Problem Set 8
Nonlinear Optics (NLO)

Solution: 17 June 2015

1) Electro-optic Mach-Zehnder modulator

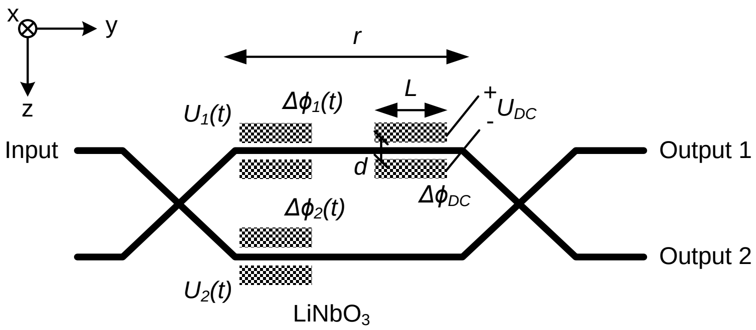


Figure 1: Dual-drive Mach-Zehnder modulator

Figure 1 shows a dual-drive Mach-Zehnder modulator. The device consists of a waveguide-based Mach-Zehnder interferometer having voltage-controlled phase shifters in each arm. For high-speed modulation, time-dependent voltages $U_1(t)$ and $U_2(t)$ are applied to two phase-shifters ($\Delta\phi_1$ and $\Delta\phi_2$) in the upper and lower arm, respectively, whereas a third phase shifter ($\Delta\phi_{DC}$) operated by a constant DC bias voltage U_{DC} is used to set the operating point. The device is made of lithium-niobate ($LiNbO_3$) using x-cut geometry. The principal axes[1] are the x, y and z axes shown in Figure 1. The propagating light of wavelength $\lambda = 1.55$ µm is polarized along the z axis. The refractive indices are $n_o = 2.211$ and $n_e = 2.138$. The electro-optic coefficients, measured at a wavelength of 0.5 µm are $r_{13} = 9.6\,\mathrm{pm/V}$, $r_{22} = 6.8\,\mathrm{pm/V}$, $r_{33} = 30.9\,\mathrm{pm/V}$, and $r_{42} = 32.6\,\mathrm{pm/V}$. Assume that these values are also valid at the wavelength of 1.55 µm.

1. Consider that $U_1(t) = U_2(t) = 0$ and an external voltage U_{DC} applied to the two parallel metal contacts (length $L = 2$ mm, distance $d = 5$ µm), inducing a phase shift $\Delta\phi_{DC}$ in the upper arm. What voltage $U_{\pi,DC}$ is needed for a phase shift of π between both arms?

   Hint: Start by calculating the change of refractive index as a function of the applied voltage U_{DC} and approximate the modulating electric field along the z-direction by $E_z^{(el)} \approx U_{DC} / d$.

2. Express the general amplitude transfer function at the two outputs of the device as a function of the applied phase shifts $\Delta\phi_1$, $\Delta\phi_2$, and $\Delta\phi_{DC}$. For the situation considered in

[1] The symmetry group of $LiNbO_3$ is $C_{3v} = 3m$. The convention used here is that the z axis is parallel to the threefold rotational axis of the crystal.

part 1 ($U_1(t) = U_2(t) = 0$), sketch the amplitude and power transfer function over the applied voltage normalized to U_π.

Hint: Assume the device to consist of lumped elements with individual scattering matrices. Using the input and output amplitudes a_i and b_i of a symmetric 2x2 directional coupler as indicated in Figure 2, its scattering matrix $\mathbf{S}_{2\mathrm{x}2}$ can be written as

$$\mathbf{S}_{2\mathrm{x}2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 & -\mathrm{j} \\ 0 & 0 & -\mathrm{j} & 1 \\ 1 & -\mathrm{j} & 0 & 0 \\ -\mathrm{j} & 1 & 0 & 0 \end{pmatrix}$$

where $b_m = S_{mn} \cdot a_n$

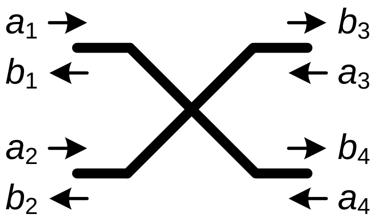


Figure 2: Definition of input and output amplitudes of a 2x2 directional coupler

The modulator is said to be operated in push-pull mode when the voltages applied to both arms have equal magnitude but with opposite sign, $U_1(t) = - U_2(t) = U(\mathrm{t})$ and therefore $\Delta\phi_1 = - \Delta\phi_2 = \Delta\phi$.

