\Pi\partial^\mu\Pi]$
➢ leading 4-pion interaction term $\frac{4}{f^2}\,\mathrm{Tr}[\partial_\mu\Pi\partial^\mu\Pi\,\Pi^2]$ etc.

Summary

Study goal: Goldstone bosons

- ➢ Goldstone theorem
- ➢ non-linear sigma model
- ➢ QCD and chiral Lagrangian



Reading assignment

- chapter 2–2.2 of C. Csaki, S. Lombardo, O. Telem, *TASI Lectures on Non-Supersymmetric BSM Models*, `arXiv:1811.04279`