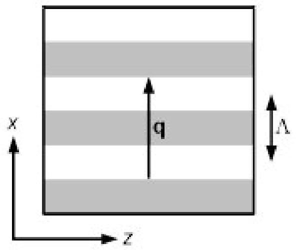
Problem Set 11
Nonlinear Optics (NLO)

Due: 12. July 2016

1) Acousto-Optic Modulator



Consider a material in which a sound wave is travelling in x-direction, with wave vector $\mathbf{q}$ and frequency Ω. The associated strain induces a refractive index grating that scatters an incoming optical wave. In Eq. (4.14) of the lecture notes, we derived a coupled-wave relation for the space-dependent amplitudes $\underline{E}(\mathbf{r},\omega_l)$ of the incoming wave $(l=0)$ at frequency ω_0 and the various scattered waves at frequencies ω_l. Assume that all waves are polarized along the y-direction, i.e. $e_l = e_y \; \forall\, l$. The scalar coupled-wave equation can then be written as

$$\sum_l -2\mathrm{j}\mathbf{k}_l \cdot \nabla \underline{E}(\mathbf{r},\omega_l) e^{\mathrm{j}(\omega_l t - \mathbf{k}_l \mathbf{r})} = \frac{2n_0}{c^2} \sum_l \frac{\partial^2}{\partial t^2}\left(\Delta n(\mathbf{r},t)\underline{E}(\mathbf{r},\omega_l) e^{\mathrm{j}(\omega_l t - \mathbf{k}_l \mathbf{r})}\right), \tag{1.1}$$

where the index variation $\Delta n(\mathbf{r},t)$ is given by

$$\Delta n(\mathbf{r},t) = \Delta n_0 \cos(\Omega t - \mathbf{qr}).$$

1. For a monochromatic incident optical wave at frequency ω_0, the right-hand side of Eq. (1.1) contains frequency components at $\omega_{\pm 1} = \omega_0 \pm \Omega$. In Eq. (1.1), consider only the expressions with $l=0$ and $l=1$, and derive two coupled differential equations for the wave amplitudes $\underline{E}(\mathbf{r},\omega_0)$ and $\underline{E}(\mathbf{r},\omega_1)$ by comparing the coefficients associated with the same frequency on the left-hand side and right-hand side of Eq. (1.1).

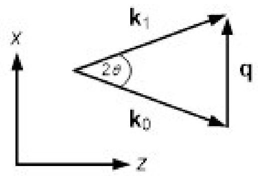
2. Consider the case where both the crystal and the optical waves are infinitely extended in x- and y-direction, which implies $\frac{\partial \underline{E}}{\partial x} = 0$ and $\frac{\partial \underline{E}}{\partial y} = 0$. Assume further that the z-components of the $\mathbf{k}$-vector for both optical waves are equal, i.e. $k_{0z} = k_{1z} = k_z$. Using these simplifications, show that the two coupled differential equations can be written as:

$$\frac{\partial \underline{E}(z,\omega_1)}{\partial z} = -\mathrm{j}\kappa \underline{E}(z,\omega_0) e^{-\mathrm{j}\Delta \mathbf{kr}}$$

$$\frac{\partial \underline{E}(z,\omega_0)}{\partial z} = -\mathrm{j}\kappa \underline{E}(z,\omega_1) e^{\mathrm{j}\Delta \mathbf{kr}}$$

with $\kappa = \frac{k_z \Delta n_0}{2n_0}$ and $\Delta\mathbf{k} = \mathbf{k}_0 + \mathbf{q} - \mathbf{k}_1$.

3. Solve the differential equations assuming perfect phase matching, i.e. $\Delta\mathbf{k} = 0$ and using the boundary conditions $\underline{E}(0,\omega_0) = E_0$ and $\underline{E}(0,\omega_1) = 0$. Sketch the evolution of the intensities of the incident and the deflected wave along z. How long should the crystal extend in the z-direction for maximum intensity of the deflected wave?



4. What is the angle of diffraction for light at 632.8 nm in a $LiNbO_3$ cell that is driven at a frequency of 1 GHz? (speed of sound: v_s = 4.1 km/s, refractive index n_0 = 2.3)

Questions and Comments:

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