Problem Set 8
Nonlinear Optics (NLO)

Due: 3. July 2018

1) Electro-optic Mach-Zehnder modulator

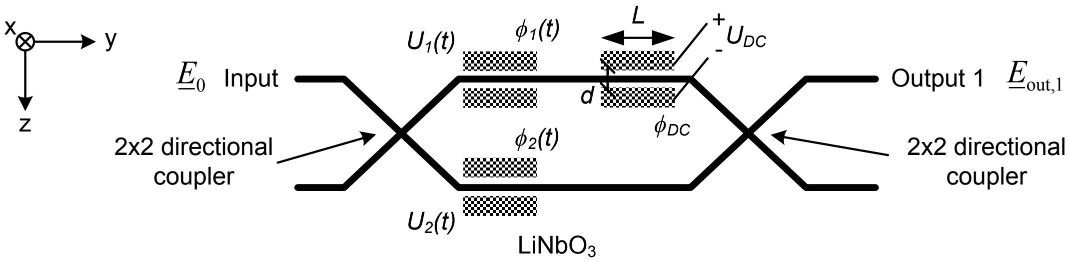


Figure 1: Dual-drive Mach-Zehnder modulator

Figure 1 shows a dual-drive Mach-Zehnder modulator. The device consists of a waveguide-based Mach-Zehnder interferometer having voltage-controlled phase shifters in each arm. For high-speed modulation, time-dependent voltages $U_1(t)$ and $U_2(t)$ are applied to two phase-shifters ($\phi_1(t)$ and $\phi_2(t)$) in the upper and lower arm, respectively, whereas a third phase shifter (ϕ_{DC}) operated by a constant DC bias voltage U_{DC} is used to set the operating point. The device is made of lithium-niobate ($LiNbO_3$) using x-cut geometry. The principal axes[1] are the x, y and z axes shown in Figure 1. The propagating light of wavelength $\lambda = 1.55\ \mu\text{m}$ is polarized along the z axis. The refractive indices are $n_o = 2.211$ and $n_e = 2.138$. The electro-optic coefficients, measured at a wavelength of 0.5 µm are $r_{13} = 9.6\,\text{pm/V}$, $r_{22} = 6.8\,\text{pm/V}$, $r_{33} = 30.9\,\text{pm/V}$, and $r_{42} = 32.6\,\text{pm/V}$. Assume that these values are also valid at the wavelength of 1.55 µm.

1. Consider that $U_1(t) = U_2(t) = 0$, and an external voltage U_{DC} is applied to the two parallel metal contacts (length $L = 2$ mm, distance $d = 5$ µm), inducing a phase shift ϕ_{DC} in the upper arm. What voltage $U_{\pi,DC}$ is needed for a phase shift of π between both arms?

   Hint: Start by calculating the change of refractive index as a function of the applied voltage U_{DC} and approximate the modulating electric field along the z-direction by the field of a parallel plate capacitor with electrode spacing d, i.e., $E_z^{(el)} \approx U_{DC} / d$.

2. Consider the initial complex amplitude $\underline{E}_0$ at input 1. Express the general amplitude transfer function for output 1 of the device, $T_{out,1} = \underline{E}_{out,1} / \underline{E}_0$, as a function of the applied phase shifts $\phi_1(t)$, $\phi_2(t)$, and ϕ_{DC}. Next, consider the situation from part 1

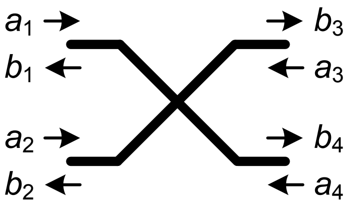
[1] The symmetry group of $LiNbO_3$ is $C_{3v} = 3m$. The convention used here is that the z axis is parallel to the threefold rotational axis of the crystal.

($U_1(t) = U_2(t) = 0$) and sketch the amplitude transfer function $T_{out,1}$ and the power transfer function $\left|T_{out,1}\right|^2$ versus the normalized applied voltage U_{DC} / U_{π}, .

