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Solution to Problem Set 12
Nonlinear Optics (NLO)

1) Stimulated Raman Scattering

Stimulated Raman scattering (SRS) is an important nonlinear effect that can e.g., be used for broadband fiber amplifiers. However, in wavelength-division multiplexing (WDM) systems, SRS can also reduce the performance, as it causes an energy transfer (crosstalk) between different channels.

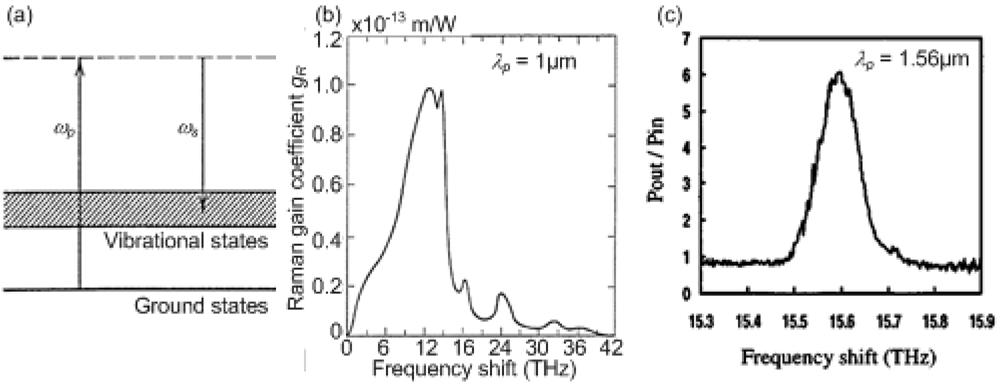

Figure 1: (a) Energy-level representation of SRS. (b) Raman gain spectrum in a fused silica fiber for a pump wavelength of 1000 nm [1]. (c) Raman gain spectrum in a silicon-on-insulator waveguide for a pump wavelength of 1557.4 nm [2]. Note that on the **vertical** axis in (b) and (c) two different quantities have been measured and therefore the numbers are not comparable.

Raman effect

1. Explain the Raman effect using your own words and relate your explanation to Figure 1. What is the difference between spontaneous and stimulated Raman scattering?

2. The Raman gain spectra of fused silica and crystalline silicon are depicted in Figure 1(a) and Figure 1(b). Explain the differences. How are they related to the material structure?

Solution:

1. Inelastic scattering of light (photons) on lattice vibrations (opt. phonons). Part of the energy of one photon ($\hbar\omega_P$) is transferred to the phonon, creating a ‘Stokes’ photon with lower energy, $\omega_S < \omega_P$.
   Spontaneous Raman scattering: the Stokes photon is emitted into one state (‘mode’) out of all possible states. The amplification is broadband and spatially isotropic.
   Stimulated Raman scattering: if there is already a photon present at the Stokes frequency, it induces a stimulated transition. This coherently creates an identical second photon that has the same phase and propagation constant.

2. Spectral bandwidth of the Raman gain: a fiber has an amorphous structure. Because of this disorder, there are a lot of different possible vibrational modes present. This leads to a broad gain spectrum. In the case of silica it has a bandwidth of ~ 40 THz. Silicon in contrast has a crystalline structure with well-defined vibrational modes. This leads to a small bandwidth < 1 THz of the Raman gain.

Stimulated Raman scattering

The nonlinear interaction of a continuous pump wave with the Stokes wave in a fiber is described by the following set of coupled differential equations:

$$\frac{dI_s}{dz} = g_R I_p I_S - \alpha_s I_S \tag{1}$$

$$\frac{dI_p}{dz} = -\frac{\omega_p}{\omega_s} g_R I_p I_S - \alpha_p I_p \,, \tag{2}$$

where I_p and I_S are pump- and Stokes-intensity at the frequencies ω_p and ω_S, respectively. g_R is the Raman gain coefficient and α_p and α_S describe the fiber losses.

3. Show that the total number of photons of the pump and the Stokes wave is conserved in the absence of fiber losses ($\alpha_p = \alpha_s = 0$).

4. Solve the coupled differential equations under the assumption that the power of the pump wave is not significantly attenuated by SRS ($I_S << I_p$, undepleted-pump approximation). Neglect the corresponding term in Eq. (1.2).

5. Calculate the Raman gain factor $G_A = \frac{I_s(L)}{I_s(0)e^{-\alpha_s L}}$ for a fiber amplifier of length $L = 1000\,\text{m}$. Assume that both the pump and the Stokes wave experience a propagation loss of $\alpha = 0.2\,\text{dB/km}$. To do this, take the maximum value of the Raman gain coefficient from Figure 1(b) and assume an input pump power of $P_{in} = I_P A_m = 240\,\text{mW}$, where $A_m = 10\,\mu\text{m}^2$ denotes the effective modal cross section. Can the gain factor be increased arbitrarily by increasing the length of the fiber?

