Problem Set 13
Nonlinear Optics (NLO)

Due: July 26, 2017, 08:00 AM

1) Nonlinear Schrödinger Equation

The nonlinear Schrödinger-equation (NLSE) of an optical fiber was derived in the lecture:

$$\frac{\partial}{\partial z}\underline{A}(z,t)+\beta_c^{(1)}\frac{\partial}{\partial t}\underline{A}(z,t)-\frac{1}{2}\mathrm{j}\beta_c^{(2)}\frac{\partial^2}{\partial t^2}\underline{A}(z,t)=-\frac{\alpha}{2}\underline{A}(z,t)-\mathrm{j}\gamma\left|\underline{A}(z,t)\right|^2\underline{A}(z,t) \quad (1.1)$$

1. Explain the parameters $\beta_c^{(1)}$, $\beta_c^{(2)}$, α and γ.
2. For optical fibers, the parameter $D=\mathrm{d}\beta_c^{(1)}/\mathrm{d}\lambda$ is usually specified instead of $\beta_c^{(2)}$. What is the connection between D and $\beta_c^{(2)}$? A standard single-mode fiber (SSMF) has D = 18 ps/(nm·km) at the vacuum wavelength of λ = 1.55 µm. Calculate $\beta_c^{(2)}$ and explain the meaning of D.
3. Consider the new coordinate system t', z' generated by the following transformation as well as the new function $\underline{A}'$:
$$t'=t-\beta_c^{(1)}z$$
$$z'=z$$
$$\underline{A}'(z',t')=\underline{A}(z,t)$$
Imagine for the moment that $\underline{A}(z,t)$ represents a pulse moving along z with velocity $1/\beta_c^{(1)}$. Sketch the functions $\underline{A}(z,t)$ and $\underline{A}'(z',t')$ as a function of t and t' for two different positions of z and z'. Explain why the (z',t') coordinate system is usually referred to as a retarded time frame.
4. Find a formulation of the NLSE for $\underline{A}'$. Notice that the term $\beta_c^{(1)}\frac{\partial}{\partial t'}\underline{A}'(z',t')$ does no longer appear in the differential equation. In the following, we will omit the primes keeping in mind that the time dependence is given with respect to a retarded reference time frame.
5. We will now assume that there are no losses ($\alpha=0$) and search for solutions describing fundamental solitons, i.e., waveforms which do not change their shape as they propagate along z. We therefore require the magnitude of the complex amplitude $\underline{A}'(z,t)$ to be independent of z, but still allow for a z-dependent phase shift. Substitute the ansatz $\underline{A}'(z,t)=A_0(t)\exp(-\mathrm{j}Kz)$ in the NLSE. Assuming further that $A_0(t)$ is a real function, show that the following differential equation holds for $A_0(t)$:

$$\frac{1}{2}\beta_c^{(2)}\frac{1}{A_0(t)}\frac{\partial^2 A_0(t)}{\partial t^2}-\gamma A_0^2(t)=-K \quad . \tag{1.2}$$

6. Show that $A_0(t)=A_1\operatorname{sech}\left(\frac{t}{T}\right)=A_1/\cosh\left(\frac{t}{T}\right)$ is a valid solution ansatz for the differential equation (1.2). Remember that $\cosh^2-\sinh^2=1$, and that the derivative of sinh(x) is cosh(x) and vice versa. Show in particular that the pulse amplitude A_1 and the pulse duration T must fulfill the following relations:

$$K=\frac{1}{2}\gamma A_1^2 \tag{1.3}$$

$$A_1^2=-\frac{\beta_c^{(2)}}{\gamma T^2} \quad . \tag{1.4}$$

7. Eq. 1.2 can be reformulated as:

$$\frac{1}{2}\beta_c^{(2)}\frac{\partial^2 A_0(t)}{\partial t^2}=\left(\gamma A_0^2(t)-K\right)A_0(t). \tag{1.5}$$

   How can this relation be interpreted taking into account the interplay of dispersion and self-phase modulation? If a pulse gets shorter, do you expect that it must have a larger or smaller peak intensity for building a soliton? Check your answer with the help of Eq. (1.4).

Questions and Comments:

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