<u>Hint</u>: Assume the device to consist of lumped elements with individual scattering matrices. Using the input and output amplitudes a_i and b_i of a symmetric 2x2 directional coupler as indicated in Figure 2, its scattering matrix $\mathbf{S}_{2x2}$ can be written as

$$\mathbf{S}_{2x2} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 & -j \\ 0 & 0 & -j & 1 \\ 1 & -j & 0 & 0 \\ -j & 1 & 0 & 0 \end{pmatrix}$$

where $b_m = S_{mn} \cdot a_n$



$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \mathbf{S}_{2x2}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$$

Figure 2: Definition of input and output amplitudes of a 2x2 directional coupler

For operating the modulator in so-called push-pull mode, voltages of equal amplitude but opposite signs are applied to the two arms, $U_1(t) = -U_2(t) = U(t)$. The phase difference between both arms then amounts to $\Delta\phi(t) = \phi_1(t) - \phi_2(t)$.

3. Adapt the expression of $T_{out,1}$ from part 2 to the push-pull configuration. Sketch the amplitude transfer function as a function of the voltage $U(t)$ normalized to the pi-voltage $U_{\pi,AC}$ of the high-frequency electrodes for $\phi_{DC} = 0$. $U_{\pi,AC}$ is the voltage required to generate a phase difference of $\Delta\phi(t) = \pi$ between the two arms.

4. In some applications it is important to have a **linear** relationship between small variations $\delta U(t)$ of the **input voltage** and the associated variations $\delta \underline{E}_{out,1}$ of the **optical amplitude** at output 1. This can be achieved by choosing a suitable DC bias, U_{DC}. Which bias voltage would you choose for this case?

5. For the case that $\phi_{DC} = 0$, sketch the power transfer function at output 1 as a function of the normalized voltage $U(t) / U_{\pi,AC}$. Which output power is obtained when adjusting the bias voltage according to part 4?

6. In other applications it is important to have a **linear** relationship between small variations of the **input voltage** and the associated variations of the **optical power** $P \propto \left|\underline{E}_{out,1}\right|^2$. Which bias voltage would you choose for this case?

Solution:

1. Following the hint, to deduce the change of refractive index as a function of U_{DC} we need to first compute the induced change of the impermeability tensor from Eq.(3.24) of the lecture notes. In our given problem, we already know the orientation of the external electric field as well as the polarization state of the optical field and orientation of nonlinear crystal:

   - External electric field is oriented in z-direction → only r_{i3} contributes. Therefore the impermeability tensor will have the form:

$$\eta = \begin{pmatrix} \frac{1}{n_o^2} + r_{13}E_z & 0 & 0 \\ 0 & \frac{1}{n_o^2} + r_{13}E_z & 0 \\ 0 & 0 & \frac{1}{n_e^2} + r_{33}E_z \end{pmatrix}$$

   - The optical field is polarized in z-direction and the coordinate axes are orientated along the principle axis of crystal → only r_{33} contributes, the optical field will experience only the effect of the extraordinary index $\frac{1}{(n_\mathrm{e}')^2} = \frac{1}{n_\mathrm{e}^2} + r_{33}E_z^{(el)}$

In this geometrical configuration and with the given fields, the indicatrix assumes the form:

$$\frac{1}{n_\mathrm{o}^2}X^2 + \frac{1}{n_\mathrm{o}^2}Y^2 + \left(\frac{1}{n_\mathrm{e}^2} + r_{33}E_z^{(el)}\right)Z^2 = 1 \tag{0.1}$$

We calculate the refractive index change by

$$\begin{aligned} &\frac{1}{n_e'^2} = \frac{1}{n_\mathrm{e}^2} + r_{33}E_z^{(el)} \\ &\Rightarrow n_e' = n_\mathrm{e}^2\sqrt{\frac{1}{1+n_\mathrm{e}^2 r_{33}E_z^{(el)}}} \approx n_e(1-\frac{1}{2}n_\mathrm{e}^2 r_{33}E_z^{(el)}) \\ &\Rightarrow \Delta n = -\frac{1}{2}n_\mathrm{e}^3 r_{33}E_z^{(el)} = -\frac{1}{2}n_\mathrm{e}^3 r_{33}\frac{U}{d} \end{aligned} \tag{0.2}$$