   Hint: The fiber propagation loss in dB/m has to be converted into linear units by [1]

$$\alpha(\text{dB/km}) = -\frac{10}{L}\log_{10}\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \approx 4.343\alpha$$

6. Even if no light at the Stokes wavelength is coupled into the fiber, a strong Stokes signal can build up from spontaneously emitted Stokes photons that are subsequently amplified by SRS. What is the consequence for the transport of high optical powers over a single-mode fiber?

7. Explain three major differences between Raman and Brillouin scattering.

Solution:

3. Conservation of the number of photons requires the losses to be negligible. We take (1.1) and (1.2) and by dividing by ω_S and ω_P we get:

$$\frac{1}{\omega_S}\frac{dI_s}{dz} = \frac{1}{\omega_S} g_R I_p I_S$$

$$\frac{1}{\omega_P}\frac{dI_p}{dz} = -\frac{1}{\omega_S} g_R I_p I_S$$

$$\frac{d}{dz}\underbrace{\left(\frac{I_S}{\omega_S} + \frac{I_P}{\omega_P}\right)}_{const} = 0.$$

So the total number of photons does not change along the propagation.

4. In a small signal approximation the power of the pump wave is not significantly attenuated by the nonlinear interaction; in other words pump depletion can be neglected. In this case the set of differential equations becomes:

$$\frac{dI_s}{dz} = g_R I_p I_S - \alpha_s I_S$$

$$\frac{dI_p}{dz} = \underbrace{-\frac{\omega_p}{\omega_s} g_R I_p I_S}_{0} - \alpha_p I_p,$$

from which we can calculate $I_P(z)$ by integration:

$$I_P(z) = I_{P0} e^{-\alpha_P z}. \tag{1.3}$$

The Stokes wave is then obtained by integrating:

$$\frac{dI_S}{dz} = \left(g_R I_{P0} e^{-\alpha_P z} - \alpha_S\right) I_S$$

$$\ln\frac{I_S(L)}{I_S(0)} = \int_0^L \left(g_R I_{P0} e^{-\alpha_P z} - \alpha_S\right) dz \tag{1.4}$$

$$I_S(L) = \underbrace{I_S(0) e^{-\alpha_S L}}_{\text{Losses at } \omega_s} \cdot \underbrace{e^{\int_0^L g_R I_{P0} e^{-\alpha_P z} dz}}_{\text{Raman gain factor } G_A}$$

The integral in the exponent can be evaluated easily and takes the following form

$$g_R I_{P0} \int_0^L e^{-\alpha_P z} dz = g_R I_{P0}\left[-\frac{e^{-\alpha_P z}}{\alpha_P}\right]_0^L = g_R I_{P0} \underbrace{\frac{1-e^{-\alpha_P L}}{\alpha_P}}_{L_{\text{eff}}}, \tag{1.5}$$

where we have introduced the effective waveguide length L_{eff}. Therefore, the Raman amplified Stokes wave has the intensity

$$I_S(L) = I_S(0)e^{-\alpha_S L} e^{g_R I_{P0} L_{\text{eff}}}. \quad (1.6)$$

5. The Raman gain can be calculated from (1.6):

$$G_A = e^{g_R I_{P0} L_{\text{eff}}}. \quad (1.7)$$

From Figure 1(b) we get a value of $g_R = 10^{-13}\,\text{m/W}$ in the gain maximum. The input intensity is $I_{P0} = \frac{240\text{mW}}{10\mu\text{m}^2} = 2.4\cdot 10^{10}\,\frac{\text{W}}{\text{m}^2}$. For typical fiber losses of 0.2 dB/km the fiber of 1 km length is reduced to an effective length of 977 m. This leads to a Raman gain factor of $G_A = 10.43\text{dB}$.

By using a longer fiber the gain cannot be increased arbitrarily. For long fiber length L_{eff} converges to $L_{\text{eff}} = 1/\alpha_p$. Additionally at some point the signal power would become comparable to the pump power and the small-signal approximation does not hold anymore.

6. The amplification of the spontaneously emitted Stokes-photons transfers energy from the pump wave into the Stokes wave. This effect can become quite significant, especially in standard single-mode fibers. The wave intensities can become very high because of the small effective area of these fibers. After a certain threshold the added energy is nearly completely transferred to the amplified spontaneous Stokes waves. This creates an upper limit (typically several Watts) of power that can be transmitted over single-mode fibers, known as the Raman threshold.

7.

   a. Brillouin scattering occurs on acoustic phonons, whereas Raman scattering occurs on optical phonons.

   b. Energy of optical phonons is much larger than the energy of acoustic phonons, therefore Raman frequency shift is much larger that the Brillouin frequency shift.

   c. In an optical fiber, Brillouin scattering can occur only in backward direction, while Raman scattering can occur in both forward and backward direction.

References

[1] G. P. Agrawal, '*Fiber-optic communcations systems*', John Wiley & Sons, 2002.

[2] T. K. Liang and H. K. Tsang, '*Efficient Raman amplification in silicon-on-insulator waveguides*', Appl. Phys. Lett., Vol. 85, No. 16, 2004

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