Problem Set 10
Nonlinear Optics (NLO)

Due: July 10, 2018, 09:30 AM

Optical Parametric Amplifier (OPA)

Optical parametric amplifiers (OPA) exploit difference-frequency generation (DFG) in a second-order nonlinear crystal. In this process, a strong external pump at frequency ω_p is used to amplify a signal at frequency ω_s, thereby generating a new, so-called idler wave at frequency $\omega_i = \omega_p - \omega_s$. All waves are linearly polarized and monochromatic:

$$\mathbf{E}_{p,s,i}(z,t) = \frac{1}{2}\left(\underline{E}_{p,s,i}(z)\, e^{\mathrm{j}\left(\omega_{p,s,i} t - k_{p,s,i} z\right)} \mathbf{e}_{p,s,i} + c.c. \right), \tag{1.1}$$

where the subscripts p, s, and i refer to the pump, signal and idler wave. Assuming a fixed set of polarizations $\mathbf{e}_{\mathrm{p,s,i}}$, we can replace the second-order nonlinear tensor by its effective value d_{eff} according to Eq. (3.50) in the lecture notes. The interaction of the three waves is then given by the following system of coupled differential equations with $k_m = \dfrac{\omega_m}{cn\left(\omega_m\right)}$:

$$\frac{\partial \underline{E}\left(z,t,\omega_p\right)}{\partial z} = -\mathrm{j} k_p d_{\mathrm{eff}} \underline{E}\left(z,t,\omega_i\right) \underline{E}\left(z,t,\omega_s\right) e^{-\mathrm{j}\Delta k z} \tag{1.2}$$

$$\frac{\partial \underline{E}\left(z,t,\omega_s\right)}{\partial z} = -\mathrm{j} k_s d_{\mathrm{eff}} \underline{E}\left(z,t,\omega_p\right) \underline{E}^*\left(z,t,\omega_i\right) e^{\mathrm{j}\Delta k z} \tag{1.3}$$

$$\frac{\partial \underline{E}\left(z,t,\omega_i\right)}{\partial z} = -\mathrm{j} k_i d_{\mathrm{eff}} \underline{E}\left(z,t,\omega_p\right) \underline{E}^*\left(z,t,\omega_s\right) e^{\mathrm{j}\Delta k z} \tag{1.4}$$

1. Calculate the evolution of the signal and the idler amplitudes along the propagation direction under the assumption of perfect phase matching, $\Delta k = 0$. Assume further that the pump is much stronger than the signal and the idler, and that we can therefore neglect pump depletion. Sketch the intensities of signal and idler waves along the propagation direction z. Derive an expression for the amplification (power gain) of the signal wave. Assume that the idler amplitude is zero at $z = 0$.

2. As a second-order nonlinear material, we use Beta-Barium-Borate (BBO), a negative-uniaxial crystal, for which we can achieve phase matching by exploiting birefringence. Let us assume that all three waves propagate in the same direction, at an angle θ with respect to the optical axis. For type-2 phase matching, the signal and the idler waves propagate in different normal modes with orthogonal polarization states. Formulate the phase matching condition for the refractive indices of the various waves, assuming that the signal is propagating in ordinary polarization, whereas the idler and the pump are in extraordinary polarization and experience angle-dependent refractive indices.

3. For BBO the dispersion relations for the ordinary and extraordinary polarization in the range of 0.64 µm to 3.18 µm are given by (λ in µm):

$$n_o^2(\lambda) = 2.7359 + \frac{0.01878}{\lambda^2 - 0.01822} - 0.01471\lambda^2,$$
$$n_e^2(\lambda) = 2.3753 + \frac{0.01224}{\lambda^2 - 0.01667} - 0.01627\lambda^2. \quad (1.5)$$

   The pump wavelength is 800 nm. Assuming the configuration described in question 2, plot the phase matching angle θ as a function of the signal wavelength in the range between 1100 nm and 1600 nm. Use a mathematical software package (e.g. MATLAB) to generate the plot.

The following two parts are slightly more challenging. They will not be considered for the bonus system, but provide additional insight.

4. The effective nonlinear coefficient d_{eff} for BBO (symmetry group 3m or C_{3V}) as a function of the angles φ and θ is given in the following paper:

   D. N. Nikogosyan. *Beta Barium Borate (BBO) - A Review of Its Properties and Applications*. Appl. Phys. A **52** (1991) 359-386.

   From the KIT network you can download this paper here[1]. Identify the relevant relations and determine d_{eff} for the calculated type-2 phase matching for a signal wavelength of 1500 nm. Choose an angle φ that maximizes d_{eff}.

5. A signal at a wavelength of 1500 nm propagates through the crystal along with a pump wave of intensity $I_\text{p} = 40\frac{\text{GW}}{\text{cm}^2}$. Calculate the amplification that the signal experiences if the interaction length within the nonlinear crystal amounts to 2 mm.

Questions and Comments:

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[1] http://link.springer.com/article/10.1007/BF00323647