We then compute the phase shift that is accumulated in the modulated arm along the propagation length L:

$$\Delta\varphi = \Delta n k_0 L = -k_0 L \frac{1}{2} n_\mathrm{e}^3 r_{33}\frac{U}{d} \tag{0.3}$$

Finally, the voltage $U_{\pi,\mathrm{DC}}$ needed to introduce a phase shift of π is:

$$U_\pi = \frac{d\lambda}{L n_e^3 r_{33}} = 12{,}83V \tag{0.4}$$

2. We start at Input 1 with the initial complex amplitude $\underline{E}_0$ and the corresponding intensity $I_0 \sim \underline{E}_0 \underline{E}_0^*$. The input vector a of the first coupler containing the amplitudes is $\left(\underline{E}_0, 0, 0, 0\right)^T$.

After the first coupler the power is split equally into the modulated upper (b_3) and the lower arm (b_4), where the lower arm receives a phase factor:

Amplitude in the upper arm: $b_3 = \frac{1}{\sqrt{2}} \underline{E}_0$

Amplitude in the lower arm: $b_4 = -j \frac{1}{\sqrt{2}} \underline{E}_0$

Both amplitudes accumulate a different phase shift during the propagation in the respective arms.

Amplitude after upper arm:

$$\frac{1}{\sqrt{2}} \underline{E}_0 \exp\left(-j\phi_1(t)\right) \exp\left(-j\phi_{DC}\right) \exp\left(-jnk_0 r\right)$$

Amplitude after lower arm:

$$-j \frac{1}{\sqrt{2}} \underline{E}_0 \exp\left(-j\phi_2(t)\right) \exp\left(-jnk_0 r\right)$$

The second directional coupler has now two input amplitudes:

$$a = \left(\frac{1}{\sqrt{2}} \underline{E}_0 \exp\left(-j\phi_1(t)\right) \exp\left(-j\phi_{DC}\right) \exp\left(-jnk_0 r\right), -j \frac{1}{\sqrt{2}} \underline{E}_0 \exp\left(-j\phi_2(t)\right) \exp\left(-jnk_0 r\right), 0, 0 \right)^T$$

The amplitudes at the Outputs 1 and 2 of the complete device is given by the last two entries of the output vector of the 2nd directional coupler:

$$\underline{E}_{out,1} = \frac{1}{2} \underline{E}_0 \exp\left(-jnk_0 r\right) \left[\exp\left(-j\phi_1(t) - j\phi_{DC}\right) - \exp\left(-j\phi_2(t)\right) \right]$$

$$\underline{E}_{out,2} = -j \frac{1}{2} \underline{E}_0 \exp\left(-jnk_0 r\right) \left[\exp\left(-j\phi_1(t) - j\phi_{DC}\right) + \exp\left(-j\phi_2(t)\right) \right]$$

The amplitude transfer function of the output 1 is given by:

$$T_{out,1} = \frac{\underline{E}_{out,2}}{\underline{E}_0} = \frac{1}{2}\exp\left(-jnk_0 r\right)\left[\exp\left(-j\Delta\phi_1(t) - j\Delta\phi_{DC}\right) - \exp\left(-j\Delta\phi_2(t)\right)\right]$$

$$= \frac{1}{2}\exp\left(-jnk_0 r\right)\left[\exp\left(-j\Delta\phi_1(t) - j\Delta\phi_{DC}\right) - \exp\left(-j\Delta\phi_2(t)\right)\right]$$

$$= \frac{1}{2}\exp\left(-jnk_0 r\right)\left\{\begin{array}{l}\exp\left[-j/2\left(\Delta\phi_1(t)+\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right]\exp\left[-j/2\left(\Delta\phi_1(t)-\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right] \\ -\exp\left[-j/2\left(\Delta\phi_1(t)+\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right]\exp\left[-j/2\left(-\Delta\phi_1(t)+\Delta\phi_2(t)-\Delta\phi_{DC}\right)\right]\end{array}\right\}$$

$$= -j\sin\left[\frac{1}{2}\left(\Delta\phi_1(t)-\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right]\exp\left[-j/2\left(\Delta\phi_1(t)+\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right]\exp\left(-jnk_0 r\right)$$

