Karlsruhe Institute of Technology

Particle Physics 1

Lecture 10: Electroweak interactions

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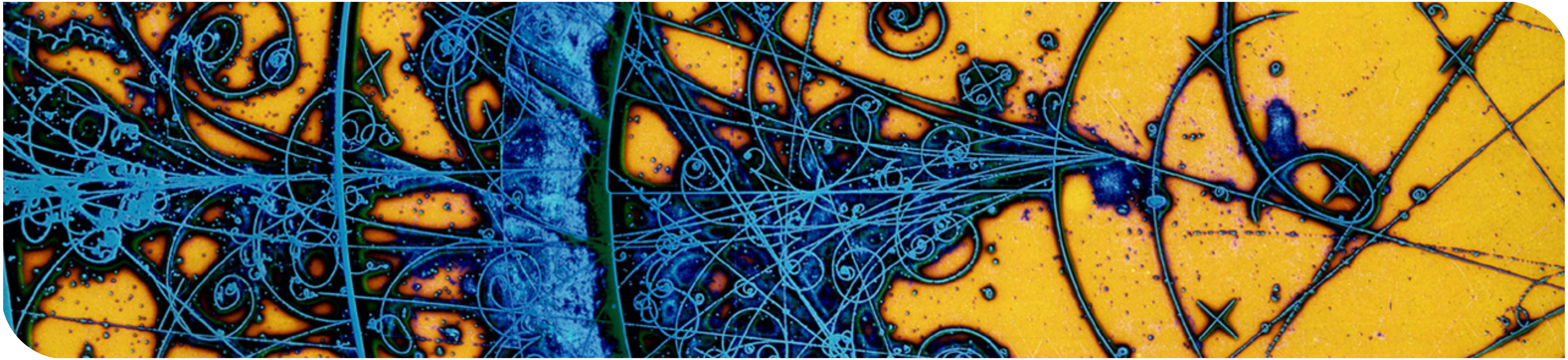
Wintersemester 2023/2024



Credit: CERN

Questions from past lectures

KCETA Colloquium

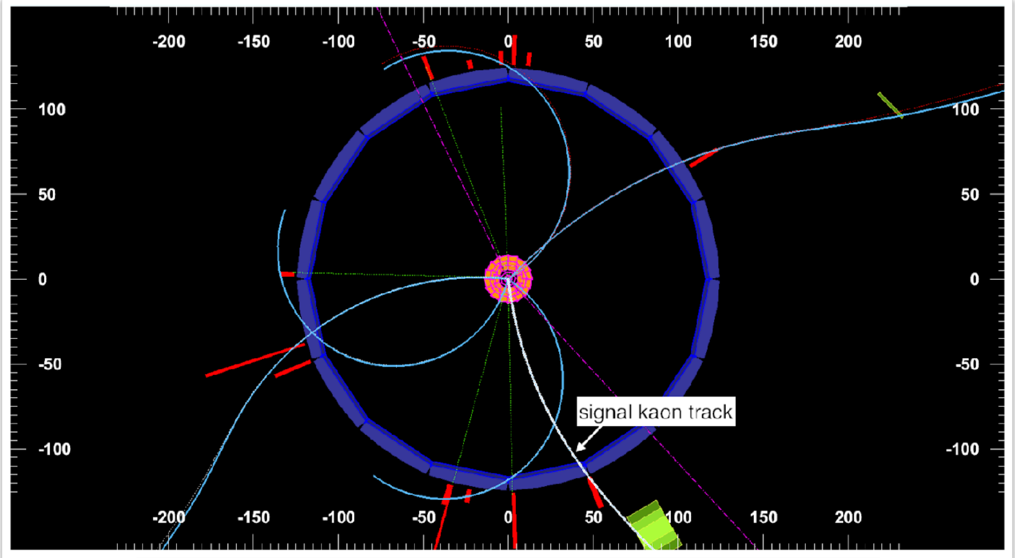
Evidence for a rare *B* decay with two invisible neutrinos at Belle II

Thursday, November 30, 2023
Kleiner Hörsaal A (CS) 15:45 - 17:00

Dr. Slavomira Stefkova
(Institute of Elementary Particle Physics, Karlsruhe Institute of Techology)

The decay of $B^+ \to K^+\nu\bar{\nu}$ is mediated by a flavor-changing neutral current. In the Standard Model, the rate for this elusive process is predicted to be 6×10^{-5}, while enhancements are foreseen in many New Physics scenarios. Searching for $B^+ \to K^+\nu\bar{\nu}$ decays is, however, experimentally challenging as these decays are not only rare but also contain two neutrinos, leaving no signature in the detector.

In this colloquium, I will show you details of the newest measurement of the rate of $B^+ \to K^+\nu\bar{\nu}$ decays, which is based on $362\ \mathrm{fb}^{-1}$ of SuperKEKB e^+e^- collision data collected at the $\Upsilon(4S)$ resonance by the Belle II experiment in Tsukuba, Japan. Using two different reconstruction techniques, we found, for the first time, evidence for the $B^+ \to K^+\nu\bar{\nu}$ process. At the end of my talk, I will also highlight future opportunities in B-decays with invisible signatures.



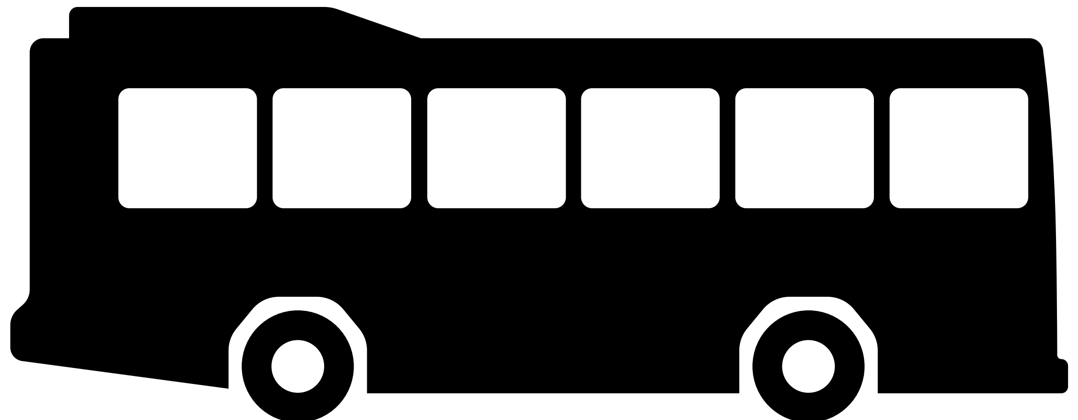
Please note:
The colloquium will also be live-streamed to Seminarraum 410 in Bld. 401 (CN).

KIT Center Elementary Particle and Astroparticle Physics (KCETA)
www.kceta.kit.edu

Karlsruher Institut für Technologie

Trip to CERN

- Limit of 45 people from lectures reached
  - Additional people on the waiting list already
- Priority was given to TP1, TP2 and detector physics students that participate in exercises.
- If you signed up successfully and got my email this morning: Please pay 65 Euro in the next days!