3. Express the amplitude transfer function for output 1and 2 as a function of the phase shifts $\Delta\phi$ and $\Delta\phi_{\mathrm{DC}}$. Sketch the amplitude transfer function as a function of the voltage $U(t)$ normalized to the pi-voltage $U_{\pi,\mathrm{AC}}$ of the high-frequency electrodes for $\Delta\phi_{\mathrm{DC}} = 0$.

4. In some applications it is important to have a linear relationship between small variations of the input voltage and the associated variations of the optical amplitude at the output. This can be achieved by choosing a suitable DC bias, U_{DC}. Which bias voltage would you choose for this case?

5. The power transfer function can be obtained simply by squaring the amplitude transfer function. Sketch the power transfer function as a function of the voltage $U(t)$ normalized to $U_{\pi,\mathrm{AC}}$ for $\Delta\phi_{\mathrm{DC}} = 0$. Which output power is obtained when adjusting the bias voltage according to part 4.

Solution:

1. Following the hint, to deduce the change of refractive index as a function of U_{DC} we need to first compute the induced change of the impermeability tensor from Eq.(3.24) of the lecture notes. In our given problem, we already know the orientation of the external electric field as well as the polarization state of the optical field and orientation of nonlinear crystal:

- External electric field is oriented in *z*-direction → only r_{i3} contributes. Therefore the impermeability tensor will have the form:

$$\eta = \begin{pmatrix} \frac{1}{n_o^2} + r_{13}E_z & 0 & 0 \\ 0 & \frac{1}{n_o^2} + r_{13}E_z & 0 \\ 0 & 0 & \frac{1}{n_e^2} + r_{33}E_z \end{pmatrix}$$

- The optical field is polarized in *z*-direction and the coordinate axes are orientated along the principle axis of crystal → only r_{33} contributes, the optical field will experience only the effect of the extraordinary index $\frac{1}{(n_e')^2} = \frac{1}{n_e^2} + r_{33}E_z^{(el)}$

In this geometrical configuration and with the given fields, the indicatrix assumes the form:

$$\frac{1}{n_o^2}X^2 + \frac{1}{n_o^2}Y^2 + \left(\frac{1}{n_e^2} + r_{33}E_z^{(el)}\right)Z^2 = 1 \tag{0.1}$$

We calculate the refractive index change by

$$\begin{aligned} &\frac{1}{n_e'^2} = \frac{1}{n_e^2} + r_{33}E_z^{(el)} \\ &\Rightarrow n_e' = n_e^2\sqrt{\frac{1}{1 + n_e^2 r_{33}E_z^{(el)}}} \approx n_e(1 - \frac{1}{2}n_e^2 r_{33}E_z^{(el)}) \\ &\Rightarrow \Delta n = -\frac{1}{2}n_e^3 r_{33}E_z^{(el)} = -\frac{1}{2}n_e^3 r_{33}\frac{U}{d} \end{aligned} \tag{0.2}$$

We then compute the phase shift that is accumulated in the modulated arm along the propagation length L:

$$\Delta\varphi = \Delta n k_0 L = -k_0 L \frac{1}{2} n_e^3 r_{33} \frac{U}{d} \tag{0.3}$$

Finally, the voltage $U_{\pi,\mathrm{DC}}$ needed to introduce a phase shift of π is:

$$U_\pi = \frac{d\lambda}{L n_e^3 r_{33}} = 12{,}83V \tag{0.4}$$

2. We start at Input 1 with the initial complex amplitude $\underline{E}_0$ and the corresponding intensity $I_0 \sim \underline{E}_0 \underline{E}_0^*$. The input vector a of the first coupler containing the amplitudes is $\left(\underline{E}_0, 0, 0, 0\right)^T$.

After the first coupler the power is split equally into the modulated upper (b_3) and the lower arm (b_4), where the lower arm receives a phase factor:

Amplitude in the upper arm: $b_3 = \frac{1}{\sqrt{2}} \underline{E}_0$

Amplitude in the lower arm: $b_4 = -j \frac{1}{\sqrt{2}} \underline{E}_0$

Both amplitudes accumulate a different phase shift during the propagation in the respective arms.