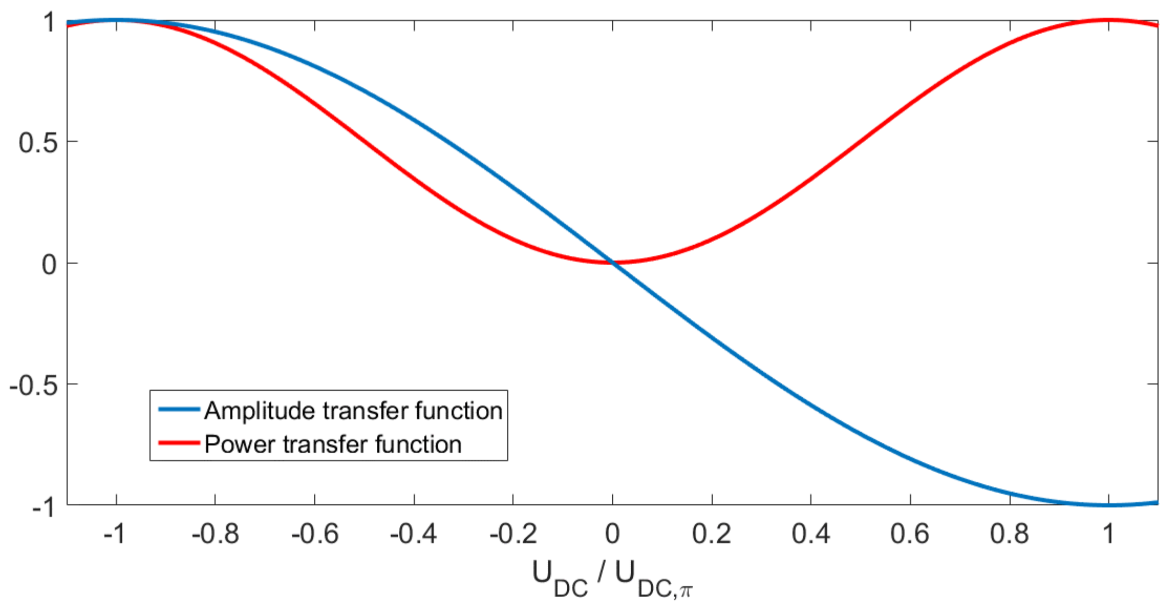
The amplitude transfer function of the output 2 is given by:

$$T_{out,2} = \frac{\underline{E}_{out,2}}{\underline{E}_0} = -j\frac{1}{2}\exp\left(-jnk_0 r\right)\left[\exp\left(-j\Delta\phi_1(t) - j\Delta\phi_{DC}\right) + \exp\left(-j\Delta\phi_2(t)\right)\right]$$

$$= -j\cos\left[\frac{1}{2}\left(\Delta\phi_1(t)-\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right]\exp\left[-j/2\left(\Delta\phi_1(t)+\Delta\phi_2(t)+\Delta\phi_{DC}\right)\right]\exp\left(-jnk_0 r\right)$$

$$T_{out,1} = -e^{-jnk_0 r}e^{-j\phi_{DC}/2}\,j\sin\left(\frac{1}{2}\phi_{DC}\right) = -e^{-jnk_0 r}e^{-j\phi_{DC}/2}\,j\sin\left(\frac{1}{2}\frac{U_{DC}}{U_{DC,\pi}}\pi\right)$$

$$\left|T_{out,1}\right|^2 = \sin^2\left(\frac{1}{2}\phi_{DC}\right) = \sin^2\left(\frac{1}{2}\frac{U_{DC}}{U_{DC,\pi}}\pi\right)$$



Where the first term correspond to a constant phase term and we could neglect as it only influences the absolute position of the curve. If the phase term depended on ϕ_{DC} is to be considered, we compute the real part of the amplitude transfer function as:

$$e^{-j\phi_{DC}/2}\sin\left(\frac{1}{2}\frac{U_{DC}}{U_{DC,\pi}}\pi\right)=\cos\left(\frac{1}{2}\frac{U_{DC}}{U_{DC,\pi}}\pi\right)\sin\left(\frac{1}{2}\frac{U_{DC}}{U_{DC,\pi}}\pi\right)=\sin\left(\frac{U_{DC}}{U_{DC,\pi}}\pi\right)$$

