Problem Set 8
Nonlinear Optics (NLO)

Due: 17. June 2015

1) Electro-optic Mach-Zehnder modulator

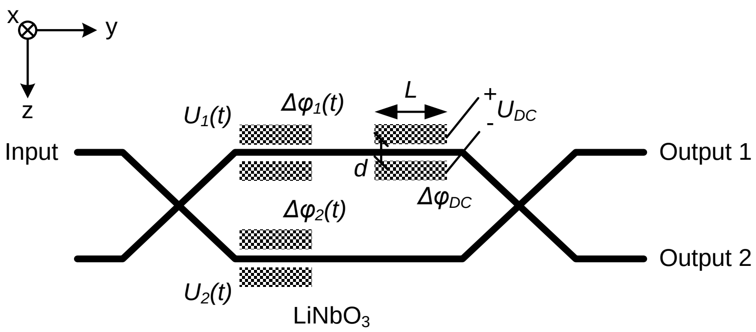

Figure 1: Dual-drive Mach-Zehnder modulator

Figure 1 shows a dual-drive Mach-Zehnder modulator. The device consists of a waveguide-based Mach-Zehnder interferometer having voltage-controlled phase shifters in each arm. For high-speed modulation, time-dependent voltages $U_1(t)$ and $U_2(t)$ are applied to two phase-shifters ($\Delta\phi_1$ and $\Delta\phi_2$) in the upper and lower arm, respectively, whereas a third phase shifter ($\Delta\phi_{DC}$) operated by a constant DC bias voltage U_{DC} is used to set the operating point. The device is made of lithium-niobate ($LiNbO_3$) using x-cut geometry. The principal axes[1] are the x, y and z axes shown in Figure 1. The propagating light of wavelength $\lambda = 1.55\ \mu m$ is polarized along the z axis. The refractive indices are $n_o = 2.211$ and $n_e = 2.138$. The electro-optic coefficients, measured at a wavelength of 0.5 µm are $r_{13} = 9.6\,\mathrm{pm/V}$, $r_{22} = 6.8\,\mathrm{pm/V}$, $r_{33} = 30.9\,\mathrm{pm/V}$, and $r_{42} = 32.6\,\mathrm{pm/V}$. Assume that these values are also valid at the wavelength of 1.55 µm.

1. Consider that $U_1(t) = U_2(t) = 0$ and an external voltage U_{DC} applied to the two parallel metal contacts (length $L = 2$ mm, distance $d = 5$ µm), inducing a phase shift $\Delta\phi_{DC}$ in the upper arm. What voltage $U_{\pi,DC}$ is needed for a phase shift of π between both arms?

   Hint: Start by calculating the change of refractive index as a function of the applied voltage U_{DC} and approximate the modulating electric field along the z-direction by $E_z^{(el)} \approx U_{DC} / d$.

2. Calculate amplitude transfer function at the two outputs of the device as a function of the applied phase shifts $\Delta\phi_1$, $\Delta\phi_2$, and $\Delta\phi_{DC}$.

[1] The symmetry group of $LiNbO_3$ is $C_{3v} = 3m$. The convention used here is that the z axis is parallel to the threefold rotational axis of the crystal.

Hint: Assume the device to consist of lumped elements with individual scattering matrices. Using the input and output amplitudes a_i and b_i of a symmetric 2x2 directional coupler as indicated in Figure 2, its scattering matrix $\mathbf{S}_{2x2}$ can be written as

$$\mathbf{S}_{2x2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 & -j \\ 0 & 0 & -j & 1 \\ 1 & -j & 0 & 0 \\ -j & 1 & 0 & 0 \end{pmatrix}$$

where $b_m = S_{mn} \cdot a_n$

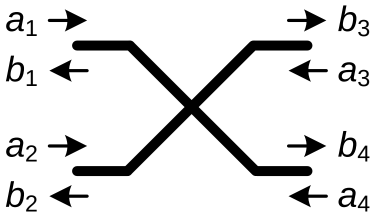

Figure 2: Definition of input and output amplitudes of a 2x2 directional coupler

The modulator is said to be operated in push-pull mode when the voltages applied to both arms have equal magnitude but with opposite sign, $U_1(t) = - U_2(t) = U(\mathrm{t})$ and therefore $\Delta\phi_1 = - \Delta\phi_2 = \Delta\phi$.

3. Express the amplitude transfer function for output 1 as a function of the phase shifts $\Delta\phi$ and $\Delta\phi_{DC}$. Sketch the amplitude transfer function as a function of the voltage $U(t)$ normalized to the pi-voltage $U_{\pi,AC}$ of the high-frequency electrodes for $\Delta\phi_{DC} = 0$.

4. In some applications it is important to have a linear relationship between small variations of the input voltage and the associated variations of the optical amplitude at the output. This can be achieved by choosing a suitable DC bias, U_{DC}. Which bias voltage would you choose for this case?

5. The power transfer function can be obtained simply by squaring the amplitude transfer function. Sketch the power transfer function as a function of the voltage $U(t)$ normalized to $U_{\pi,AC}$ for $\Delta\phi_{DC} = 0$. Which output power is obtained when adjusting the bias voltage according to part 5?

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