Extra dimensions – the basics

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Why consider extra dimensions?

- viable extension of the SM
- required for mathematical consistency of string theory
- TeV-scale extra dimensions have interesting phenomenology
  - Kaluza-Klein resonances in reach of the LHC
  - gauge-Higgs unification: Higgs field from 5D gauge symmetry
  - flavour hierarchies from localisation of fermions along 5th dimension
  - weakly coupled holographic dual to strongly coupled 4D theory

Flat 5D space-time

- described by metric

$$ds^2 = g_{MN} dx^M dx^N \qquad \text{with} \quad g_{MN} = \mathrm{diag}(+1, -1, -1, -1, -1)$$

5D indices: $M, N = 0, 1, 2, 3, 5$
4D indices: $\mu, \nu = 0, 1, 2, 3$

- fifth dimension compactified to interval

$$0 \leq y = x_5 \leq \pi R$$

5D bulk: $x_\mu, y \in [0, \pi R]$
4D branes (boundaries): $x_\mu, y = 0$; $x_\mu, y = \pi R$

Scalar field on a 5D interval

bulk action for real scalar field

$$S_{\text{bulk}} = \int d^4x \int_0^{\pi R} dy \left[\frac{1}{2}\partial^M\phi\partial_M\phi - V(\phi)\right]$$

variational principle and integration by parts

$$\begin{aligned} 0 = \delta S_{\text{bulk}} &= \int d^4x \int_0^{\pi R} dy \left[\partial^M\phi\partial_M\delta\phi - \frac{\partial V}{\partial\phi}\delta\phi\right] \\ &= \int d^4x \int_0^{\pi R} dy \left[-\partial_M\partial^M\phi - \frac{\partial V}{\partial\phi}\right]\delta\phi \\ &\quad - \int d^4x\, \partial_y\phi\delta\phi\Big|_0^{\pi R} \end{aligned}$$

➢ **bulk equations of motion (EOM) & boundary conditions (BC)**

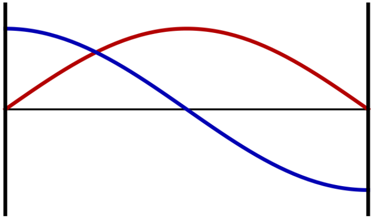
Bulk EOM & boundary conditions

Bulk EOM

$$\partial_M \partial^M \phi = -\frac{\partial V}{\partial \phi}$$

Boundary conditions

- Neumann BC: $\partial_y \phi\Big|_{\text{boundary}} = 0$
- Dirichlet BC: $\phi\Big|_{\text{boundary}} = 0$
- can differ on the two branes



Kaluza-Klein (KK) decomposition

free massive scalar field: $V(\phi) = M^2\phi^2$

ansatz: generalized Fourier decomposition

$$\phi(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \phi_n(x_\mu) f_n(y)$$

$$\partial_\mu \partial^\mu \phi_n + \underbrace{\left(-\frac{{f_n}''}{f_n} + M^2\right)}_{\text{KK mass } m_n} \phi_n = 0 \qquad {f_n}'' + \underbrace{(m_n^2 - M^2)}_{=:p_n^2} f_n = 0$$

➢ $f_n = a_n \cos p_n y + b_n \sin p_n y$ with a_n, b_n determined by BCs

- infinite tower of 4D Kaluza-Klein modes with masses $m_n^2 = M^2 + p_n^2$
- (massless) zero mode only for Neumann BCs (and $M = 0$)

Gauge theory in 5D

pure 5D gauge theory

$$S = \int d^5x \left[-\frac{1}{4} F^a_{MN} F^{MN,a} \right]$$

where $F^a_{MN} = \partial_M A^a_M - \partial_N A^a_M + g_5 f^{abc} A^b_M A^c_N$ (antisymmetric)

mass dimensions:

F^a_{MN}	5/2
A^a_M	3/2
g_5	-1/2

➢ 5D theory is non-renormalisable

➢ effective theory valid below cutoff

Gauge fixing

Goal: cancel mixing term $\partial_5 A^a_\mu \partial^\mu A^{5,a}$ ➢ add gauge fixing term

$$S_{\mathsf{GF}} = \int d^4x \int_0^{\pi R} dy \left[-\frac{1}{2\xi} (\partial_\mu A^{\mu,a} - \xi \partial_5 A^{5,a})^2 \right]$$

- A_5-independent part gives usual Lorentz gauge fixing term
- cross-term cancels mixing term from above

$$\delta S_{\mathsf{bulk}} = \int d^5x \left[\left(\partial_M F^{M\nu,a} - g_5 f^{abc} F^{M\nu,b} A^c_M + \frac{1}{\xi} \partial^\nu \partial\sigma - \partial^\nu \partial_5 A^a_5 \right) \delta A^a_\nu \right.$$
$$\left(\partial^\sigma F^a_{\sigma 5} - g_5 f^{abc} F^b_{\sigma 5} A^{\sigma,c} + \partial_5 \partial_\sigma A^{\sigma,a} - \xi \partial_5^2 A^a_5 \right) \delta A^a_5$$