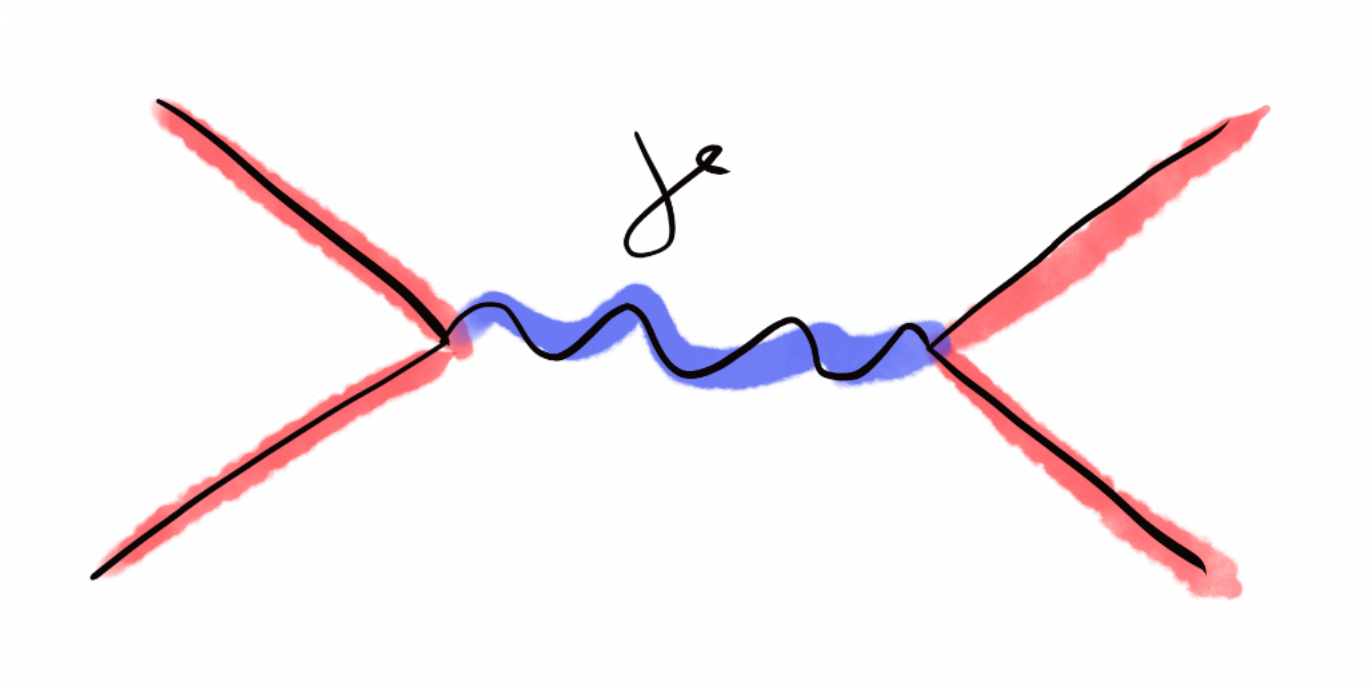


Learning goals

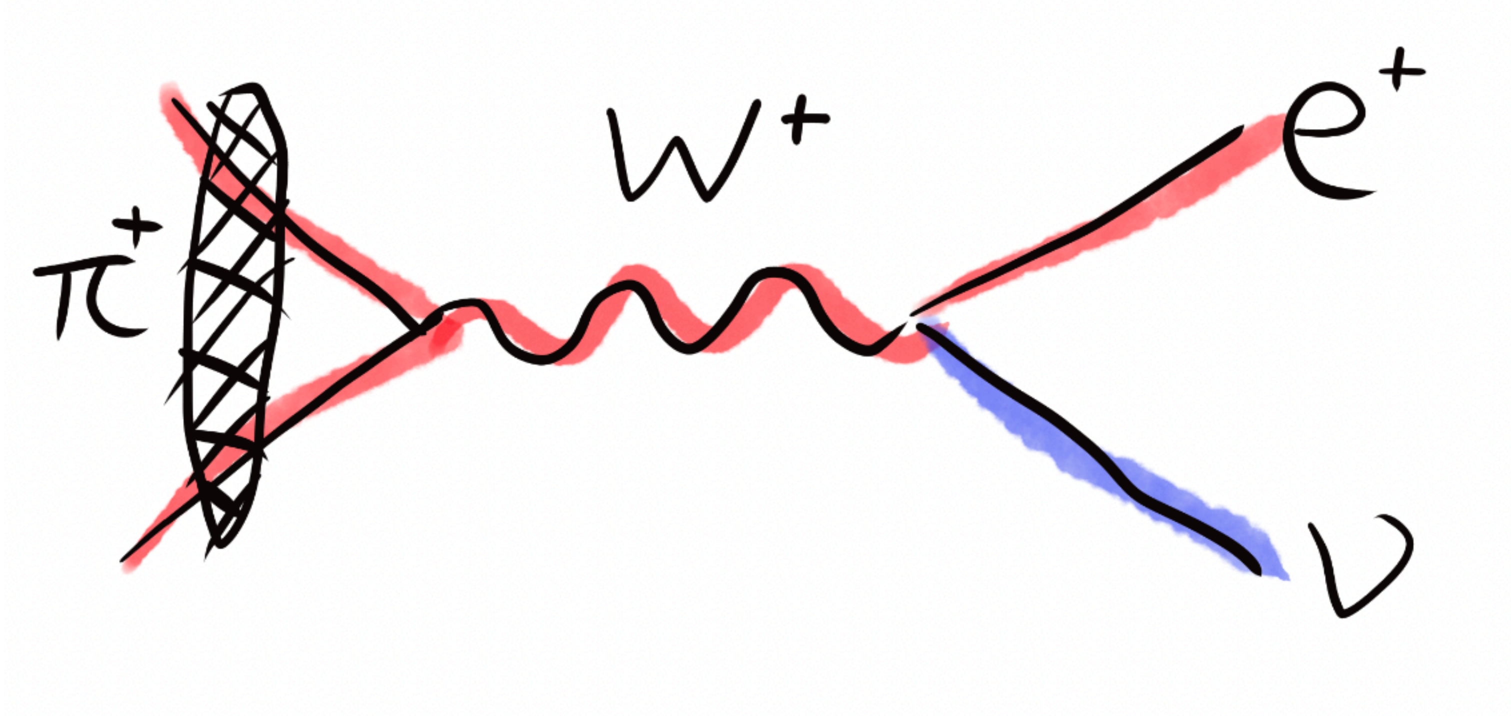
- Understand how to use symmetries to construct complex theories
- Understand electroweak unification and SU(2)×U(1)
- Understand how the postulated Higgs-mechanism works
- Understand boson and fermion mass generation

Reminder

- QED: symmetry under $U(1)$ gauge transformation → photon exchange



- QCD: symmetry under $SU(3)$ gauge transformation → gluon exchange
- Experimental observation of **parity violation** in the Wu experiment



How can we include this in a consistent theory?

Parity conservation in axial currents

- $$j_1 = \bar{u}\gamma^\mu\gamma^5 u \to \hat{P}j_1 = \begin{cases} \hat{P}j_1^0 = \bar{u}\gamma^0\gamma^0\gamma^5\gamma^0 u \overset{\gamma^0\gamma^\mu = -\gamma^\mu\gamma^0}{=} -\bar{u}\gamma^0\gamma^0\gamma^0\gamma^5 u \overset{\gamma^0\gamma^0 = 1}{=} -\bar{u}\gamma^0\gamma^5 u = -j_1^0, & \text{if } k = 0 \\ \hat{P}j_1^k = \bar{u}\gamma^0\gamma^k\gamma^5\gamma^0 u \overset{\substack{\gamma^0\gamma^\mu = -\gamma^\mu\gamma^0 \\ \text{twice}}}{=} u\gamma^0\gamma^0\gamma^k\gamma^5 u \overset{\gamma^0\gamma^0 = 1}{=} u\gamma^k\gamma^5 u = j_1^k, & \text{if } k = 1,2,3 \end{cases}$$

  and analogue for j_2

  - → the time like component changes sign, the space-like is unchanged

- $$j_1 j_2 = j_1^0 j_2^0 - j_1^k j_2^k \to \hat{P}j_1 j_2 = (-j_1^0)(-j_2^0) - (j_1^k)(j_2^k) = j_1 j_2$$

  - **→ Axial vector current conserves parity as well…** 😫

Parity conservation in combination of vector and axial currents

- Linear combination of vector and axial-vector currents with coefficients g_V and g_A: $j_1 = \bar{u}(g_V\gamma^\mu + g_A\gamma^\mu\gamma^5)u = g_V j_{V,1} + g_A j_{A,1}$ and analogous $j_2 = g_V j_{V,2} + g_A j_{A,2}$

- $j_1 j_2 = g_V^2 j_{V,1} j_{V,2} + g_A^2 j_{A,1} j_{A,2} + g_V g_A (j_{V,1} j_{A,2} + j_{V,2} j_{A,1})$

