The Randall-Sundrum model

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Why consider a warped extra dimension?

- Einstein's theory of General Relativity links gravity to curvature of space-time, induced by energy density
- in particle physics gravity is usually neglected due to its weakness
- observed 4D space-time approximately flat ➢ Lorentz invariance

➢ **warped higher-dimensional theory with Lorentz-invariant 4D effective theory?**

The Randall-Sudrum ansatz

Ansatz

- 5th dimension compactified to finite interval (or equiv. orbifold)
- non-zero cosmological constant in the 5D bulk
- 4D Lorentz-invariance preserved

$$ds^2 = e^{-A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

$e^{-A(y)}$ – warp-factor describing curvature ("warping") along extra dim.

➢ transform to conformal coordinates $e^{-A(z)/2}dz = dy$

$$ds^2 = e^{-A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

The Randall-Sundrum metric

Einstein equation with bulk cosmological constant Λ

$$G_{MN} \equiv R_{MN} - \frac{1}{2} g_{MN} R = \frac{1}{2M_*^3} \Lambda \, g_{MN}$$

M_* – 5D Planck scale

solution

$$ds^2 = \left(\frac{R}{z}\right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \qquad \text{with} \quad R^{-2} = k^2 = -\frac{\Lambda}{M_*^3}$$

requires brane tensions (energy densities on branes) $V_0 = -V_1 = 12 k M_*^3$

in conventional coordinates: $ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$

Summary: The Randall-Sundrum background

- bulk metric in conformal coordinates

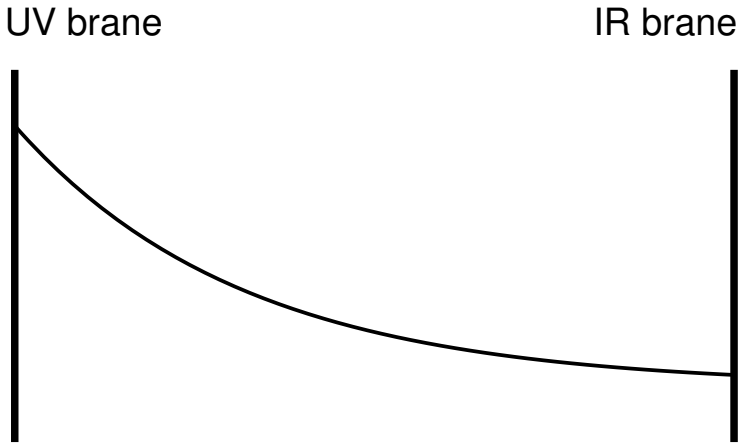
$$ds^2 = \left(\frac{R}{z}\right)^2 (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

5D Anti-de-Sitter space (AdS_5)

- confined to slice of AdS_5 to address hierarchy problem

  UV brane: $z = R \sim 1/M_{\text{Pl}}$

  IR brane: $z = R' \gg R \sim 1/\text{TeV}$



next step: RS as a solution to the hierarchy problem

Effective 4D scale of gravity

5D gravity action

$$S_g = M_*^3 \int d^5x \sqrt{g} R = \int_R^{R'} dz \left(\frac{R}{z}\right)^3 \int d^4x \sqrt{g_{(4)}} R_{(4)}$$
$$= M_*^3 \frac{R}{2} \left(1 - \frac{R^2}{R'^2}\right) \int d^4x \sqrt{g_{(4)}} R_{(4)}$$

read off 4D effective scale of gravity

$$M_{\mathsf{Pl}}^2 \sim M_*^3 R \left(1 - \frac{R^2}{R'^2}\right) \sim M_*^2 \qquad \text{for } M_* \sim R \ll R'$$

➢ **no hierarchy between 4D and 5D scale of gravity!**

Effective energy scale on the IR brane

4D Higgs field confined to IR brane

$$S_H = \int d^5x \sqrt{g} \left[g_{\mu\nu} \partial^\mu H^* \partial^\nu H - V(H) \right] \delta(z - R')$$

with $V(H) = \lambda(|H|^2 - v^2/2)^2$

$$\mathcal{L}_H = \left(\frac{R}{R'} \right)^2 \partial_\mu H^* \partial^\mu H + \left(\frac{R}{R'} \right)^4 \lambda \left(|H|^2 - \frac{v^2}{2} \right)^2$$

canonically normalising H, we find

$$\tilde{v}_{\text{4D}} = v \frac{R}{R'}$$

i.e. the effective 4D Higgs VEV is **"warped down"** by scale ratio R/R'

Original RS model

Ingredients

- RS background with only gravity propagating in the bulk
- SM fields localised on IR brane

Consequences

- warped-down effective energy scale on IR brane solves hierarchy problem
- but also serves as cutoff-scale for higher-dim. operators in EFT
  ➢ problems with precision tests, flavour, proton decay etc.!

