Collective symmetry breaking & Little Higgs

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The Simplest Little Higgs

- global $SU(3) \to SU(2)$ breaking by VEV $\Sigma_0 = (0, 0, f)^T$ in fundamental representation of $SU(3)$
- broken $SU(3)$ generators correspond to $\lambda_4 \ldots \lambda_8$

$$U_{\mathsf{NGB}} = \exp\left[\frac{i}{f}\left(\begin{array}{cc|c} \eta/\sqrt{6} & & \\ & \eta/\sqrt{6} & H \\ \hline & H^\dagger & -2\eta/\sqrt{6} \end{array}\right)\right]$$

- H identified as SM Higgs doublet, extra $SU(2)$ singlet η
- non-linear sigma model field

$$\Sigma = U_{\mathsf{NGB}}\Sigma_0 = \begin{pmatrix} iH \\ f - \frac{H^\dagger H}{2f} \end{pmatrix} + \cdots$$

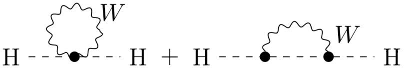
Gauging $SU(2)_L$

Gauging upper left $SU(2)$ subgroup

$$\mathcal{L}_{\mathsf{nl}\sigma\mathsf{m}} = (D_\mu \Sigma)^\dagger D^\mu \Sigma = |D_\mu H|^2 \left(1 + \frac{H^\dagger H}{f^2}\right) + \cdots$$

with $D_\mu H = \left(\partial_\mu - i\frac{g}{2} W_\mu^a \tau^a\right) H$

- gauging $SU(2)$ explicitly breaks $SU(3)$
  ➢ $|g W_\mu h|^2$ interaction breaks GB shift symmetry
- one-loop contribution to Higgs potential



$$\simeq \frac{3g^2}{64\pi^2} \Lambda^2 H^\dagger H$$

➢ reintroduces quadratic divergence! ☹

Understanding the origin of the quadratic divergence

Coleman-Weinberg potential from radiative gauge boson contributions

$$V_{\mathsf{CW}}(H) = \frac{\Lambda^2}{16\pi^2}\,\mathrm{Tr}[M_{ab}^2] + \cdots \quad \text{with } M_{ab}^2 = \frac{g^2}{4}\Sigma^\dagger \underbrace{\left(\begin{array}{c|c} \tau^a\tau^b & \\ \hline & \end{array}\right)}_{\text{spurion } P}\Sigma$$

➢ quadratically divergent because $P \neq \mathbb{1}$

Solution

extend model to global $SU(3)_1 \times SU(3)_2 \to SU(2)_1 \times SU(2)_2$ breaking by

$$\Sigma_1 = e^{i\pi_1/(\sqrt{2}f)}\begin{pmatrix}0\\0\\f\end{pmatrix} \qquad \Sigma_2 = e^{i\pi_2/(\sqrt{2}f)}\begin{pmatrix}0\\0\\f\end{pmatrix}$$

Towards collective symmetry breaking

- "double" symmetry breaking pattern to $SU(3)_1 \times SU(3)_2 \to SU(2)_1 \times SU(2)_2$ with nlσm fields Σ_1, Σ_2
- gauge diagonal subgroup $SU(3)_D$

$$\mathcal{L}_{\text{nl}\sigma\text{m}} = |D_\mu \Sigma_1|^2 + |D_\mu \Sigma_2|^2$$

➢ nlσm dynamics breaks gauged $SU(3)_D \to SU(2)_D$

- 10 GBs: 5 eaten by broken $SU(3)$ generators, 5 physical scalars $H \sim \pi_1 - \pi_2$, η
- quadratically divergent contribution to the Coleman-Weinberg potential

$$V_{\text{CW}} = \frac{\Lambda^2}{16\pi^2}\left(\text{Tr}\left[\Sigma_1^\dagger \Sigma_1\right] + \text{Tr}\left[\Sigma_2^\dagger \Sigma_2\right]\right)$$

➢ **independent of the GBs!**

More on collective breaking

Consider quadratic term: $\mathcal{L}_{\text{nl}\sigma\text{m}} \supset |gW_\mu \Sigma_1|^2 + |gW_\mu \Sigma_2|^2$

