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Problem Set 2
Nonlinear Optics (NLO)

Due: May 15, 2018, 09:45 AM

1) Lorentz oscillator model for the linear case

In a classical picture, an electron bound to an atom can be considered as a harmonic oscillator in analogy to a mass connected to a spring. This oscillator, when driven by an external electric field E_x, follows the classical equation of motion

$$m_\mathrm{e}\frac{d^2x(t)}{dt^2} = -eE_\mathrm{x}(t) - m_\mathrm{e}\omega_\mathrm{r}^2 x(t) - m_\mathrm{e}\gamma_\mathrm{r}\frac{dx(t)}{dt}, \tag{1.1}$$

where $x(t)$ is the dislocation of the electron, m_e denotes the electronic mass, $-eE_\mathrm{x}(t)$ is the driving force by the external electric field, $-m_\mathrm{e}\omega_\mathrm{r}^2 x$ is the restoring force of the oscillator and $-m_\mathrm{e}\gamma_\mathrm{r}\frac{dx}{dt}$ is a damping term, with damping constant γ_r. The parameter ω_r will turn out to be the resonance frequency of the oscillator.

1. Solve the differential equation (1.1) for a time-harmonic electric field of the form $E(t) = \frac{1}{2}\left(\underline{E}(\omega)\exp(j\omega t) + c.c.\right)$ by using an ansatz for the dislocation with the same time dependence and unknown amplitude $\underline{x}(\omega)$. Derive an expression for $\underline{x}(\omega)$.
2. The electric polarization is the dipole moment per volume,

$$\underline{P}_\mathrm{x}(\omega) = \varepsilon_0\underline{\chi}^{(1)}(\omega)\underline{E}_\mathrm{x}(\omega) = -\frac{N}{V}e\cdot\underline{x}(\omega), \tag{1.2}$$

   where $\frac{N}{V}$ is the number density of atoms in the medium and $-e\cdot\underline{x}(\omega)$ is the induced dipole moment per atom. Show that the susceptibility is given by

$$\underline{\chi}^{(1)}(\omega) = \chi_0\frac{\omega_\mathrm{r}^2}{\omega_\mathrm{r}^2 - \omega^2 + j\omega\gamma_\mathrm{r}},\quad \chi_0 = \frac{Ne^2}{Vm_\mathrm{e}\varepsilon_0\omega_\mathrm{r}^2} \tag{1.3}$$

   and separate the susceptibility into real and imaginary part.
3. Sketch the real and imaginary part of the susceptibility $\underline{\chi}^{(1)}$ around the resonance frequency ω_r. What is the consequence of this result for the real and imaginary part of the refractive index at very large frequencies, e.g. X-ray frequencies?

2) Third-order nonlinear polarization

Consider a linearly polarized plane wave $\mathbf{E}(z,t) = E(z,t)\mathbf{e}_x$ propagating in z-direction in a homogeneous medium, in which third-order nonlinear effects dominate over second and higher-order contributions, $\chi^{(3)} \neq 0$, $\chi^{(m)} = 0$ for m = 2 or m > 3. Assuming an instantaneous response of the polarization $\mathbf{P}_{\mathrm{NL}}$ to the applied electric field, we can express $\mathbf{P}_{\mathrm{NL}}$ as

$$\mathbf{P}_{\mathrm{NL}}(z,t) = P^{(3)}(z,t)\mathbf{e}_{\mathrm{x}}, \text{ with } P^{(3)}(z,t) = \varepsilon_0 \chi^{(3)} E(z,t)^3. \tag{2.1}$$

Calculate $P^{(3)}(z,t)$ considering a field $E\left(z,t\right)$ composed of two distinct frequency components ω_1 and ω_2 with their complex amplitudes $\underline{E}_1$ and $\underline{E}_2$

$$E(z,t) = \frac{1}{2}\left(\underline{E}_1 e^{\mathrm{j}(\omega_1 t - k_1 z)} + \underline{E}_2 e^{\mathrm{j}(\omega_2 t - k_2 z)} + c.c.\right). \tag{2.2}$$

Group the resulting terms with appropriate degeneracy factors according to their frequency and assign them to the following effects:

- Third-Harmonic Generation (THG)
- Third-order Sum-Frequency Generation (TSFG)
- Self-Phase Modulation (SPM)
- Cross-Phase Modulation (XPM)
- (Degenerate) Four-Wave Mixing (FWM)

Bonus Program:

Bonus system: During the term, three problem sets will be collected in the tutorial without prior announcement, and graded. If more than 70% of each of these problem sets was solved correctly, your oral examination grade will be upgraded by a bonus of 0.3 or 0.4 (except for the grades of 1.0, and 4.7 or worse). If you cannot join a tutorial, you may also submit your solutions by e-mail to nlo@ipq.kit.edu **before** the respective tutorial starts. In that case, please merge all pages into a single pdf file, and please use a scanner. Smartphone made snapshots are often illegible, and in that case your solution will not be graded.

Questions and Comments:

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