  $\rightarrow P j_1 j_2 = g_V^2 j_{V,1} j_{V,2} + g_A^2 j_{A,1} j_{A,2} - g_V g_A (j_{V,1} j_{A,2} + j_{V,2} j_{A,1})$

  - **Linear combination of vector and axial currents violates parity!** 🥳

*compare to chirality operators, VL8

- Only left-handed chirality* particles and right-handed antiparticles participate in the charged current weak interaction: $j \propto \bar{\psi}(\gamma^\mu(1-\gamma^5)\psi$

"V-A theory" (vector minus axialvector)

Glashow-Salam-Weinberg (GSW) model

Source: https://www.nobelprize.org/prizes/physics/1979/salam/facts/

Abdus Salam

Born: 29 January 1926, Jhang Maghiāna, India (now Pakistan)

Died: 21 November 1996, Oxford, United Kingdom

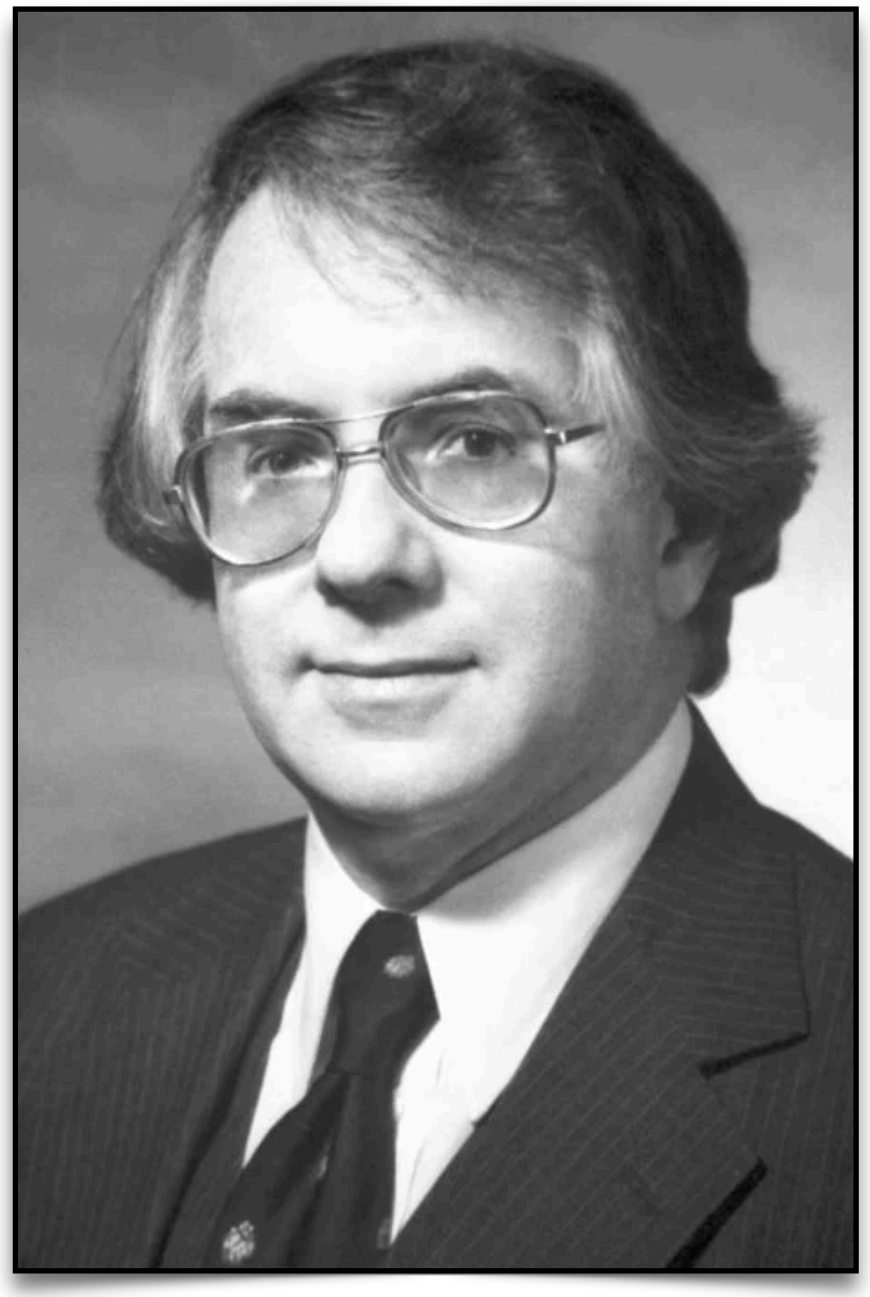
Steven Weinberg

Born: 3 May 1933, New York, NY, USA

Died: 23 July 2021, Austin, TX, USA

Source: https://www.nobelprize.org/prizes/physics/1979/weinberg/facts/

Source: https://www.nobelprize.org/prizes/physics/1979/glashow/facts/



Sheldon Glashow

Born: 5 December 1932, New York, NY, USA

SU(2)×U(1)

- Simplest (≠ simple!) combination of groups to arrive at **unified description** of electromagnetic and weak interactions: ***SU*(2)*_L*×*U*(1)*_Y***
  - $SU(2)_L$: **weak isospin I**, acts on **left-handed** particles only
  - $U(1)_Y$: **hypercharge Y**, acts on **all** particles (≠ $U(1)$ gauge group of QED)
- **Particle content:** distinguish left-handed and right-handed particles
  - Left-handed particles: weak isospin doublets ($I = 1/2$, $I_3 = \pm 1/2$)

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \ldots, \begin{pmatrix} u \\ d \end{pmatrix}_L$$

  - Right-handed particles: weak isospin singlets ($I = I_3 = 0$)

$R = e_R^-, \ldots u_R, d_R, \ldots,$ no right-handed neutrinos!

Covariant derivatives (see VL 9)

- Analogue to QED: invariance under local SU(n) transformations by introducing covariant derivatives

$$\partial_\mu \;\rightarrow\; D_\mu = \partial_\mu + igT^a A_\mu^a$$

QED:

$$\partial_\mu \;\rightarrow\; D_\mu = \partial_\mu + iqA_\mu(x)$$

$$A_\mu^a \;\rightarrow\; A_\mu^a - \frac{1}{g}\partial_\mu \alpha^a(x) - f^{abc}\alpha^b(x)A_\mu^c$$

$$A_\mu \;\rightarrow\; A_\mu - \frac{1}{q}\partial_\mu \alpha$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{i}{q}[D_\mu, D_\nu]$$

Electroweak gauge group

- Gauge transformation of SU(2):

$$\psi \rightarrow \psi' = U(x)\psi = e^{i\alpha^a(x)T^a}\psi$$

  - Typical representation of generators T^a as traceless, hermitian matrices: Pauli matrices σ^a (a=1, 2, 3) → 3 gauge bosons W_μ^a
  - Since SU(2) is not-abelian, the generators do not commutate, but fulfil $[T^a, T^b] = if^{abc}T^c$ with structure constants $f^{abc} = \epsilon^{abc}$, the totally anti-symmetric Levi-Chivita tensor

- Covariant derivative (analogous to QCD):

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + igT^aW_\mu^a$$

Electroweak gauge group: SU(2)$_L$

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma^\mu D_\mu)\psi \\ &= \bar{\psi}(i\gamma^\mu \partial_\mu)\psi - g\bar{\psi}(\gamma^\mu T^a W^a_\mu)\psi\end{aligned}$$

- This Lagrange density is invariant under gauge transformations if the new gauge fields transforms like $W^a_\mu \;\rightarrow\; W^a_\mu - \frac{1}{g}\partial_\mu \alpha^a(x) - f^{abc}\alpha^b(x) W^c_\mu$
- Field strength tensors:

