Dynamical electroweak symmetry breaking

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Goldstone bosons – the QCD example

Lessons learned

- QCD condensate breaks global chiral flavour symmetry
- GBs of $SU(3)_A$ form pseudoscalar meson octet
- GB dynamics described by non-linear sigma model

Next steps:

- introduce explicit symmetry breaking ➢ small GB mass
- include gauged electroweak (EW) symmetry

Explicit breaking – spurion approach

Sources of explicit chiral symmetry breaking

- quark masses – mass matrix $M = \mathrm{diag}(m_u, m_d, m_s)$
  treating M as bi-fundamental spurion, we obtain leading operator

$$\Delta\mathcal{L} = \mu^2\,\mathrm{Tr}\left[M(\Sigma + \Sigma^\dagger)\right] \propto \mu^2 f\,\mathrm{Tr}\left[M\left(\frac{\Pi^a T^a}{f}\right)^2\right] + \cdots$$

  ➢ shift in pion mass $\Delta m_\pi^2 \propto \mu^2 \frac{m_q}{f}$

- quark electric charges – charge matrix $Q = \mathrm{diag}(2/3, -1/3, -1/3)]$

$$\Delta\mathcal{L} = e^2\,\mathrm{Tr}[Q\Sigma^\dagger Q\Sigma]$$

  ➢ charged-neutral pion mass splitting

Gauging $SU(2)_L \times U(1)_Y$

Caveat: in three-flavour QCD, only the strange quark is present, but not charm – however s_L and c_L form $SU(2)_L$ doublet

- *formally correct procedure:* start with four flavours and integrate out charm quark at scale m_c
- *pragmatic ansatz:* focus on gauging of $SU(2)_L$ subgroup of $SU(3)_L$

➢ weakly gauge effective GB Lagrangian by replacing ∂_μ with

$$D_\mu = \partial_\mu - igW_\mu^a \left(\begin{array}{c|c} \tau^a/2 & \\ \hline & \end{array}\right) - ig' B_\mu \left(\begin{array}{c|c} 1/6 \cdot \mathbb{1} & \\ \hline & \end{array}\right)$$

Dynamical EW symmetry breaking

- gauging EW symmetry promotes leading term in chiral Lagrangian to

$$\frac{1}{4}\,\mathrm{Tr}\left[(D_\mu\Sigma)^\dagger D^\mu\Sigma\right]$$

- expand in pion fields and rotate to gauge boson mass eigenstates

$$\frac{g^2 f^2}{4} W_\mu^+ W^{\mu -} + \frac{(g^2 + g'^2) f^2}{4}\frac{1}{2} Z_\mu Z^\mu + \cdots$$

➢ dynamical chiral symmetry breaking induces spontaneous EW symmetry breaking

➢ pion decay constant f plays role of VEV v

QCD and chiral symmetry breaking – summary

Take-home message: chiral symmetry breaking provides neat example how Higgs mechanism could be realised dynamically

- global chiral symmetry broken spontaneously by strongly coupled QCD condensate
- Goldstone bosons (pions) as light scalar degrees of freedom
- explicit symmetry breaking generates small, non-zero GB mass – parametrically lighter than other strong resonances
- EW symmetry is gauged subgroup of chiral symmetry ➢ spontaneous breaking
- pions play role of Higgs field

Cutoff scale Λ

Non-linear sigma model description of GB dynamics is effective theory – region of validity characterised by cutoff scale Λ

- scale at which we expect new particles/resonances
- cutoff scale for radiative corrections to Higgs potential

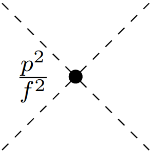
➢ **size of Λ** – energy scale at which divergent loop contributions become as large as tree-level ones

- can be estimated by naive dimensional analysis
- effective theory remains calculable

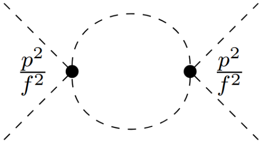
Pion-pion scattering

4-point pion-pion interaction vertex

$$\frac{1}{4}\,\mathrm{Tr}\left[(\partial_\mu\Sigma)^\dagger\partial^\mu\Sigma\right] \supset \frac{\Pi^2(\partial\Pi)^2}{f^2} \to \frac{p^2}{f^2}\text{vertex}$$



one-loop contribution



$$\sim \frac{1}{f^4}\int d^4k\,\frac{k^2p^2}{(k^2)^2} \sim \frac{p^2\Lambda^2}{16\pi^2 f^4} = \left(\frac{\Lambda}{4\pi f}\right)^2 \frac{p^2}{f^2}$$

➢ one-loop contribution must not exceed tree-level: $\boxed{\Lambda \lesssim 4\pi f}$