➢ set terms in brackets to zero for bulk EOMs of A^a_μ and A^a_5

Physical gauge degrees of freedom

KK decomposition similar to the scalar case

- KK tower of gauge fields $A_n^{\mu,a}$ with masses m_n and propagator

$$\frac{-i\delta^{ab}}{k^2 - m_n^2}\left(g^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2 - \xi m_n^2}\right)$$

- EOM for A_5^a contains $\xi\partial_5^2 A_5^a$
  - ➢ if A_5^a is not flat along y (i.e. $m_n \neq 0$), in the unitary gauge ($\xi \to \infty$) A_5^a gets infinite mass and decouples
  - ➢ KK-modes of A_5^a are unphysical
    serve as Goldstone d.o.f. for KK modes of A_μ^a
  - ➢ only A_5^a zero mode is physical (if allowed by BCs)

Boundary gauge fixing

to eliminate boundary mixing term $\partial_\mu A^{\mu,a} A_5^a\big|_0^{\pi R}$

$$S_{\text{GF,bound}} = -\frac{1}{2\xi_{\text{bound}}} \int d^4x \left(\partial_\mu A^{\mu,a} \pm \xi_{\text{bound}} A_5^a\right)^2 \Big|_0^{\pi R}$$

➢ boundary variation

$$\left(\partial_5 A^{\mu,a} + \frac{1}{\xi_{\text{bound}}} \partial_\nu \partial^\mu A^{\nu,a}\right) \delta A_\mu^a \Big|_0^{\pi R} + \left(\xi \partial_5 A_5^a \pm \xi_{\text{bound}} A_5^a\right) \delta A_5^a \Big|_0^{\pi R}$$

Neumann (Dirichlet) BC for A_μ ➢ Dirichlet (Neumann) BC for A_5

Consequences of gauge field BCs

Neumann BC for A^a_μ

- massless zero mode $A^a_{\mu,0}$ ➢ unbroken gauge symmetry in 4D
- tower of spin-1 massive KK modes $A^a_{\mu,n}$ w/ longitudinal d.o.f. $A^a_{5,n}$
- no physical scalar A^a_5 modes

Dirichlet BC for A^a_μ

- tower of spin-1 massive KK modes $A^a_{\mu,n}$ w/ longitudinal d.o.f. $A^a_{5,n}$
- no massless zero mode $A^a_{\mu,0}$ ➢ 4D gauge symmetry broken by BCs
- physical massless scalar $A^a_{5,0}$

Mixed BCs

- tower of spin-1 massive KK modes $A^a_{\mu,n}$ w/ longitudinal d.o.f. $A^a_{5,n}$
- no massless zero modes

Symmetry breaking via BCs

- unbroken 4D gauge symmetry for Neumann BCs on both branes
- broken symmetry if at least one BC is Dirichlet

  ➢ use BCs as symmetry breaking tool
- if both BCs are Dirichlet: massless scalar for each broken generator

  ➢ similar to Goldstone bosons from spontaneusly broken global symmetry

  ➢ mass protected by 5D gauge symmetry

  ➢ use A_5 as a Higgs field

gauge-Higgs unification

Interlude: Interval vs. orbifold

We introduced 5th dimension as a compact interval $y \in [0, \pi R]$.
More traditional:

Orbifold compactification

- start with infinite extra dimension
- compactify to circle y identifying $y \to y + 2\pi R$
- reduce to half-line by $\mathbb{Z}_2$ reflection $y \to -y$

➢ action required to be invariant under both projections

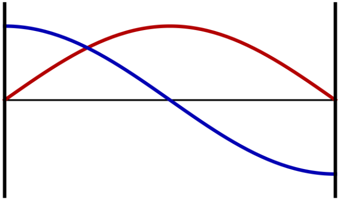
One can show: equivalent to defining orbiforld in terms of two $\mathbb{Z}_2$ reflections at $y = 0, \pi R$

- fields live on fundamental domain $y \in [0, \pi R]$
- Neumann or Dirichlet BCs on the boundaries $y = 0, \pi R$

Summary

Study goal: basics of extra dimensions

- ➢ 5D gauge and scalar fields
- ➢ KK decomposition
- ➢ boundary conditions
- ➢ symmetry breaking by BCs



Reading assignment

- chapter 2 of C. Csaki, J. Hubisz, P. Meade, *TASI Lectures on Electroweak Symmetry Breaking from Extra Dimensions*, `arXiv:hep-ph/0510275`