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g f^{abc} W^b_\mu W^c_\nu$$

Electroweak gauge group: SU(2)$_L$

$$\begin{aligned}\mathcal{L} &= \bar{\psi}_L(i\gamma^\mu D_\mu)\psi_L - \frac{1}{4}W^{a\mu\nu}W^a_{\mu\nu} \\ &= \underbrace{\bar{\psi}_L(i\gamma^\mu\partial_\mu)\psi_L}_{\text{kin. term of fermions}} - \underbrace{g\bar{\psi}_L(\gamma^\mu T^a W^a_\mu)\psi_L}_{\text{Isospin doublet coupling to } W} - \underbrace{\frac{1}{4}W^{a\mu\nu}W^a_{\mu\nu}}_{W \text{ kin. term and self interaction}}\end{aligned}$$

- Field strength tensors:

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g f^{abc} W^b_\mu W^c_\nu$$

- Mass terms $m^2 W^a_\mu W^{a,\mu}$ are not SU(2) invariant → Boson masses must be zero (analogous to QED and QCD)
  - This is not observed in nature (compare nuclear β decay), we will come back to that later

Electroweak gauge group: Forbidden mass terms

- Reminder chirality operator $P_L = \frac{1}{2}(1-\gamma^5)$ and $P_R = \frac{1}{2}(1+\gamma^5)$
- Any spinor can be decomposed into left- and right-handed components, e.g. $e_L = P_L e = \frac{1}{2}(1-\gamma^5)e$ and $e_R = P_R e = \frac{1}{2}(1+\gamma^5)e$
- Applied to massterm $m\bar{\psi}\psi$ in Lagrangian, with $\psi = e$ for an electron (as example):

$(\gamma^5)^2 = 1$

$(1-\gamma^5)(1+\gamma^5) = 1 - \gamma^5 + \gamma^5 + (\gamma^5)^2 = 0$

$$
\begin{aligned}
m\bar{e}e &= m(\bar{e}_L + \bar{e}_R)(e_L + e_R) \\
&= m(\bar{e}\frac{1}{2}(1-\gamma^5) + \bar{e}\frac{1}{2}(1+\gamma^5))(\frac{1}{2}(1-\gamma^5)e + \frac{1}{2}(1+\gamma^5)e) \\
&= m(\bar{e}_L e_R + e_L \bar{e}_R)
\end{aligned}
$$

Electroweak gauge group: Forbidden mass terms

- $m\bar{e}e = m\left(\bar{e}_L e_R + e_L \bar{e}_R\right) \rightarrow mU\bar{e}e = m\left(U\bar{e}_R e_L + \bar{e}_R U e_L\right) \neq m\bar{e}e$
- The weak charged-current interaction only couples to left-handed chiral particle states and right-handed chiral anti-particle states
- Fermion masses must be zero… 😩
  - This is definitely not observed in nature, we will come back to that later

- lecture end 28.11.2023

Particles and electroweak quantum numbers

Fermion	Chirality	Isospin (I, I_3)	Hypercharge Y	Charge Q (e)
Neutrinos	L	(1/2, +1/2)	-1	0
	R	not part of the standard model		
Charged leptons	L	(1/2, -1/2)	-1	-1
	R	(0, 0)	-2	-1
up-type quarks (u, c, t)	L	(1/2, +1/2)	+1/3	+2/3
	R	(0, 0)	+4/3	+2/3
down-type quarks (d, s, b)	L	(1/2, -1/2)	+1/3	-1/3
	R	(0, 0)	-2/3	-1/3

$$Y = 2(Q - I_3)$$

Electroweak gauge group: $U(1)_Y$

- Gauge transformation of U(1):

$$\psi \rightarrow \psi' = U(x)\psi = e^{i\alpha(x)}\psi$$

- Covariant derivative (analogous to QED):

$$\partial_\mu \;\rightarrow\; D_\mu = \partial_\mu + ig'\frac{Y}{2}B_\mu(x)$$

- Instead of the electric charge Q, introduce so-called hypercharge $Y = 2(Q - I_3)$ ("Gell-Mann-Nishijima"-equation)
  - $I_3 = \pm\frac{1}{2}$ for the weak isospin doublets, 0 for the isospin singlets
- Instead of the photon, use a gauge boson B^μ (this is **not** the photon!)

Electroweak gauge group: U(1)$_Y$

$$\begin{aligned}\mathcal{L}_Y &= \bar{\psi}(i\gamma^\mu D_\mu)\psi - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \\ &= \bar{\psi}(i\gamma^\mu \partial_\mu)\psi - g'\frac{Y}{2}\bar{\psi}(\gamma^\mu B_\mu)\psi - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}\end{aligned}$$

- Field strength tensor:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

- Since Y is different for left- and for right handed particles, this interaction is parity violating as well

$$\mathcal{L}_Y = \bar{\psi}(i\gamma^\mu\partial_\mu)\psi - \left(g'\frac{Y_R}{2}\bar{\psi}_R(\gamma^\mu B_\mu)\psi_R + g'\frac{Y_L}{2}\bar{\psi}_L(\gamma^\mu B_\mu)\psi_L\right) - \frac{1}{4}B^{\mu\nu}B_{\mu\nu},$$

Weak interactions: Interim Summary

- Solved:
  - $SU(2)_L$ and $U(1)_Y$ are parity violating
  - Structure of field strength tensor allows triple (TGC) and quartic (QGC) gauge couplings for $SU(2)_L$ like in QCD

- Problems:
  - Mediators must be massless, which contradicts experimental findings:
    - Cross section of $e\nu$-scattering goes to infinity for large collision energy
    - WW production cross section is incorrect
    - Longitudinal WW scattering is not finite
  - Fermions must be massless, which contradicts experimental findings
  - Three $SU(2)_L$ bosons with identical couplings to left-handed fermions

Unified electroweak: $SU(2)_L \times U(1)_Y$

- Lagrange density is simply the sum of the two Lagrange densities:

$$\mathcal{L} = -\frac{1}{4} W^{a\mu\nu} W^a_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \sum \bar{\psi}_L i\gamma^\mu D_\mu \psi_L + \sum \bar{\psi}_R i\gamma^\mu D_\mu \psi_R$$

  - Summation runs over all left-handed doubles and all right-handed singles

- Covariant derivatives:

$$\begin{aligned} D_{\mu,L}\psi_L &= (\partial_\mu + igT^a W^a_\mu + ig'\frac{Y}{2}B_\mu)\psi_L \\ D_{\mu,R}\psi_R &= (\partial_\mu + ig'\frac{Y}{2}B_\mu)\psi_R . \end{aligned}$$

- Reminder:
  - Left-handed particles participate in $SU(2)_L$ and $U(1)_Y$, right-handed particles only in $U(1)_Y$ (not identical with QED!)
  - Two different coupling constants g and g'
  - Mass terms for all bosons and for all fermions are forbidden!

Unified electroweak: $SU(2)_L \times U(1)_Y$

- Insert explicit expressions (Pauli matrices) for the generators:

$$T^a = \sigma^a/2$$

$$\begin{aligned}
D_\mu &= \partial_\mu + igT^a W^a_\mu + ig'\frac{Y}{2}B_\mu \\
&= \partial_\mu + \frac{ig}{2}\left(\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}W^1_\mu + \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}W^2_\mu + \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}W^3_\mu\right) + ig'\frac{Y}{2}B_\mu \\
&= \partial_\mu + \frac{ig}{2}\begin{pmatrix}W^3 & W^1 - iW^2\\ W^1 + iW^2 & -W^3\end{pmatrix}_\mu + ig'\frac{Y}{2}\begin{pmatrix}B & 0\\ 0 & B\end{pmatrix}_\mu
\end{aligned}$$

Unified electroweak: $SU(2)_L \times U(1)_Y$

- There are qualitatively different contributions:
  - One contribution that involves both isospin partners and changes flavours (“charged current”) via off-diagonal elements in the matrices
  - One contribution that does not change flavour (“neutral current”) via on-diagonal elements

