

Problem Set 2

Nonlinear Optics (NLO)

Due: 11. May 2016

1) Lorentz oscillator model for the linear case

In a classical picture, an electron bound to an atom can be considered as a harmonic oscillator in analogy to a mass connected to a spring. This oscillator, when driven by an external electric field E_x , follows the classical equation of motion

$$m_e \frac{d^2 x(t)}{dt^2} = -eE_x(t) - m_e \omega_r^2 x(t) - m_e \gamma_r \frac{dx(t)}{dt}, \quad (1.1)$$

where $x(t)$ is the dislocation of the electron, m_e denotes the electronic mass, $-eE_x(t)$ is the driving force by the external electric field, $-m_e \omega_r^2 x$ is the restoring force of the oscillator and $-m_e \gamma_r \frac{dx}{dt}$ is a damping term, with damping constant γ_r . The parameter ω_r will turn out to be the resonance frequency of the oscillator.

1. Solve the differential equation (1.1) for a time-harmonic electric field of the form $E(t) = \frac{1}{2}(\underline{E}(\omega) \exp(j\omega t) + c.c.)$ by using a similar ansatz for the dislocation. Derive an expression for $\underline{x}(\omega)$.
2. The electric polarization is the dipole moment per volume,

$$\underline{P}_x(\omega) = \varepsilon_0 \underline{\chi}^{(1)}(\omega) \underline{E}_x(\omega) = -\frac{N}{V} e \cdot \underline{x}(\omega), \quad (1.2)$$

where $\frac{N}{V}$ is the number density of atoms in the medium and $-e \cdot \underline{x}(\omega)$ is the induced dipole moment per atom. Show that the susceptibility is given by

$$\underline{\chi}^{(1)}(\omega) = \frac{Ne^2}{Vm_e \varepsilon_0} \frac{1}{\omega_r^2 - \omega^2 + j\omega\gamma_r} \quad (1.3)$$

and separate the susceptibility into real and imaginary part.

3. Sketch the real and imaginary part of the susceptibility $\underline{\chi}^{(1)}$ around the resonance frequency ω_r . What is the consequence of this result for the real and imaginary part of the refractive index at very large frequencies, e.g. X-ray frequencies?

2) Third-order nonlinear polarization

Consider a linearly polarized plane wave $\mathbf{E}(z,t) = E(z,t)\mathbf{e}_x$ propagating in z-direction in a homogeneous medium, in which third-order nonlinear effects dominate over second and higher-order contributions, $\chi^{(3)} \neq 0$, $\chi^{(m)} = 0$ for $m = 2$ or $m > 3$. Assuming an instantaneous response of the polarization P_{NL} to the applied electric field, we can express P_{NL} as

$$P_{\text{NL}}(z,t) = P^{(3)}(z,t)\mathbf{e}_x, \text{ with } P^{(3)}(z,t) = \varepsilon_0 \chi^{(3)} E(z,t)^3. \quad (2.1)$$

Calculate $P^{(3)}(z,t)$ considering a field $E(z,t)$ composed of two distinct frequency components ω_1 and ω_2 with their complex amplitudes \underline{E}_1 and \underline{E}_2

$$E(z,t) = \frac{1}{2}(\underline{E}_1 e^{j(\omega_1 t - k_1 z)} + \underline{E}_2 e^{j(\omega_2 t - k_2 z)} + c.c.). \quad (2.2)$$

Group the resulting terms with appropriate degeneracy factors according to their frequency and assign them to the following effects:

- Third-Harmonic Generation (THG)
- Third-order Sum-Frequency Generation (TSFG)
- Self-Phase Modulation (SPM)
- Cross-Phase Modulation (XPM)
- (Degenerate) Four-Wave Mixing (FWM)

Questions and Comments:

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