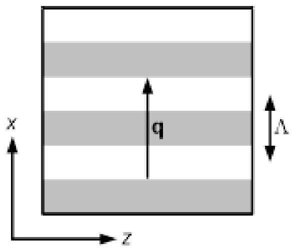
Problem Set 11
Nonlinear Optics (NLO)

Due: July 11, 2018, 08:00 AM

1) Acousto-Optic Modulator



Consider a material in which a sound wave is travelling in x-direction with wave vector $\mathbf{q}$ and frequency Ω. The associated strain induces a periodic change of the refractive index that scatters an incoming optical wave. In Eq. (4.14) of the lecture notes, we derived a coupled-wave relation for the space-dependent amplitudes $\underline{E}(\mathbf{r},\omega_l)$ of the incoming optical wave $(l=0)$ at frequency ω_0 and the various scattered optical waves at frequencies ω_l. Assume that all optical waves are polarized along the y-direction, i.e., $\mathbf{e}_l = \mathbf{e}_y \; \forall\, l$. The scalar coupled-wave equation can then be written as

$$\sum_l -2\mathrm{j}\mathbf{k}_l \cdot \nabla \underline{E}(\mathbf{r},\omega_l) e^{\mathrm{j}(\omega_l t - \mathbf{k}_l \mathbf{r})} = \frac{2n_0}{c^2} \sum_l \frac{\partial^2}{\partial t^2} \left(\Delta n(\mathbf{r},t) \underline{E}(\mathbf{r},\omega_l) e^{\mathrm{j}(\omega_l t - \mathbf{k}_l \mathbf{r})} \right), \tag{1.1}$$

where the index variation $\Delta n(\mathbf{r},t)$ is given by

$$\Delta n(\mathbf{r},t) = \Delta n_0 \cos(\Omega t - \mathbf{q}\mathbf{r}).$$

1. For a monochromatic incident optical wave at frequency ω_0, the right-hand side of Eq. (1.1) contains frequency components at $\omega_{\pm 1} = \omega_0 \pm \Omega$. In Eq. (1.1), consider only the expressions with $l = 0$ and $l = 1$, and derive two coupled differential equations for the optical wave amplitudes $\underline{E}(\mathbf{r},\omega_0)$ and $\underline{E}(\mathbf{r},\omega_1)$ by comparing the coefficients associated with the same frequency on the left- and right-hand side of Eq. (1.1).

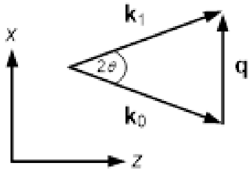
2. Consider the case where both the crystal and the optical waves are infinitely extended in x- and y-direction, which implies $\frac{\partial \underline{E}}{\partial x} = 0$ and $\frac{\partial \underline{E}}{\partial y} = 0$. Assume further that the z-components of the $\mathbf{k}$-vector for both optical waves are equal, i.e. $k_{0z} = k_{1z} = k_z$. Using these simplifications, show that the two coupled differential equations can be written as:

$$\frac{\partial \underline{E}(z,\omega_1)}{\partial z} = -\mathrm{j}\kappa \underline{E}(z,\omega_0) e^{-\mathrm{j}\Delta\mathbf{k}\mathbf{r}}$$

$$\frac{\partial \underline{E}(z,\omega_0)}{\partial z} = -\mathrm{j}\kappa \underline{E}(z,\omega_1) e^{\mathrm{j}\Delta\mathbf{k}\mathbf{r}}$$

with $\kappa = \frac{k_z \Delta n_0}{2n_0}$ and $\Delta\mathbf{k} = \mathbf{k}_0 + \mathbf{q} - \mathbf{k}_1$.

3. Solve the differential equations assuming perfect phase matching, i.e. $\Delta\mathbf{k} = 0$ and using the boundary conditions $\underline{E}(0, \omega_0) = E_0$ and $\underline{E}(0, \omega_1) = 0$. Sketch the evolution of the intensities of the incident and the deflected wave along z. How long should the crystal extend in the z-direction for maximum intensity of the deflected wave?



4. What is the angle of diffraction for light at a vacuum wavelength of 632.8 nm in a $LiNbO_3$ cell that is driven at a frequency $\Omega / 2\pi$ = 1 GHz? (speed of sound: $v_s = 4.1$ km/s, refractive index $n_0 = 2.3$)

2) Nonlinear absorption in a material

Semiconductors such as silicon show losses beyond regular linear losses. This is caused by multi-photon absorption, i.e., the simultaneous absorption of multiple photons, where the overall photon energy is sufficient to overcome the bandgap and hence to transfer electrons from the valence to the conduction band. In the following, we consider only the case of two-photon absorption. It can be described by the imaginary part of a complex third-order susceptibility $\underline{\chi}^{(3)} = \chi^{(3)} + \mathrm{j}\chi_i^{(3)}$. If only a single monochromatic wave is propagating in the medium, we can use the equation derived for the case of self-phase modulation (SPM) to describe the evolution of the field along the propagation direction z (see also Lecture Notes Eq. (1.116)):

$$\frac{\partial \underline{E}(z,\omega)}{\partial z} = -\mathrm{j}\frac{3\omega\underline{\chi}^{(3)}(\omega:\omega,-\omega,\omega)}{8cn_0}\left|\underline{E}(z,\omega)\right|^2 \underline{E}(z,\omega). \tag{1.2}$$

When considering the evolution of the intensity, $I(z,\omega) = \frac{1}{2}\frac{n_0}{Z_0}\left|\underline{E}(z,\omega)\right|^2$, rather than the electric field, two-photon absorption (TPA) is described by the TPA coefficient β_{TPA} that is defined through the decay of the intensity according to

$$\frac{\partial I(z,\omega)}{\partial z} = -\beta_{TPA} I(z,\omega)^2. \tag{1.3}$$

1. Express the TPA coefficient β_{TPA} by the susceptibility $\underline{\chi}^{(3)}$. Use Eqs. (1.2) and (1.3).

2. Solve the differential equation (1.3) with the initial condition $I(0) = I_0$ and sketch the intensity as a function of the input intensity I_0 at a fixed position z. Derive an upper bound of the power that can be observed at the output of the material.

Questions and Comments:

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