Unified electroweak charged current: D_μ^W

- Separating the off-diagonal elements to identify charged currents

$$\begin{aligned} W^\pm &= \frac{1}{\sqrt{2}}(W^1 \mp iW^2) \\ T^\pm &= \frac{1}{\sqrt{2}}(T^1 \pm iT^2), \end{aligned}$$

$$T^+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad T^- = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\begin{aligned} D_\mu &= \partial_\mu + \frac{ig}{2}\begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix}_\mu + \frac{i}{2}\begin{pmatrix} gW^3 + g'YB & 0 \\ 0 & -gW^3 + g'YB \end{pmatrix}_\mu \\ &= \partial_\mu + ig(T^+W^+ + T^-W^-)_\mu + \left(igT^3W^3 + ig'\frac{Y}{2}B\right)_\mu \end{aligned}$$

purely off-diagonal:
flavour changing charged currents

D_μ^W

example ve CC:

$$\begin{aligned} \mathcal{L}_L &= ig\bar{\psi}_L i\gamma^\mu(T^+W^+ + T^-W^-)_\mu \psi_L \\ &= \frac{1}{\sqrt{2}} ig(\bar{\nu}_e, \bar{e})_L i\gamma^\mu \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix}_\mu \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \\ &= -\frac{1}{\sqrt{2}} g(\bar{\nu}_{eL}\gamma^\mu W_\mu^+ e_L + \bar{e}_L \gamma^\mu W_\mu^- \nu_{eL}). \end{aligned}$$

Unified electroweak neutral current: $D_\mu^{\gamma,Z}$

- Remaining part with both left- and right interactions by W^3 and B:

$$\mathcal{L}_{\gamma,Z} = \mathcal{L}_{W^3,B} = \bar{\psi}_L i\gamma^\mu i \left(g T^3 W_\mu^3 + g' \frac{Y}{2} B_\mu \right) \psi_L + \bar{\psi}_R i\gamma^\mu i \left(g' \frac{Y}{2} B_\mu \right) \psi_R$$

$$= \sum_{\psi = e_L, e_R, \nu_L, \nu_R} \bar{\psi} i\gamma^\mu i \left(g T^3 W_\mu^3 + g' \frac{Y}{2} B_\mu \right) \psi$$

- Remember $T^3 = \frac{\sigma_3}{2}$, $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

- Express $W^3$and B by two new fields (right now somewhat arbitrary but it does not change physics), by rotation by the weak mixing angle θ_W:

$$A_\mu = +B_\mu \cos\theta_\mathrm{W} + W_\mu^{(3)} \sin\theta_\mathrm{W}$$
$$Z_\mu = -B_\mu \sin\theta_\mathrm{W} + W_\mu^{(3)} \cos\theta_\mathrm{W}$$

Unified electroweak neutral current: $D_\mu^{\gamma,Z}$

$$\mathcal{L}_{\gamma,Z} = \sum_{e\nu} i\bar{\psi} i\gamma^\mu \left(g \sin\theta_W I_3 + g' \cos\theta_W \frac{Y}{2} \right) A_\mu \psi$$
$$+ i\bar{\psi} i\gamma^\mu \left(g \cos\theta_W I_3 - g' \sin\theta_W \frac{Y}{2} \right) Z_\mu \psi.$$

- Identify A_μ with the QED photon:

$$eQ = g \sin\theta_W I_3 + g' \cos\theta_W \frac{Y}{2}$$
$$eQ = e I_3 + e\frac{Y}{2},$$

$\Rightarrow$ $e = g \sin\theta_W = g' \cos\theta_W$

Unified electroweak: $SU(2)_L \times U(1)_Y$

$$\partial_\mu + D_\mu^W + D_\mu^{\gamma Z}$$

- $\bar{\psi} i \gamma^\mu \partial_\mu \psi$: kinetic term for massless* fermions
- $\bar{\psi} i \gamma^\mu D_\mu^W \psi$: charged current interactions with massless* charged W bosons
- $\bar{\psi} i \gamma^\mu D_\mu^{\gamma,Z} \psi$: neutral current interactions with massless* bosons**

*This will be fixed as soon as we introduce the spontaneous symmetry breaking.

** These are not quite the Z boson and the photon, but almost…

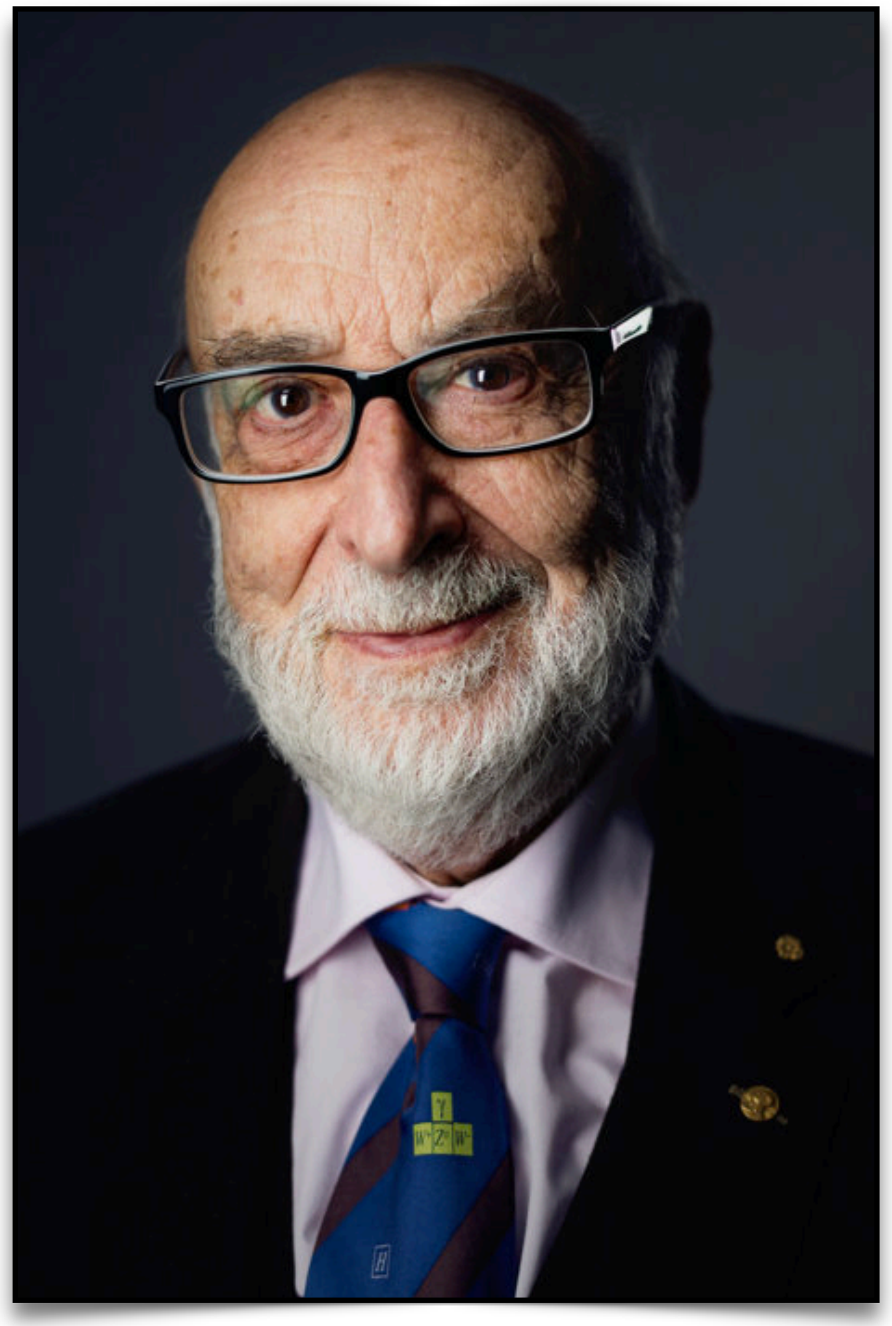
Weak interactions: Interim Summary

- Solved:
  - $SU(2)_L$ and $U(1)_Y$ are parity violating
  - Structure of field strength tensor lead triple (TGC) and quartic (QGC) gauge couplings for $SU(2)_L$ like in QCD
  - **2 charged bosons $W^{\pm}$, one neutral boson Z^0, one neutral boson γ (with identical properties as the QED photon)**
- Problems:
  - Mediators must be massless, which contradicts experimental findings:
    - Cross section of eν scattering goes to infinity for large collision energy
    - WW production cross section is incorrect
    - Longitudinal WW scattering is not finite
  - Fermions must be massless, which contradicts experimental findings
  - ~~Three $SU(2)_L$ bosons with identical couplings to left-handed fermions~~

