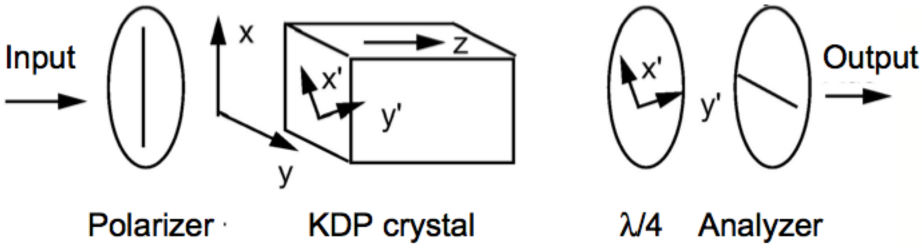
Problem Set 7
Nonlinear Optics (NLO)

Due: 16. June 2015

1) Pockels cells



A Pockels cell is an electro-optic modulator, where the modulating electric field is applied in the direction of light propagation. Incoming light is first linearly polarized by a polarizer and then launched into a nonlinear crystal (e.g., potassium dihydrogen phosphate, KDP). The light then passes through a quarter-wave plate and a polarization analyzer that is rotated by 90° with respect to the input polarizer. Depending on the applied electric field, the nonlinear crystal changes the polarization state of the light, and the output power after the polarization analyzer is varied. In the following, we assume that the light can be described by a completely polarized plane wave that is propagating in z-direction.

1. A completely polarized plane wave is described by:

$$\mathbf{E}(z,t) = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \mathrm{Re}\left\{ \begin{pmatrix} \underline{E}_x \\ \underline{E}_y \end{pmatrix} e^{\mathrm{j}(\omega t - kz)} \right\} = \mathrm{Re}\left\{ \underline{\mathbf{E}}\, e^{\mathrm{j}(\omega t - kz)} \right\}, \text{ with } k = n\omega / c\,. \tag{1.1}$$

   The state of polarization is defined by the relative phase and the amplitudes of the complex values $\underline{E}_x$, $\underline{E}_y$. What kind of pattern does the endpoint of vector $\mathbf{E}(z,t)$ make in (x,y) plane, if $|\underline{E}_x| = |\underline{E}_y|$ and the relative phase difference $\arg\left(\frac{\underline{E}_y}{\underline{E}_x}\right)$ between both components takes a value of 0, π/2, π, and 3π/2?

2. Only the components of the electric field that are oriented parallel to the axis of a polarizer can pass. Show that the complex field vector $\mathbf{E}_{out}$ at the output of a polarizer is given by $\mathbf{E}_{out} = \mathbf{M}_\alpha \mathbf{E}_{in}$, where the following matrix $\mathbf{M}_\alpha$ (also called Jones matrix) describes a polarizer with an axis that forms an angle of α with the x-axis:

$$\mathbf{M}_\alpha = \begin{pmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{pmatrix}. \tag{1.2}$$

When we apply a voltage U between the input and output facet of the KDP crystal, the nonlinear medium becomes birefringent. The orientation of the crystal is such that the principal axes x' and y' of the index ellipsoid are rotated by -45° with respect to x and y. This leads to a phase difference between the x'- and y'-components of the electric field (see Eq. (3.35) in the lecture notes):

$$\Delta\Phi = r_{63} n_o^3 E_z k_0 L = r_{63} n_o^3 k_0 U \,, \tag{1.3}$$

where L is the length of the crystal, and E_z is the applied field along the crystal (induced by the voltage U).

3. Derive the transfer matrix $\mathbf{M}_{\mathrm{KDP}}$ of the KDP crystal that relates the complex field vector $\mathbf{E}_{out}$ to the input field $\mathbf{E}_{in}$. Use the $x'y'$ coordinate system as your basis.

4. The λ/4 wave plate causes an additional phase shift of π/2 (λ/4 light path difference) between x'- and y'-components of the field. Its slow x' and fast y' axes have the same orientation as the respective principal axes of KDP. Derive the corresponding transfer matrix $\mathbf{M}_{\lambda/4}$ in the $x'y'$ coordinate system.

5. Derive the transfer matrix of the complete system.

   Hint: The coordinate transformations between $x'y'$ and xy can be formulated by 2x2 rotation matrices.

6. Calculate the power transmission

$$T = \frac{I_{out}}{I_{in}} = \frac{|\mathbf{E}_{out}|^2}{|\mathbf{E}_{in}|^2} \tag{1.3}$$

   of a plane wave that is linearly polarized in x-direction.

7. Sketch the dependence of the transmission T on the voltage U. What is the purpose of the λ/4 wave plate?

Questions and Comments:

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