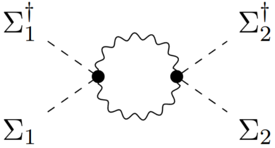
- $SU(3)_D$ gauge transformation:

$$W_\mu \to e^{i\alpha^a T^a} W_\mu e^{-i\alpha^a T^a} \quad \Rightarrow \quad \Sigma_1 \to e^{i\alpha^a T^a}\Sigma_1\,,\ \Sigma_2 \to e^{i\alpha^a T^a}\Sigma_2$$

➢ global $SU(3) \times SU(3)$ broken as **both** Σ_1 and Σ_2 couple to W

collective symmetry breaking

- quadratic contribution to Higgs potential must involve **both** terms



➢ logarithmically divergent

Who cancelled the divergence?

- *recall SUSY:* superpartners cancel quadratic divergence
- gauging of $SU(3)_D$ implies extra gauge bosons

$$\begin{pmatrix} & W^+ & X^+ \\ W- & & Y^1+1Y^2 \\ X^- & Y^1-iY^2 & \end{pmatrix} + W^3, A^8 \text{ on the diagonal}$$

- nlσm Lagrangian yields

$$\frac{g^2}{4}H^\dagger H\left[2W_\mu^+W^{-\mu}+W_\mu^3W^{3\mu}-X_\mu^+X^{-\mu}-\frac{1}{2}(Y_\mu^1Y^{1\mu}+Y_\mu^2Y^{2\mu})-A_\mu^8A^{8\mu}\right]$$

  ➢ prefactors and relative signs imply cancellation of quadratic divergence by new $SU(3)/SU(2)$ gauge bosons, i. e. particles with same spin!

The top sector, collectively

Top quark yields dominant contribution to quadratic divergence

➢ introduce top partners in complete rep.s of $SU(3)_1 \times SU(3)_2$

- formally start from $(\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})$
- keep only degrees of freedom in diagonal $SU(3)_D$ (c. f. gauge bosons)

$$\Psi = \begin{pmatrix} t_L \\ b_L \\ T_L \end{pmatrix}$$

- two right-handed $SU(3)_D$ singlets: t_1, t_2
- top Yukawa

$$\mathcal{L}_{\text{top}} = \lambda_1 \bar{\Psi} \Sigma_1 t_1 + \lambda_2 \bar{\Psi} \Sigma_2 t_2$$

➢ **collective symmetry breaking in the top sector**

The top and its partner

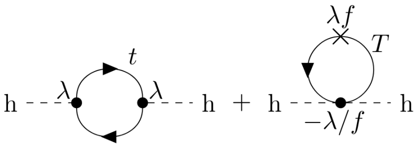
- expanding $\mathcal{L}_{\text{top}}$ in H, setting $\lambda_1 = \lambda_2 = \lambda$ and rotating to the mass eigenbasis

$$\mathcal{L}_{\text{top}} = \lambda \bar{Q}_L H t_R + \lambda f \left(1 - \frac{H^\dagger H}{2f^2}\right) \bar{T}_L T_R$$

with $t_R = 1/\sqrt{2}(t_1 - t_2)$, $T_R = 1/\sqrt{2}(t_1 + t_2)$

- SM top quark t + heavy top partner T with mass λf

- cancellation of quadratic divergence



The Littlest Higgs – basics

Littlest Higgs – a simple group model

- $SU(5) \to SO(5)$ symmetry breaking triggered by VEV

$$\Sigma_0 = f \begin{pmatrix} 0 & 0 & \mathbb{1} \\ 0 & 1 & 0 \\ \mathbb{1} & 0 & 0 \end{pmatrix}$$

➢ 14 Goldstone bosons

$$U_{\text{NGB}} = \exp\left[\frac{i}{f}\left(\begin{array}{c|c|c} & H & \Phi \\ \hline H^\dagger & & H^T \\ \hline \Phi^* & H^* & \end{array}\right)\right] + \text{eaten GBs}$$

H: SM Higgs doublet, Φ: scalar $SU(2)$ triplet

- $SU(5)/SO(5)$ coset parametrised by $\Sigma = U_{\text{NGB}}\Sigma_0 U^\dagger_{\text{NGB}}$