Emy (by Y. Klügl)

Englert-Brout-Higgs-Guralnik-Hagen-Kibble-Mechanism

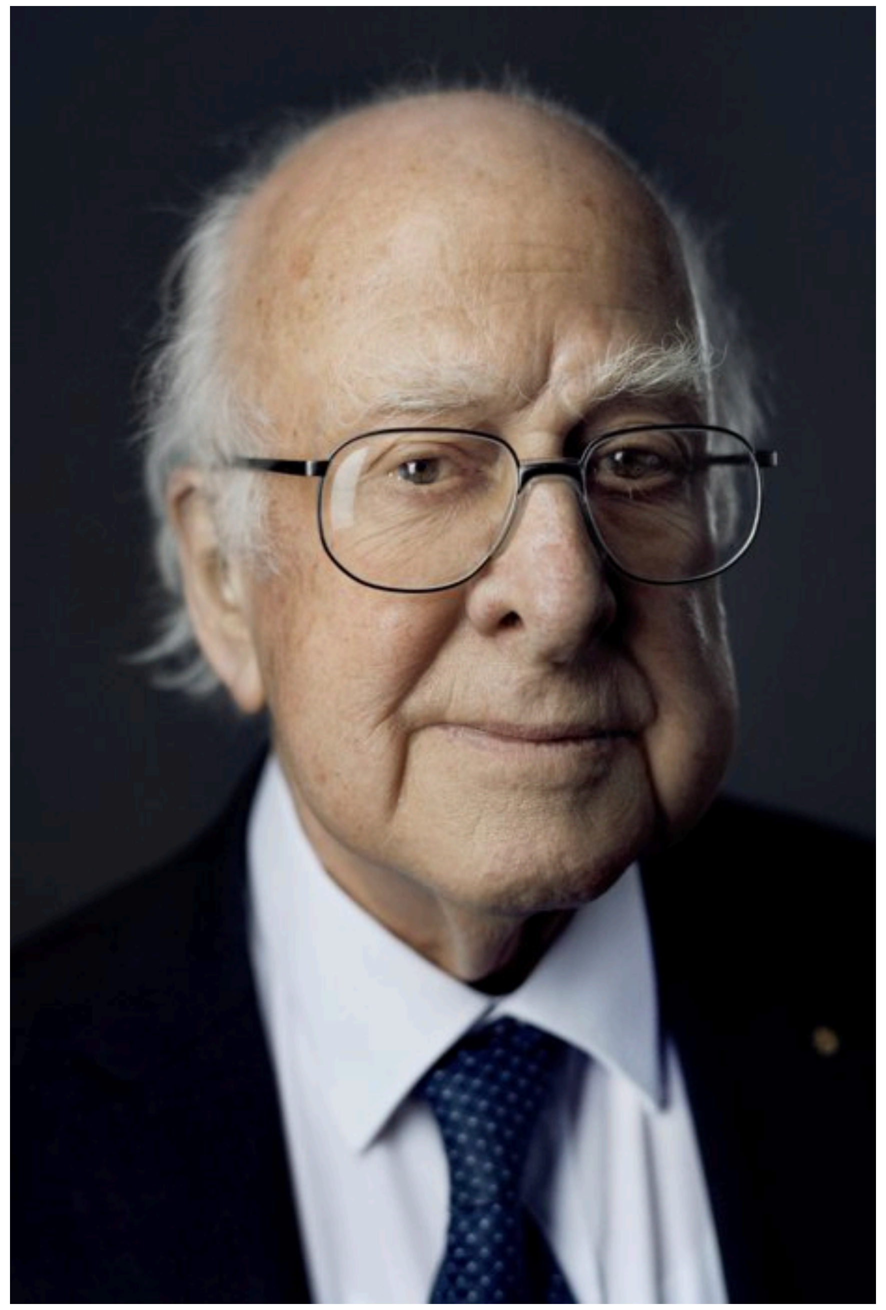
Source: https://www.nobelprize.org/prizes/physics/2013/englert/facts/



Francois Englert

Born: 6 November 1932, Etterbeek, Belgium

Source: https://www.nobelprize.org/prizes/physics/2013/higgs/facts/



Peter Higgs

Born: 29 May 1929, Newcastle upon Tyne, United Kingdom

- Independently discovered and published by three groups in 1964:
  - P. Higgs
  - R. Brout (died 2011) and F. Englert
  - T.W.B. Kibble, C. R. Hagen, G. Guralnik

Englert-Brout-Higgs-Mechanism: Idea and plan

- Need a mechanism to introduce gauge boson mass terms in Lagrangian without violating gauge invariance
- Brout, Englert; Higgs: spontaneous symmetry breaking (SSB)
  - SSB: ground state ("vacuum") of electroweak theory does not follow symmetry of Lagrangian
  - SSB by adding new scalar field to Lagrangian
  - Goldstone theorem: each generator of a broken symmetry is accompanied by a massless boson ("Nambu-Goldstone boson")
  - Exception if SSB is applied to theories with local gauge invariance: use freedom of gauge choice to "gauge away" Nambu–Goldstone boson
  - Consequence: degrees of freedom of Nambu-Goldstone bosons transferred to gauge bosons (gauge bosons "eat up NG bosons") → massive gauge bosons

Higgs field

- Postulate existence of a new **complex scalar** field ϕ
  - Symmetric under under $SU(2)_L \times U(1)_Y$
  - $SU(2)_L$ doublet $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$

	I_3	Y	Q
ϕ^+	1/2	1	1
ϕ^0	-1/2	1	0

  - **scalar** = spin 0 → only spin 0 field of the standard model (i.e. propagation is described by Klein-Gordon equation and not the Dirac equation)
  - **complex**: four degrees of freedom $$|\phi|^2 = \phi^\dagger\phi = (\phi^+, \phi^0)^* \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = |\phi^+|^2 + |\phi^0|^2 = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2$$

Spontaneous symmetry breaking

- Covariant derivative (identical to EW):

$$D_\mu = \partial_\mu + igT^a W^a_\mu + ig' \frac{Y}{2} B_\mu$$

- Lagrange density:

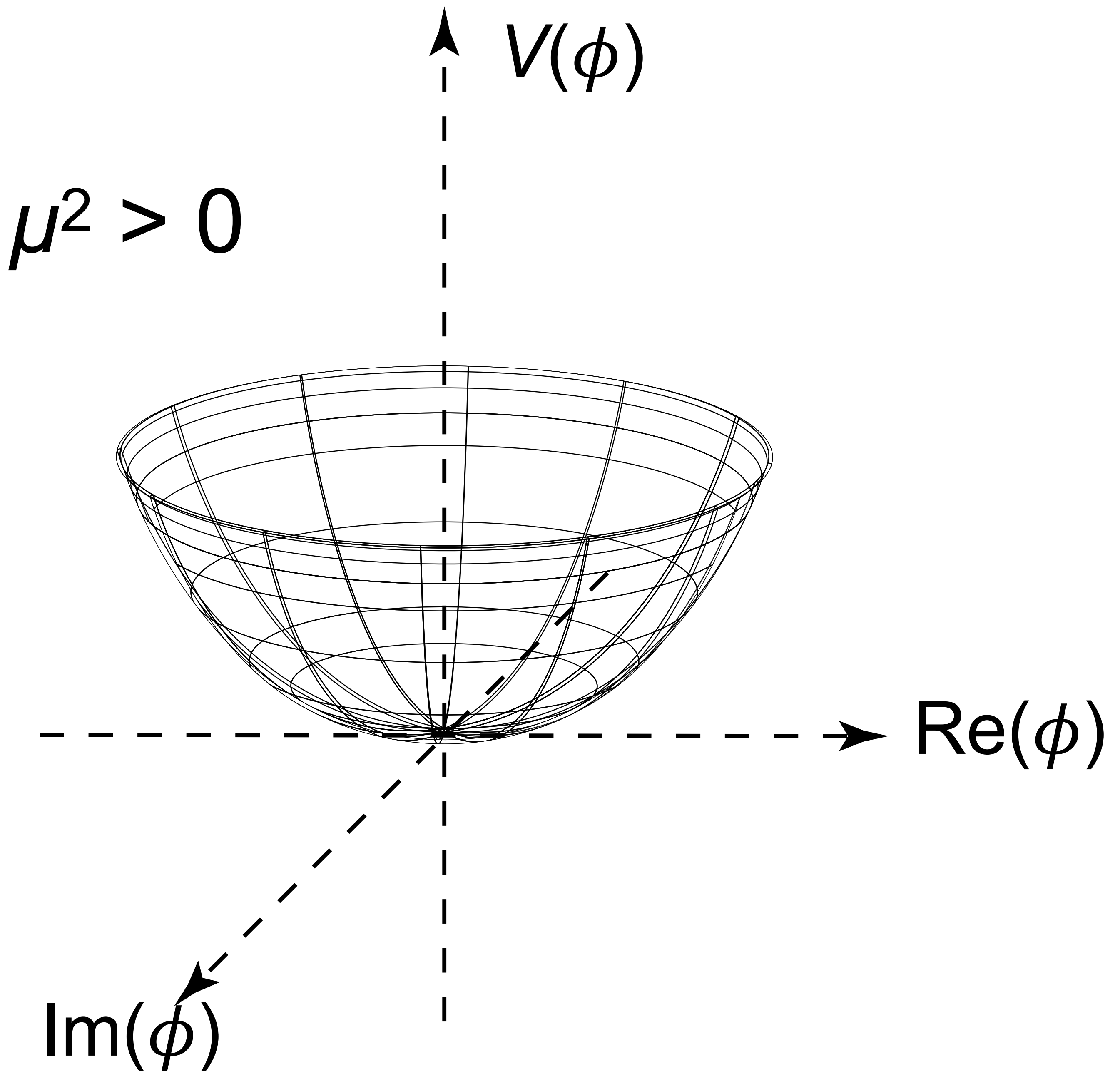
$$\mathcal{L}_\phi = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi) = |D_\mu \phi|^2 - V(\phi)$$

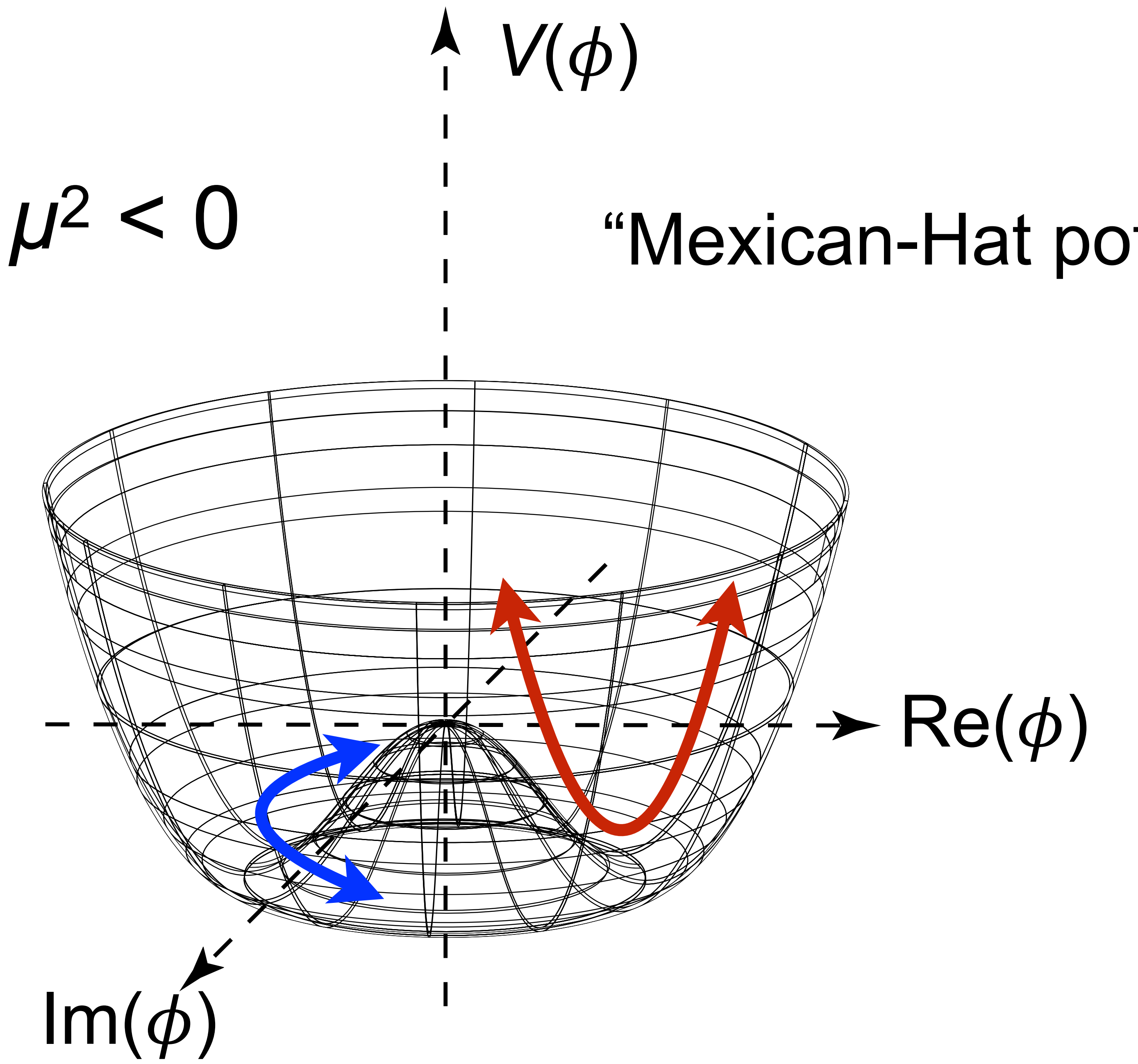
- Postulate potential that is invariant under gauge transformations (rotations and changes of complex phases → can only depend on $|\phi|^2$ and $|\phi|^4$. $|\phi|^6$ and higher are not renormalizable (here without proof)

$$V(\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4$$

Spontaneous symmetry breaking

minimum (=ground state) at
$|\phi| = 0$





minimum after symmetry breaking at
$|\phi| = \sqrt{\frac{-\mu^2}{2\lambda}}$

- Motion in minimum → massless Nambu–Goldstone bosons
- Motion perpendicular to minimum → massive particles

Vacuum expectation value (vev)

- Make specific choice for the ground state, e.g. the neutral one:

  $$\phi_{vac} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$ with $v = \sqrt{\frac{-\mu^2}{2\lambda}}$, i.e. only ϕ_3 remains after SU(2) gauge transformation in which all $\alpha(x)$ are chosen so that $\phi_1 = \phi_2 = \phi_4 = 0$

- For the quantization of the field, expand field around minimum v:

  $$\phi_{vac} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$, where the excitation are interpreted as real Higgs particles

Higgs potential

- Recall potential: $V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$

- Insert the field into the potential:

$$\begin{aligned} V(\phi) &= \underbrace{\frac{1}{2}\mu^2(v+H)^2}_{m_H=\sqrt{-2\mu^2}} + \frac{1}{4}\lambda(v+H)^4 \\ &= (\mu^2 + \lambda v^2)vH + \frac{1}{2}(\mu^2 + 3\lambda v^2)H^2 + \lambda v H^3 + \frac{1}{4}\lambda H^4 \end{aligned}$$

$$V(\phi) = -\mu^2 H^2 + \lambda v H^3 + \frac{1}{4}\lambda H^4$$

with $v = \sqrt{-\mu^2/\lambda}$

- As usual, we will identify terms with H^2 with the Higgs mass term, and terms with H^3 and H^4 as Higgs self-interactions with three and four Higgs vertices