3. In push-pull-mode, the amplitude transfer function is:

$$T_{out,1}=-je^{-jnk_0r}e^{-j/2(\phi_1+\phi_2+\phi_{DC})}\sin\left(\frac{1}{2}(\phi_1-\phi_2+\phi_{DC})\right)$$

$$=-je^{-jnk_0r}e^{-j/2(\phi_1+\phi_2+\phi_{DC})}\sin\left(\frac{1}{2}\Delta\phi\right)=e^{j\theta}\sin\left(\frac{U_{AC}}{U_{AC,\pi}}\pi\right)$$

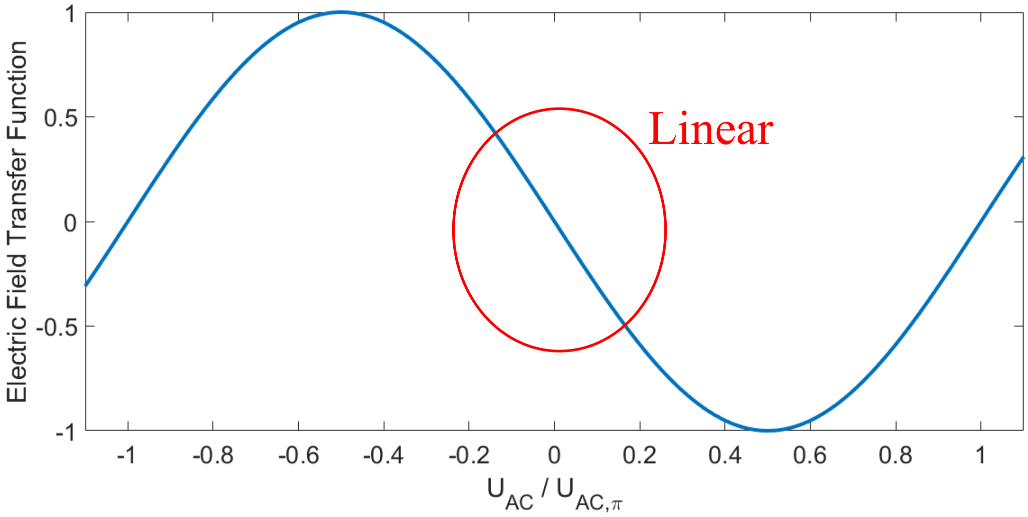
For $\Delta\phi_{DC}=0$:

$$T_{out,1}=e^{-jnk_0r}e^{-j\pi/2}\sin\left(\frac{U_{AC}}{U_{AC,\pi}}\pi\right)$$

Where $\exp(-jnk_0r)\exp(-j\pi/2)$ is a constant phase term.

$$\Delta\phi(t)=\frac{2U(t)}{U_{\pi,AC}}\pi$$

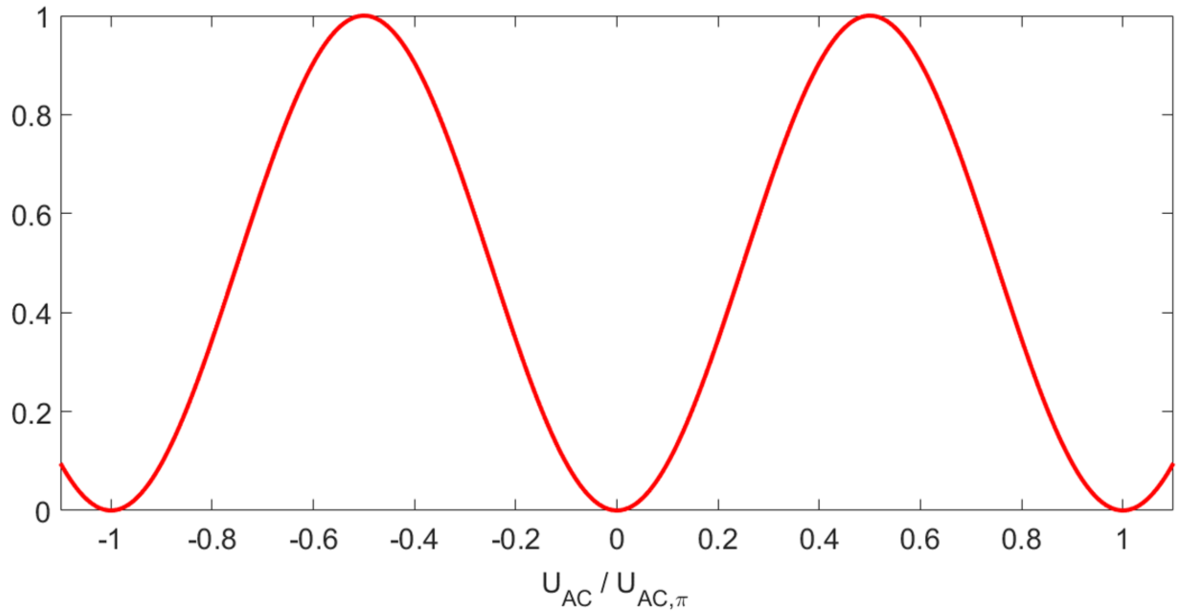
For output 1 and 2 the amplitude transfer function has oscillating behaviour. Note that the absolute position of such curves may be influenced by the previously mentioned constant phase terms.



