Fermions in extra dimensions

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Fields in 5D

Last time: gauge and scalar fields in 5D

- 5D bulk action
- boundary conditions on 4D branes
- KK decomposition
- conditions for presence of massless zero modes

Today: fermions in 5D

- Clifford algebra in 5D
- zero modes and their localization
- towards flavour hierarchies

Warmup: Fermions in 4D

- chiral (Weyl) spinor

$$\Psi = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}$$

  where $\chi, \bar{\psi}$ are two-component left- and right-handed Weyl spinors

- chiral basis for γ matrices ($\sigma^0 = \bar{\sigma}^0 = -1, \sigma^i = -\bar{\sigma}^i$ Pauli matrices)

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

  Clifford algebra: $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$

- projection operators $P_{L,R} = \frac{1}{2}(1 \mp \gamma^5)$ single out $\chi, \bar{\psi}$

Clifford algebra in 5D

Goal: extend 4D set of gamma matrices γ^μ to 5D set Γ^M such that

$$\{\Gamma^M, \Gamma^N\} = 2g^{MN}$$

➢ easy to see: $\Gamma^M = \gamma^\mu, \gamma^5$

Consequence

- 5D fermion Lagrangian couples χ and ψ even for $M = 0$
- simplest fermion representation in 5D: 4-component Dirac spinor
  ➢ non-chiral bulk fermions – in contrast to chiral fermions in the SM

Bulk spinor in 5D

- 5D bulk action

$$S = \int d^5x \left[\frac{i}{2}(\bar{\Psi}\Gamma^M \partial_M \Psi - \partial_M \bar{\Psi}\Gamma^M \Psi) - M\bar{\Psi}\Psi \right]$$

- decomposition in terms of Weyl spinors ($\overleftrightarrow{\partial_5} = \overrightarrow{\partial_5} - \overleftarrow{\partial_5}$)

$$S = \int d^5x \Big[-i\bar{\chi}\bar{\sigma}^\mu \partial_\mu \chi - i\psi\sigma^\mu \partial_\mu \bar{\psi} \\ + \frac{1}{2}(\psi \overleftrightarrow{\partial_5} \chi - \bar{\chi} \overleftrightarrow{\partial_5} \bar{\psi}) + M(\psi\chi + \bar{\chi}\bar{\psi}) \Big]$$

- bulk EOM couple Weyl spinors $\chi, \bar{\psi}$ even for $M = 0$

$$\begin{aligned} -i\bar{\sigma}^\mu \partial_\mu \chi - \partial_5 \bar{\psi} + M\bar{\psi} &= 0 \\ -i\sigma^\mu \partial_\mu \bar{\psi} + \partial_5 \chi + M\chi &= 0 \end{aligned}$$

Consistent boundary conditions

χ **and** $\bar{\psi}$ **not independent of each other** ➢ only *one* BC on each brane

variations of S including relevant y-derivatives

$$\delta S = \int d^5x \, \frac{1}{2} \left(\delta\psi \overleftrightarrow{\partial_5} \chi + \psi \overleftrightarrow{\partial_5} \delta\chi - \delta\bar{\chi} \overleftrightarrow{\partial_5} \bar{\psi} - \bar{\chi} \overleftrightarrow{\partial_5} \delta\bar{\psi} \right)$$

integration by part yields most general BC

$$-\delta\psi\chi + \psi\delta\chi + \delta\bar{\chi}\bar{\psi} - \bar{\chi}\delta\bar{\psi} = 0$$

➢ solution

$$\psi_\alpha = M_\alpha^\beta \chi_\beta + N_{\alpha\dot{\beta}} \bar{\chi}^{\dot{\beta}}$$

may be restricted by additional symmetries

Simple and common BC

Choose Dirichlet BCs for one Weyl component, e. g. $\psi\Big|_0^{\pi R} = 0$

➢ BC for other Weyl component: $(\partial_5 + M)\chi\Big|_0^{\pi R} = 0$

Resulting KK spectrum

- vectorlike KK modes $(\chi, \bar{\psi})_n$
- zero mode for χ, but not for ψ
- analogously, zero mode for ψ can be obtained

➢ although obtained from a non-chiral 5D theory,
the low energy effective theory is chiral
crucial for phenomenologically viable models!

KK decomposition

KK decomposition of Weyl spinors

$$\chi = \sum_n g_n(y)\chi_n(x) \qquad \bar{\psi} = \sum_n f_n(y)\bar{\psi}_n(x)$$

- KK modes fulfill 4D Dirac equation with KK mass m_n

$$\begin{aligned} -i\bar{\sigma}^\mu \partial_\mu \chi_n + m_n \bar{\psi}_n &= 0 \\ -i\bar{\sigma}^\mu \partial_\mu \bar{\psi}_n + m_n \chi_n &= 0 \end{aligned}$$

- insertion into bulk EOM ($h = f, g$)

$$h_n'' + (m_n^2 - M^2)h_n = 0$$

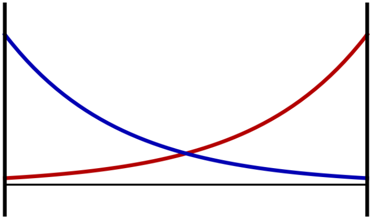
➢ solved again by sum of $\sin$ and $\cos$

Zero mode profile

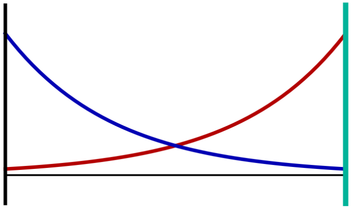
bulk fermion with BCs $\psi|_{0,\pi R} = 0$ ➢ zero mode χ_0 with profile

$$g_0(y) = g_0 e^{-My}$$

➢ fermion zero-mode profile not flat, grows or decays exponentially, depending on sign of bulk mass M



Kaplan-Tait model – a model of flavour hierarchies



- 5D toy model with bulk gauge fields and bulk fermions
- Higgs introduced as 4D field confined to one brane
  ➢ brane-localised couplings to bulk fields
- Yukawa couplings in 5D theory

$$(\pi R) Y_u \bar{u}_R H Q_L \delta(y - \pi R)$$

  with anarchic coupling Y_u
- effective zero-mode Yukawa couplings are products of fundamental coupling and fermion profiles evaluated at $y = \pi R$
  ➢ **hierarchies from exponential dependence on bulk mass M**

Kaplan-Tait model – FCNCs

coupling of fermion zero modes to gauge boson modes obtained from integral over 5th dimensions

$$g_n^0 \propto \int dy \underbrace{g_0(y)^2}_{\text{fermion}} \underbrace{f_n(y)}_{\text{gauge}}$$

➢ overlap of profiles determines coupling strength

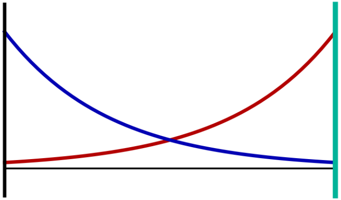
Observations

- gauge zero mode profile is flat ➢ couplings to fermions universal (as predicted by gauge symmetry)
- gauge KK profiles induce flavour-specific couplings
  ➢ large tree-level FCNCs when rotated to mass eigenbasis

Summary

Study goal: fermions in extra dimensions

- 5D Clifford algebra and chirality
- KK decomposition
- zero modes
- Kaplan-Tait model



Reading assignment

- chapter 4 until p.40 of C. Csaki, J. Hubisz, P. Meade, *TASI Lectures on Electroweak Symmetry Breaking from Extra Dimensions*, `arXiv:hep-ph/0510275`