Gauge boson masses

- Reminder: Massive gauge boson were forbidden in unbroken SU(2)×U(1)
- But Higgs Lagrangian introduces terms with both Higgs and Gauge bosons:

$$\begin{aligned} \mathcal{L}_\phi &= \left(D^\mu \phi\right)^\dagger \left(D_\mu \phi\right) - V(\phi) = \left|D_\mu \phi\right|^2 - V(\phi) \\ &= \left|\left(\partial_\mu + igT^a W^a_\mu + ig' \frac{Y}{2} B_\mu\right) \phi\right|^2 - V(\phi). \end{aligned}$$

insert covariant derivative

- Use Pauli-matrices and insert expansion of Higgs field:

$$\begin{aligned} D_\mu \phi &= \left(\partial_\mu + igT^a W^a_\mu + ig' \frac{Y}{2} B_\mu\right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} i\frac{g}{2}(W^1_\mu - iW^2_\mu)(v+H) \\ \partial_\mu H(x) - \frac{i}{2}(gW^3_\mu - g' B_\mu)(v+H) \end{pmatrix} \end{aligned}$$

use:

$$\boxed{\partial_\mu (v + H(x)) = \partial_\mu H(x)}$$

Gauge boson masses

- Multiply the two vectors after taking the hermitian conjugate:

$$
\begin{aligned}
(D^\mu\phi)^\dagger (D_\mu\phi) &= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) \\
&+\frac{1}{8}g^2(v+H)^2\left(W^{1,\mu}-iW^{2,\mu}\right)\left(W^1_\mu-iW^2_\mu\right) \\
&+\frac{1}{8}(v+H)^2\left(g'B^\mu-gW^{3,\mu}\right)\left(g'B_\mu-gW^3_\mu\right)
\end{aligned}
$$

The mixed terms for the lower vector components vanish because $(a+ib)(a-ib)=a^2+b^2$

- Rewrite using $W^\pm$ instead of $W^{1,2}$:

$$
\begin{aligned}
(D^\mu\phi)^\dagger (D_\mu\phi) &= \frac{1}{2}(\partial^\mu H)(\partial_\mu H) \\
&+\frac{1}{4}g^2(v+H)^2 W^{+,\mu}W^-_\mu \\
&+\frac{1}{8}(v+H)^2\left(W^{3,\mu},\ B^\mu\right)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2\end{pmatrix}\begin{pmatrix} W^3_\mu \\ B_\mu\end{pmatrix}
\end{aligned}
$$

Gauge boson masses: Final

$\frac{1}{2}(\partial^\mu H)(\partial_\mu H)$: this is the kinetic energy for a single real scalar field H. The excitation quantum of this field is the Higgs particle.

$\frac{1}{4}g^2v^2W^{+,\mu}W^-_\mu + \frac{1}{2}(g^2+g'^2)v^2Z^\mu Z_\mu = m_W^2 W^{+,\mu}W^-_\mu + \frac{1}{2}m_Z^2 Z^\mu Z_\mu$: this means that the masses of the W and Z bosons are fixed by the weak coupling constants and the Higgs potential to be

$$m_W = \frac{1}{2}gv \quad \text{and} \quad m_Z = \frac{1}{2}v\sqrt{g^2+g'^2}$$

$$\mathcal{L}_\phi = \frac{1}{2}(\partial^\mu H)(\partial_\mu H) + \frac{1}{4}g^2(v+H)^2W^{+,\mu}W^-_\mu + \frac{1}{2}(g^2+g'^2)(v+H)^2Z^\mu Z_\mu - V(\phi)$$

$\frac{1}{2}g^2vW^{+,\mu}W^-_\mu H + \frac{1}{4}(g^2+g'^2)vZ^\mu Z_\mu H$: these are interactions between the Higgs and the W and Z bosons. Comparing to the mass terms above, we see that the couplings are proportional to the W, Z boson masses.

$\frac{1}{4}g^2W^{+,\mu}W^-_\mu HH + \frac{1}{2}(g^2+g'^2)Z^\mu Z_\mu HH$: these are four-vertices between bosons and Higgs particle.

Gauge boson masses: Final

- The masses of Z and W, and the vev are not independent:

$$m_Z^2 = \frac{1}{4}v^2(g^2 + g'^2) = \frac{m_W^2}{\cos^2\theta_W}$$

$$\frac{m_W}{m_Z} = \cos\theta_W.$$

$$v = 246\ \mathrm{GeV}$$

Weak interactions: Interim Summary

- Solved:
  - $SU(2)_L$ and $U(1)_Y$ are parity violating
  - Structure of field strength tensor lead triple (TGC) and quartic (QGC) gauge couplings for $SU(2)_L$ like in QCD
  - 2 charged bosons $W^{\pm}$, one neutral boson Z^0, one neutral boson γ (with identical properties as the QED photon)
  - **Massive gauge bosons $W^{\pm}$ and Z^0, massless γ**

- Remaining problem:
  - Fermions must be massless, which contradicts experimental findings

Fermion masses

- Remember left and right handed fields: $L = Q_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \ldots, \begin{pmatrix} u \\ d \end{pmatrix}_L,$
  $R = e_R^-, \ldots u_R, d_R, \ldots$
- Postulate fermion-Higgs interactions, gauge invariant under $SU(2)_L \times U(1)_Y$ with a Yukawa-type coupling $\bar{\psi}\phi\psi$: $-c_d(\bar{Q}_L \phi d_R + \bar{d}_R \phi^\dagger Q_L)$
- Insert Higgs fields after SBB: $\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$

$$\begin{aligned} \mathcal{L} &= -c_d \frac{1}{\sqrt{2}} (\bar{d}_L (v + H) d_R + \bar{d}_R (v + H) d_L) \\ &= -c_d \frac{v}{\sqrt{2}} (\bar{d}_L d_R + \bar{d}_R d_L) - c_d \frac{1}{\sqrt{2}} H (\bar{d}_L d_R + \bar{d}_R d_L) \end{aligned}$$

$\Rightarrow$ $\mathcal{L} = -m_d \bar{d} d - \frac{m_d}{v} H \bar{d} d$

$$m_d = c_d \frac{v}{\sqrt{2}}$$

Fermion masses

- Fermions acquire mass due to interactions with the Higgs field
- Coupling is proportional to the mass of the particle - but these masses themselves are not predicted by the theory
  - This is conceptually different from the boson masses, whose masses are predicted by the theory via the couplings g and g' that can be measured in weak decays
- Neutrinos ("up-type leptons") are massless
- Up-type quarks require charge-conjugate Higgs doublet with hypercharge Y=-1 which then works identically:

$$\phi_C = -i\sigma_2\phi^* = \begin{pmatrix} \phi^{0\dagger} \\ -\phi^{+\dagger} \end{pmatrix}$$

Weak interactions and SBB: Summary

- Solved:
  - $SU(2)_L$ and $U(1)_Y$ are parity violating
  - Structure of field strength tensor lead triple (TGC) and quartic (QGC) gauge couplings for $SU(2)_L$
  - 2 charged bosons $W^{\pm}$, one neutral boson Z^0, one neutral boson γ (with identical properties as the QED photon)
  - After SBB: Photon massless, charged bosons $W^{\pm}$ and one neutral boson Z^0 are massive, Higgs boson massive. **Boson mass values are predicted by SM** and given by coupling constants. **Higgs mass value is not predicted by SM but must be measured**
  - After SBB: Neutrinos massless, all other fermions massive by interaction with the Higgs field. **Fermion mass values are not predicted by SM but must be measured**
- Two ingredients missing: Neutrino mixing and Quark mixing (later)

What questions do you have?