The Littlest Higgs – gauge sector

- gauged $[SU(2) \times U(1)]^2$ subgroup with generators

$$Q_1^a = \begin{pmatrix} \sigma^a/2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sigma^{a*}/2 \end{pmatrix}$$

$$Y_1 = \frac{1}{10}\,\mathrm{diag}(3,3,-2,-2,-2) \qquad Y_2 = \frac{1}{10}\,\mathrm{diag}(2,2,2,-3,-3)$$

- explicity breaks global symmetry ➢ GBs acquire mass
- symmetry broken collectively – each $SU(2) \times U(1)$ factor preserves global $SU(3)$
- global $SU(5) \to SO(5)$ breaking induces

$$[SU(2) \times U(1)]^2 \to SU(2) \times U(1)$$

➢ $W^{\pm}$, Z, photon partners at the scale f

The Littlest Higgs – top sector

Analogously to Simplest Little Higgs

- left-handed third generation doublet Q_L embedded into $SU(3)$ triplet

$$\bar\Psi = (\bar t_L, \bar b_L, \bar T_L) \equiv (\bar Q_L, \bar T_L)$$

➢ explicit breaking of $SU(5)$ global symmetry

- two right-handed $SU(3)$ singlets t_1, t_2
- top Yukawa $(i,j,k = 1,2,3;\ m,n = 4,5)$

$$\mathcal{L}_{\text{top}} = -\frac{\lambda_1 f}{2}\bar\Psi_i \epsilon_{ijk}\epsilon_{mn}\Sigma_{jm}\Sigma_{kn} t_1 - \lambda_2 f \bar T_L t_2$$

➢ each term preserves one $SU(3)$ subgroup – collective breaking

The Littlest Higgs – Higgs potential

Higgs potential generated radiatively – Coleman-Weinberg potential

$$V_{\mathsf{CW}}^{\mathsf{quad}} = a\frac{\Lambda^2}{16\pi^2} f^2 \sum_{j=1,2} \left(g_j^2 \,\mathrm{Tr}\left[Q_j^a \Sigma Q_j^{a*} \Sigma^*\right] + {g'_j}^2 \,\mathrm{Tr}\left[Y_j \Sigma Y_J^* \Sigma^*\right] \right)$$

g_j, g'_j: $[SU(2)\times U(1)]_j$ gauge couplings
a: $\mathcal{O}(1)$ coefficient

➢ expanding in H and Φ

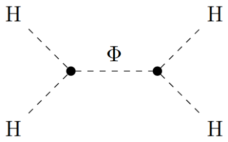
$$V_{\mathsf{CW}}^{\mathsf{quad}}(H,\Phi) = af^2 \left[(g_1 + g'_1)^2 \left| \Phi_{ij} + \frac{i}{4f}(H_i H_j + H_j H_i) \right|^2 + (g_2 + g'_2)^2 \left| \Phi_{ij} - \frac{i}{4f}(H_i H_j + H_j H_i) \right|^2 \right]$$

The Littlest Higgs – an $\mathcal{O}(1)$ quartic

Features of $V_{\mathsf{CW}}^{\mathsf{quad}}(H,\Phi)$

- quadratic divergence for H cancels as expected
- quadratically divergent mass term for Φ and $H\Phi\Phi$ coupling remain
- integrating out Φ below $M_\Phi \sim af$

$$\lambda = a\frac{(g_1^2 + {g_1'}^2)(g_2^2 + {g_2'}^2)}{g_1^2 + {g_1'}^2 + g_2^2 + {g_2'}^2}$$



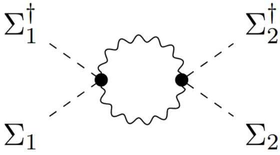
➢ $\mathcal{O}(1)$ Higgs quartic generated at tree level

- similar Higgs quartic from fermion sector
- interesting prediction: $m_h = \sqrt{2\lambda}v \sim \sqrt{2}g_{\mathsf{SM}}v \sim (200 - 300)$ GeV
  ➢ some tuning required to reconcile model with measured Higgs mass

Summary

Study goal: Little Higgs models

- Simplest Little Higgs
- collective symmetry breaking
- top and gauge partners
- Littlest Higgs model



Reading assignment

- chapter 3–3.3.1 of C. Csaki, S. Lombardo, O. Telem, *TASI Lectures on Non-Supersymmetric BSM Models*, `arXiv:1811.04279`
- chapter 1–3 of M. Perelstein, *Little Higgs Models and Their Phenomenology*, `arXiv:hep-ph/0512128`