Problem Set 1
Nonlinear Optics (NLO)

Due: May 16, 2017, 09:45 AM

1) Refractive Index, Extinction Coefficient and Absorption

Express the real and imaginary part of the complex refractive index

$$\underline{n} = n - \mathrm{j}n_i \tag{1.1}$$

using the real and imaginary part of the complex susceptibility

$$\underline{\chi}^{(1)} = \chi + \mathrm{j}\chi_i \,. \tag{1.2}$$

Simplify the results in the case of low losses, $|\chi_i| << |\chi|$, and derive an expression for the power attenuation coefficient α, that is experienced by a plane wave in a homogeneous medium.

2) Kramers-Kronig Relations

The polarization $\mathbf{P}(t)$ of a medium does not only depend on the interaction with a field $\mathbf{E}(t)$ at one particular point in time t, but it also depends on the history of the interaction. For a linear time-invariant medium, this can be expressed as a convolution with the impulse response $\chi(t)$ in the time domain. In the frequency domain this corresponds to a multiplication by the frequency dependent complex susceptibility $\underline{\chi}(\omega) = F\left[\chi(t)\right]$:

$$\mathbf{P}(t) = \varepsilon_0 \int_{-\infty}^{+\infty} \chi(\tau)\mathbf{E}(t-\tau)d\tau \tag{2.1}$$

$$\tilde{\mathbf{P}}(\omega) = \varepsilon_0 \underline{\chi}(\omega)\tilde{\mathbf{E}}(\omega). \tag{2.2}$$

1. The reaction of a medium to an electric field is causal, as there cannot be any polarization prior to the application of the electric field to the medium. Explain why for this case the following identity holds, where *H(t)* is the Heaviside function:

$$\chi(t) = \chi(t)\, H(t) \quad \text{with} \quad H(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0. \end{cases} \tag{2.3}$$

2. Causality in the time domain corresponds to an equivalent relation in the frequency domain. Transform (2.3) to its frequency domain equivalent. Use the Fourier transform of the Heaviside function:

$$\tilde{H}(\omega) = \frac{1}{j\omega} + \pi\delta(\omega). \tag{2.4}$$

Note: In this course the following definitions of the Fourier transform are used:

$$\tilde{x}(\omega) = F\left[x(t)\right] = \int_{-\infty}^{+\infty} x(t) e^{-i\omega t} dt$$

$$x(t) = F^{-1}\left[\tilde{x}(\omega)\right] = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{x}(\omega) e^{i\omega t} d\omega$$

$$F\left[x(t) \cdot y(t)\right] = \frac{1}{2\pi} \tilde{x}(\omega) * \tilde{y}(\omega) .$$

In order to calculate the convolution of $f(x)$ and $\frac{1}{x}$, the Cauchy principal value has to be introduced: $f(x) * \frac{1}{x} = \mathcal{P}\int_{-\infty}^{+\infty} \frac{f(x')}{x-x'} dx'$.

3. The susceptibility is complex, $\underline{\chi}(\omega) = \chi(\omega) + \mathrm{j}\chi_i(\omega)$. Use the previous result to derive a general relation between the real part $\chi(\omega)$ and the imaginary part $\chi_i(\omega)$ of the susceptibility. This relation is known as the "Kramers-Kronig relation" (after the discoverers H. A. Kramers and R. de Laer Kronig). Note that $\tilde{\chi}(\omega)$ is an even, and $\tilde{\chi}_i(\omega)$ is an odd function, since $\chi(t)$ is a real function.

4. Sketch the frequency dependence of the real and imaginary part of the susceptibility if the medium has a sharp, symmetric absorption line at a frequency ω_0. To do so, assume that $n_i(\omega)$ is predominantly affected by $\chi_i(\omega)$.

Bonus Program:

During the term, three problem sets will be collected in the tutorial without prior announcement, and graded. If more than 70% of each of these problem sets was solved correctly, your oral examination grade will be upgraded by a bonus of 0.3 or 0.4 (except for the grades of 1.0, and 4.7 or worse). If you cannot join a tutorial, you may also submit your solutions by e-mail to the teaching assistants (see contact details below) **before** the respective tutorial starts. In that case, please merge all pages into a single pdf file, and please use a scanner. Smartphone made snapshots are often illegible, and in that case your solution will not be graded.

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