Naive dimensional analysis (NDA)

NDA limit: $\Lambda = 4\pi f$ realised if all resonances strongly coupled, $g_* \sim 4\pi$

- resonance mass $m_\rho \sim g_* f \sim 4\pi f$
- lower resonances possible if their coupling is weaker

NDA estimate for generic term in chiral Lagrangian

- every GB field suppressed by $1/f$
- remaining dimensions made up by $m_\rho = g_* f$
- define dim.less $x = \Pi/f = g_*\Pi/m_\rho$, $y = \partial/m_\rho$
- construct dim.less function $\tilde{\mathcal{L}}(x, y)$
- obtain dim. 4 Lagrangian as $\mathcal{L} = m_\rho^4/g_*^2 \tilde{\mathcal{L}}(x, y)$

➢ **easy-to-follow prescription to determine size of contribution**

First steps towards a composite Higgs model

Starting point: mimic QCD chiral symmetry breaking at higher scale

Technicolour models

- strong dynamics with chiral symmetry breaking pattern $SU(2)_L \times SU(2)_R \to SU(2)_V$
- dynamical scale Λ_{TC} gives rise to condensate $\langle \bar{q} q \rangle = \Lambda^3_{\mathsf{TC}}$ (q: techniquarks)
- QCD results rescaled by $\Lambda_{\mathsf{QCD}} \to \Lambda_{\mathsf{TC}}$
- 3 GBs eaten by $W^{\pm}$, Z, hence no physical light Higgs boson

➢ **excluded by discovery of SM-like Higgs boson at $m_h \sim 125\,$GeV**

Ingredients for a realistic model

Shortcoming of technicolour

GBs transform as $SU(2)_L$ adjoint, hence three degrees of freedom (d.o.f.)
recall: SM higgs field is a complex $SU(2)_L$ fundamental, i. e. four d.o.f.

➢ **needed:** more complicated symmetry breaking pattern that generates GBs in the correct EW rrepresentation

Desired pattern

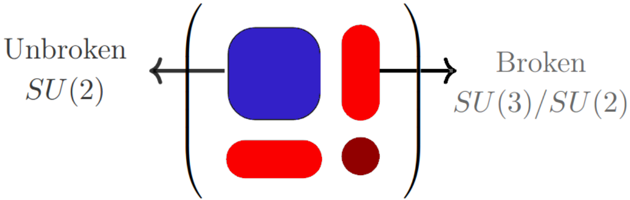
- two-step symmetry breaking
  1. spontaneous breaking of global symmetry generates light GBs in the EW $\mathbf{2}_{1/2}$ representation
  2. EW symmetry breaking from radiatively generated Higgs potential
- separation of scales $\Lambda_{\text{strong}} \gg 4\pi m_h$ to comply with precision data

Choice of symmetry breaking pattern

Model requirements

- coset G/H contains $SU(2)_L$ doublet to be identified as pNGB Higgs
- G explicitly broken, generating Higgs mass term and quartic

Minimal choice: $G = SU(3) \to H = SU(2)$



➢ Higgs as $SU(2)$ doublet, additional $SU(2)$ singlet

More formally

- global $SU(3) \to SU(2)$ breaking by VEV $\Sigma_0 = (0,0,f)^T$ in fundamental representation of $SU(3)$
- number of GBs: $N_{\text{GB}} = (3^2-1)-(2^2-1) = 5$
- broken $SU(3)$ generators correspond to $\lambda_4 \dots \lambda_8$

$$U_{\text{NGB}} = \exp\left[\frac{i}{f}\left(\begin{array}{cc|c} \eta/\sqrt{6} & & \\ & \eta/\sqrt{6} & H \\ \hline \multicolumn{2}{c|}{H^\dagger} & -2\eta/\sqrt{6} \end{array}\right)\right]$$

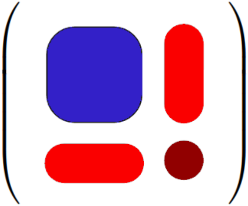
- H identified as SM Higgs doublet
- non-linear sigma model field

$$\Sigma = U_{\text{NGB}}\Sigma_0 = \begin{pmatrix} iH \\ f - \frac{H^\dagger H}{2f} \end{pmatrix} + \cdots$$

Summary

Study goal: dynamical electroweak symmetry breaking

- ➢ QCD example
- ➢ impact of explicit breaking
- ➢ naive dimensional analysis
- ➢ technicolour
- ➢ $SU(3) \to SU(2)$ symmetry breaking pattern



Reading assignment

- chapter 2.3–3.0 (until p.17) of C. Csaki, S. Lombardo, O. Telem, *TASI Lectures on Non-Supersymmetric BSM Models*, `arXiv:1811.04279`