Amplitude after upper arm:

$$\frac{1}{\sqrt{2}} \underline{E}_0 \exp\left(-j\Delta\phi_1(t)\right) \exp\left(-j\Delta\phi_{DC}\right) \exp\left(-jnk_0 r\right)$$

Amplitude after lower arm:

$$-j \frac{1}{\sqrt{2}} \underline{E}_0 \exp\left(-j\Delta\phi_2(t)\right) \exp\left(-jnk_0 r\right)$$

The second directional coupler has now two input amplitudes:

$$a = \left(\frac{1}{\sqrt{2}} \underline{E}_0 \exp\left(-j\Delta\phi_1(t)\right) \exp\left(-j\Delta\phi_{DC}\right) \exp\left(-jnk_0 r\right), -j \frac{1}{\sqrt{2}} \underline{E}_0 \exp\left(-j\Delta\phi_2(t)\right) \exp\left(-jnk_0 r\right), 0, 0 \right)^T$$

The amplitudes at the Outputs 1 and 2 of the complete device is given by the last two entries of the output vector of the 2nd directional coupler:

$$\underline{E}_{out,1} = \frac{1}{2} \underline{E}_0 \exp\left(-jnk_0 r\right) \left[\exp\left(-j\Delta\phi_1(t) - j\Delta\phi_{DC}\right) - \exp\left(-j\Delta\phi_2(t)\right) \right]$$

$$\underline{E}_{out,2} = -j \frac{1}{2} \underline{E}_0 \exp\left(-jnk_0 r\right) \left[\exp\left(-j\Delta\phi_1(t) - j\Delta\phi_{DC}\right) + \exp\left(-j\Delta\phi_2(t)\right) \right]$$

The amplitude transfer function of the output 1 is given by:

$$
\begin{aligned}
t_{out,1} = \frac{\underline{E}_{out,2}}{\underline{E}_0} &= \frac{1}{2}\exp\left(-jnk_0 r\right)\left[\exp\left(-j\Delta\phi_1(t) - j\Delta\phi_{DC}\right) - \exp\left(-j\Delta\phi_2(t)\right)\right] \\
&= \frac{1}{2}\exp\left(-jnk_0 r\right)\left[\exp\left(-j\Delta\phi_1(t) - j\Delta\phi_{DC}\right) - \exp\left(-j\Delta\phi_2(t)\right)\right] \\
&= \frac{1}{2}\exp\left(-jnk_0 r\right)\begin{Bmatrix} \exp\left[-j/2\left(\Delta\phi_1(t)+\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right]\exp\left[-j/2\left(\Delta\phi_1(t)-\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right] \\ -\exp\left[-j/2\left(\Delta\phi_1(t)+\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right]\exp\left[-j/2\left(-\Delta\phi_1(t)+\Delta\phi_2(t)-\Delta\phi_{DC}\right)\right] \end{Bmatrix} \\
&= -j\sin\left[\frac{1}{2}\left(\Delta\phi_1(t)-\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right]\exp\left[-j/2\left(\Delta\phi_1(t)+\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right]\exp\left(-jnk_0 r\right)
\end{aligned}
$$

The amplitude transfer function of the output 2 is given by:

$$
\begin{aligned}
t_{out,2} = \frac{\underline{E}_{out,2}}{\underline{E}_0} &= -j\frac{1}{2}\exp\left(-jnk_0 r\right)\left[\exp\left(-j\Delta\phi_1(t) - j\Delta\phi_{DC}\right) + \exp\left(-j\Delta\phi_2(t)\right)\right] \\
&= -j\cos\left[\frac{1}{2}\left(\Delta\phi_1(t)-\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right]\exp\left[-j/2\left(\Delta\phi_1(t)+\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right]\exp\left(-jnk_0 r\right)
\end{aligned}
$$

3. In push-pull-mode, the amplitude transfer functions are:

$$
t_{out,1,push-pull} = -j\sin\left[\Delta\phi(t) + \frac{1}{2}\Delta\phi_{DC}\right]\exp\left[-j/2\left(\Delta\phi_{DC}\right)\right]\exp\left(-jnk_0 r\right)
$$

$$
t_{out,2,push-pull} = -j\cos\left[\Delta\phi(t) + \frac{1}{2}\Delta\phi_{DC}\right]\exp\left[-j/2\left(\Delta\phi_{DC}\right)\right]\exp\left(-jnk_0 r\right)
$$

For $\Delta\phi_{DC} = 0$:

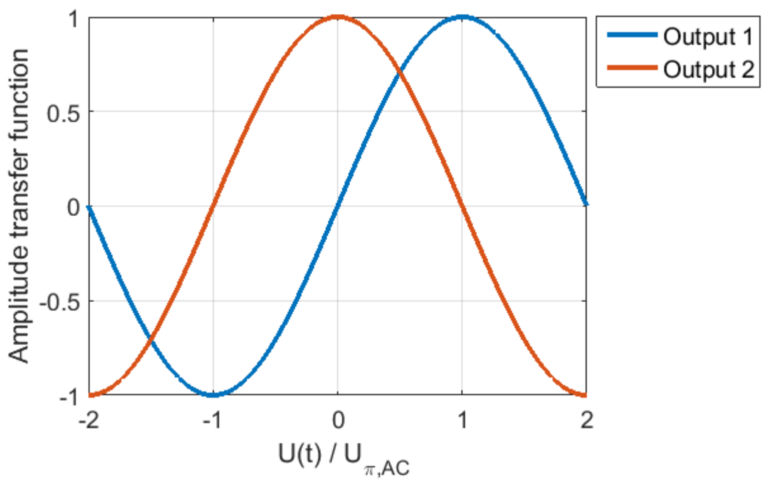
$$
\begin{aligned}
t_{out,1,push-pull} &= \sin\left[\Delta\phi(t)\right]\exp\left(-jnk_0 r\right)\exp(-j\pi/2) \\
t_{out,2,push-pull} &= \cos\left[\Delta\phi(t)\right]\exp\left(-jnk_0 r\right)\exp(-j\pi/2)
\end{aligned}
$$

Where $\exp\left(-jnk_0 r\right)\exp(-j\pi/2)$ is a constant phase term.

$U_{\pi,AC}$ is the voltage needed to generated a phase different of pi between both arms, $2\Delta\phi(t) = \pi$. Therefore:

$$
\Delta\phi(t) = \frac{U(t)}{U_{\pi,AC}}\frac{\pi}{2}
$$

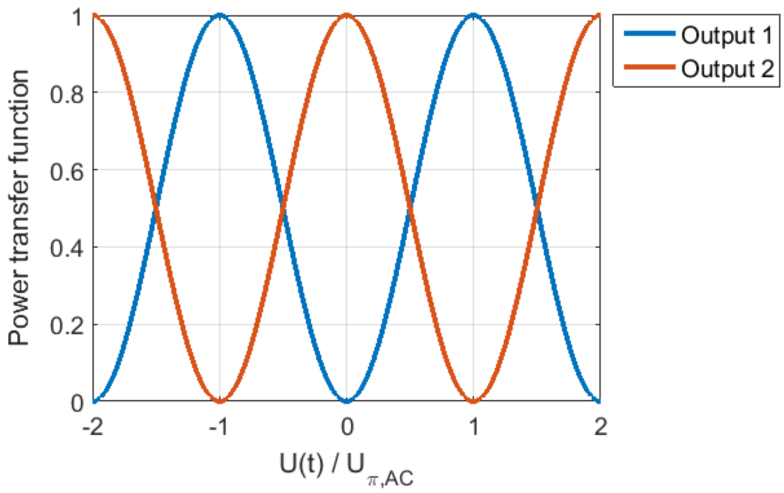
For output 1 and 2 the amplitude transfer function has oscillating behaviour. Note that the absolute position of such curves may be influenced by the previously mentioned constant phase terms.



4. For a linear relationship the device needs to be operated at the so-called null point, where the amplitude transfer function is near to zero. For the output 1 the DC bias should be null, whereas for the output 2, should be $U_{\pi,\mathrm{AC}}$.

5. The power transfer functions for the ouput 1 and 2 are:

$$T_{out,1} = t_{out,1} t^*_{out,1} = \sin^2\left[\Delta\phi(t)\right]$$
$$T_{out,2} = t_{out,2} t^*_{out,2} = \cos^2\left[\Delta\phi(t)\right]$$



When adjusting the bias at the null point, the output power is zero.

Questions and Comments:

Philipp-Immanuel Dietrich
Building: 30.10, Room: 1.23
Phone: 0721/608-47173
p-i.dietrich@kit.edu

Pablo Marin
Room 2.33
42487
pablo.marin@kit.edu

Jörg Pfeifle
Room: 2.33
42487
joerg.pfeifle@kit.edu