Problem Set 1
Nonlinear Optics (NLO)

Due: 10. May 2016

1) Refractive Index, Extinction Coefficient and Absorption

Express the real and imaginary part of the complex refractive index

$$\underline{n} = n - \mathrm{j}n_i \tag{1.1}$$

using the real and imaginary part of the complex susceptibility

$$\underline{\chi}^{(1)} = \chi + \mathrm{j}\chi_i . \tag{1.2}$$

Simplify the results in the case of low losses, $\chi_i << \chi$, and derive an expression for the power attenuation coefficient α, that is experienced by a plane wave in a homogeneous medium.

2) Kramers-Kronig Relations

The polarization $\mathbf{P}(t)$ of a medium does not only depend on the interaction with a field $\mathbf{E}(t)$ at one particular point in time t, but it also depends on the history of the interaction. For a linear time-invariant medium, this can be expressed as a convolution with the impulse response $\chi(t)$ in the time domain. In the frequency domain this corresponds to a multiplication with the frequency dependent complex susceptibility $\underline{\chi}(\omega) = F\left[\chi(t)\right]$:

$$\mathbf{P}(t) = \varepsilon_0 \int_{-\infty}^{+\infty} \chi(\tau)\mathbf{E}(t-\tau)d\tau \tag{1.3}$$

$$\tilde{\mathbf{P}}(\omega) = \varepsilon_0 \underline{\chi}(\omega)\tilde{\mathbf{E}}(\omega). \tag{1.4}$$

1. The reaction of a medium to an electric field is causal, as there cannot be any polarization prior to the application of the electric field to the medium. Explain why for this case the following identity holds, where $H(t)$ is the Heaviside function.

$$\chi(t) = \chi(t)\, H(t) \quad \text{with} \quad H(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0. \end{cases} \tag{1.5}$$

2. Causality in time domain corresponds to an equivalent relation in frequency domain. Transform (1.5) to the frequency domain. Use the Fourier transform of the Heaviside function:

$$\tilde{H}(\omega) = \frac{1}{j\omega} + \pi\delta(\omega) \quad \text{with} \quad \tilde{H}(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0. \end{cases} \tag{1.6}$$

Note: In this course the following definitions of the Fourier transform are used:

$$\tilde{x}(\omega) = F\left[x(t)\right] = \int_{-\infty}^{+\infty} x(t)e^{-i\omega t}dt$$

$$x(t) = F^{-1}\left[\tilde{x}(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \tilde{x}(\omega)e^{i\omega t}d\omega$$

$$F\left[x(t)\cdot y(t)\right] = \frac{1}{2\pi}\tilde{x}(\omega) * \tilde{y}(\omega)$$

In order to calculate the convolution of $f(x)$ and $\frac{1}{x}$, the Cauchy principal value has to be introduced: $f(x) * \frac{1}{x} = \mathcal{P}\int_{-\infty}^{+\infty} \frac{f(x')}{x-x'}dx'$.

3. The susceptibility is complex, $\underline{\chi}(\omega) = \chi(\omega) + \mathrm{j}\chi_i(\omega)$. Use the previous result to derive a general relation between the real part $\chi(\omega)$ and the imaginary part $\chi_i(\omega)$ of the susceptibility. This relation is known as the "Kramers-Kronig relation" (after the discoverers H. A. Kramers und R. de Laer Kronig.). Note that $\tilde{\chi}(\omega)$ is an even, $\tilde{\chi}_i(\omega)$ an odd function, since $\chi(t)$ is a real function.

4. Sketch the frequency dependence of the real and imaginary part of the susceptibility if the medium has a sharp, symmetric absorption line at a frequency ω_0. To do so, assume that $n_i(\omega)$ is affected mostly by $\chi_i(\omega)$.

Bonus Program:

At three randomly chosen tutorials we will collect your solutions before the session starts. The solutions will be marked. If you have 70% or more of each collected problem set completed correctly, your oral examination grade will be upgraded by 0.3 or 0.4 (except grades of 1.0 and 4.7 or worse). If you cannot join a tutorial, you may also hand in your solutions by email to the teaching assistants (see contact details below) **before** the respective session. Please attach all pages in one pdf-file with white background. Students, who handed in a problem set, will be chosen randomly to present their solution.

Questions and Comments:

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