Problem Set 8
Nonlinear Optics (NLO)

Due: July 03, 2018, 09:45 AM

1) Electro-optic Mach-Zehnder modulator

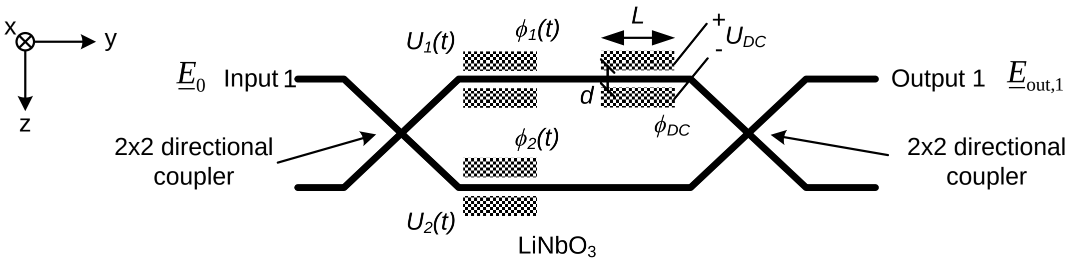

Figure 1: Dual-drive Mach-Zehnder modulator

Figure 1 shows a dual-drive Mach-Zehnder modulator. The device consists of a waveguide-based Mach-Zehnder interferometer having voltage-controlled phase shifters in each arm. For high-speed modulation, time-dependent voltages $U_1(t)$ and $U_2(t)$ are applied to two phase-shifters ($\phi_1(t)$ and $\phi_2(t)$) in the upper and lower arm, respectively, whereas a third phase shifter (ϕ_{DC}) operated by a constant DC bias voltage U_{DC} is used to set the operating point. The device is made of lithium-niobate ($LiNbO_3$) using x-cut geometry. The principal axes[1] are the x, y and z axes shown in Figure 1. The propagating light of wavelength $\lambda = 1.55\ \mu m$ is polarized along the z axis. The refractive indices are $n_o = 2.211$ and $n_e = 2.138$. The electro-optic coefficients, measured at a wavelength of 0.5 µm are $r_{13} = 9.6\,\text{pm/V}$, $r_{22} = 6.8\,\text{pm/V}$, $r_{33} = 30.9\,\text{pm/V}$, and $r_{42} = 32.6\,\text{pm/V}$. Assume that these values are also valid at the wavelength of 1.55 µm.

1. Consider that $U_1(t) = U_2(t) = 0$, and an external voltage U_{DC} is applied to the two parallel metal contacts (length $L = 2$ mm, distance $d = 5$ µm), inducing a phase shift ϕ_{DC} in the upper arm. What voltage $U_{\pi,DC}$ is needed for a phase shift of π between both arms?

   Hint: Start by calculating the change of refractive index as a function of the applied voltage U_{DC} and approximate the modulating electric field along the z-direction by the field of a parallel plate capacitor with electrode spacing d, i.e., $E_z^{(el)} \approx U_{DC} / d$.

2. Consider the initial complex amplitude $\underline{E}_0$ at Input 1. Express the general amplitude transfer function for output 1 of the device, $T_{out,1} = \underline{E}_{out,1} / \underline{E}_0$, as a function of the applied phase shifts $\phi_1(t)$, $\phi_2(t)$, and ϕ_{DC}. Next, consider the situation from part 1 ($U_1(t) = U_2(t) = 0$) and sketch the amplitude transfer function $T_{out,1}$ and the power transfer function $|T_{out,1}|^2$ versus the normalized applied voltage U_{DC} / U_π.

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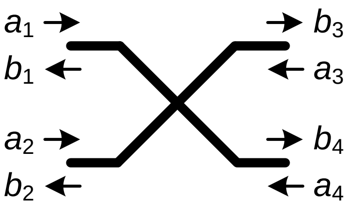
[1] The symmetry group of $LiNbO_3$ is $C_{3v} = 3m$. The convention used here is that the z-axis is parallel to the threefold rotational axis of the crystal.

Hint: Assume that the device consists of lumped elements with individual scattering matrices. Using the input and output amplitudes a_i and b_i of a symmetric 2x2 directional coupler as indicated in Figure 2, its scattering matrix $\mathbf{S}_{2x2}$ can be written as

$$\mathbf{S}_{2x2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 & -j \\ 0 & 0 & -j & 1 \\ 1 & -j & 0 & 0 \\ -j & 1 & 0 & 0 \end{pmatrix}$$

where $b_m = S_{mn} \cdot a_n$.



$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \mathbf{S}_{2x2} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$$

Figure 2: Definition of input and output amplitudes of a 2x2 directional coupler

For operating the modulator in so-called push-pull mode, voltages of equal amplitude but opposite signs are applied to the two arms, $U_1(t) = -U_2(t) = U(t)$. The phase difference between both arms then amounts to $\Delta\phi(t) = \phi_1(t) - \phi_2(t)$.

3. Adapt the expression of $T_{out,1}$ from part 2 to the push-pull configuration. Sketch the amplitude transfer function as a function of the voltage $U(t)$ normalized to the π-voltage $U_{\pi,AC}$ of the high-frequency phase-shifters for $\phi_{DC} = 0$. $U_{\pi,AC}$ is the voltage required to generate a phase difference of $\Delta\phi(t) = \pi$ between the two arms.

4. In some applications it is important to have a **linear** relationship between small variations $\delta U(t)$ of the **input voltage** and the associated variations $\delta \underline{E}_{out,1}$ of the **optical amplitude** at output 1. This can be achieved by choosing a suitable DC bias, U_{DC}. Which bias voltage would you choose for this case?

5. For the case that $\phi_{DC} = 0$, sketch the power transfer function at output 1 as a function of the normalized voltage $U(t) / U_{\pi,AC}$. Which output power is obtained when adjusting the bias voltage according to part 4?

6. In other applications it is important to have a **linear** relationship between small variations of the **input voltage** and the associated variations of the **optical power** $P \propto \left|\underline{E}_{out,1}\right|^2$. Which bias voltage would you choose for this case?

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