4. For a linear relationship the device needs to be operated at the so-called null point, where the amplitude transfer function is near to zero. For the output 1 the DC bias should be null.

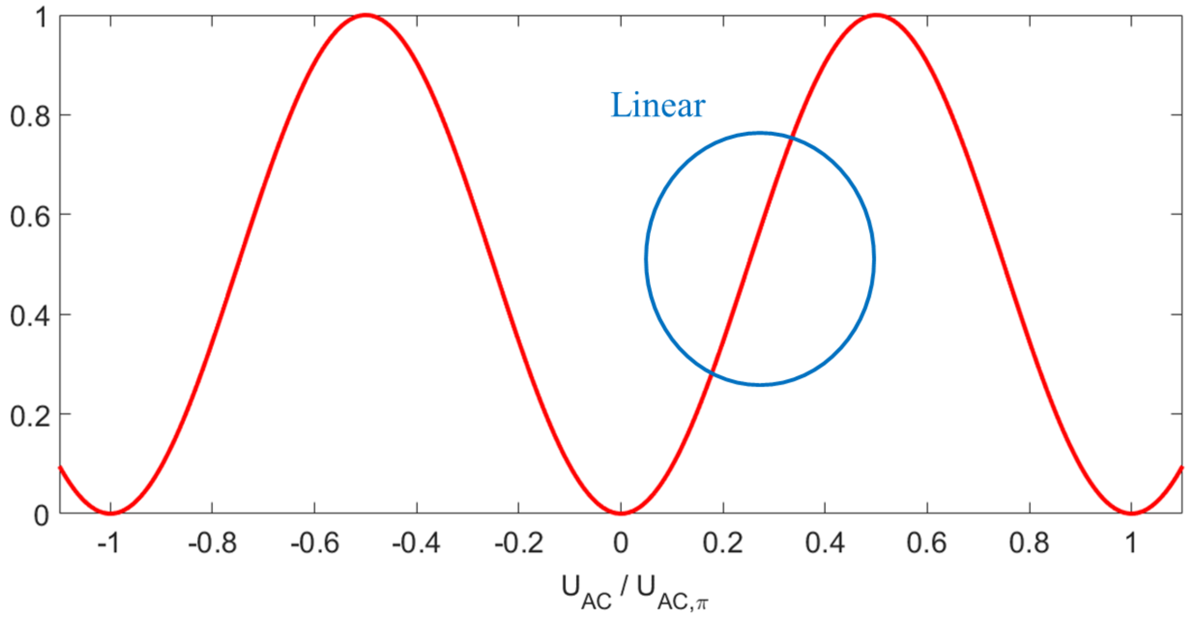
5. The power transfer functions:

$$\left|T_{out,1}\right|^2 = \sin^2\left(\frac{1}{2}\Delta\phi\right) = \sin^2\left(\frac{U_{AC}}{U_{AC,\pi}}\pi\right)$$



When adjusting the bias at the null point, the output power is zero.

6. For a linear relation between small variations of the input voltage and the associated variations of the optical power, the device needs to be operated at the "quadrature point", at half the maximum intensity.



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