➢ **we need bulk fields!**

Bulk fields in warped space

In a nutshell: "the devil is in the details"

- same basic principles as in flat 5D
- effects of warped metric alter resulting KK spectrum

RS bulk scalar

$$S = \int d^4x \int_R^{R'} dz \sqrt{g} \left[g^{MN} (\partial_M \phi)^\dagger \partial_N \phi - M^2 |\phi|^2 \right]$$

➢ bulk EOM

$$\partial_M (\sqrt{g} g^{MN} \partial_N \phi) + \sqrt{g} M^2 \phi = 0$$

➢ Dirichlet or Neumann BCs as in flat space

KK decomposition

usual ansatz

$$\phi(x,z) = \frac{1}{\sqrt{R}} \sum_n \phi^{(n)}(x) f^{(n)}(z)$$

➢ bulk profiles in terms of Bessel functions J_α, Y_α with $\alpha = \sqrt{4 + M^2 R^2}$

$$f^{(n)}(z) = z^2 \left[A_n J_\alpha(m_n z) + B_n Y_\alpha(m_n z) \right]$$

KK spectrum

- massive KK modes appear above $m_{\text{KK}} \sim 1/R'$
- for large z, $m_n \neq 0$: $f^{(n)}(z) \sim z^{3/2} \sin(m_n z)$
  oscillating, grow towards IR brane
  ➢ enhanced interaction with other states localised on/near IR brane
- zero mode localisation controlled by bulk mass parameter

Gauge fields in the RS bulk

$$S_{\text{gauge}} = \int d^4x \int_R^{R'} dz \, \sqrt{g} \left[-\frac{1}{4} g^{MR} g^{NS} F_{MN} F_{RS} \right]$$

Main results from KK decomposition

- massive KK modes for A_μ with masses $\gtrsim 1/R'$, peaked near IR brane
- A_5 KK modes unphysical, eaten by corresponding A_μ modes
- A_μ zero mode for Neumann BCs on both branes:
  flat along z (as required by gauge invariance)
- for Dirichlet BCs for A_μ:
  massless A_5 zero mode, localized near IR brane

Fermions in the RS bulk

$$S_{\text{fermion}} = \int d^5x \sqrt{g} \left[\frac{i}{2} \left(\bar{\Psi} e_a^M \gamma^a D_M \Psi - D_M \bar{\Psi} e_a^M \gamma^a \Psi \right) - M \bar{\Psi} \Psi \right]$$

vielbein e_a^M links flat-space γ matrices to warped space

Main results from KK decomposition

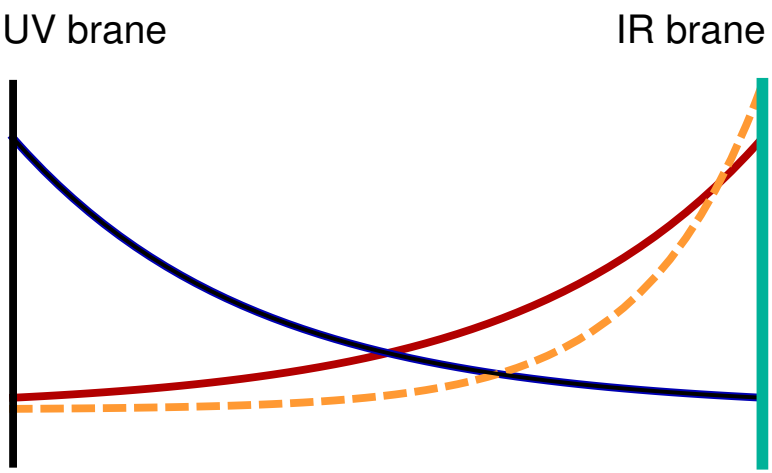
- massive KK modes again localised near IR brane
- chiral zero mode depending on choice of BCs
- zero mode bulk profile (for LH fermion zero mode)

$$g_0(z) = A_0 \left(\frac{z}{R} \right)^{2-c}$$

$c = MR$ – dimensionless bulk mass parameter

Towards a realistic model

- SM gauge bosons and fermions as 5D fields, with appropriate BCs
- 4D Higgs field confined to IR brane, to solve hierarchy problem



- fermion zero mode localisations as in Kaplan-Tait model
  ➢ flavour hierarchies from anarchy
- KK modes localised near IR brane
  ➢ coupling to fermion zero modes governed by the same hierarchies as the SM Yukawa couplings

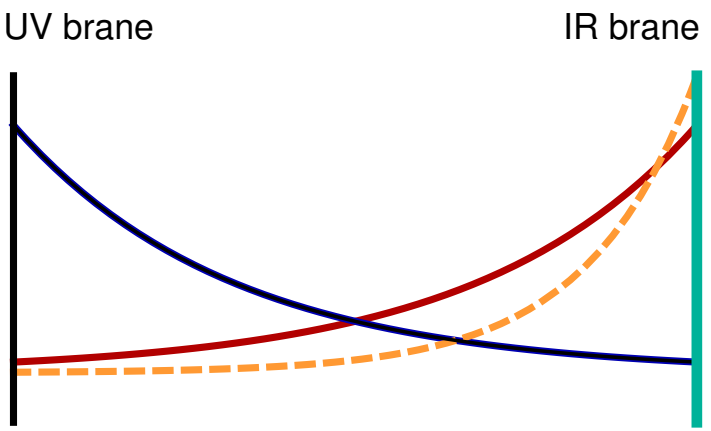
  "RS-GIM mechanism"

➢ EW precision constraints: $m_{\text{KK}} \gtrsim 10\,\text{TeV}$
little hierarchy problem remains unsolved

Summary

Study goal: Randall-Sundrum model

- warped 5D metric
- effective energy scales & hierarchy problem
- 5D fields & KK decomposition
- SM in the RS bulk w/ IR-localised Higgs



Reading assignment

- chapter 4.5, 4.6, 4.8 of C. Csaki, S. Lombardo, O. Telem, *TASI Lectures on Non-Supersymmetric BSM Models*, `arXiv:1811.04279`
- chapters 4.1, 5.4 of c. Csaki, *TASI Lectures on Extra Dimensions and Branes*, `arXiv:hep-ph/0404096`