Karlsruhe Institute of Technology

Particle Physics 1
Lecture 12: Electroweak precision

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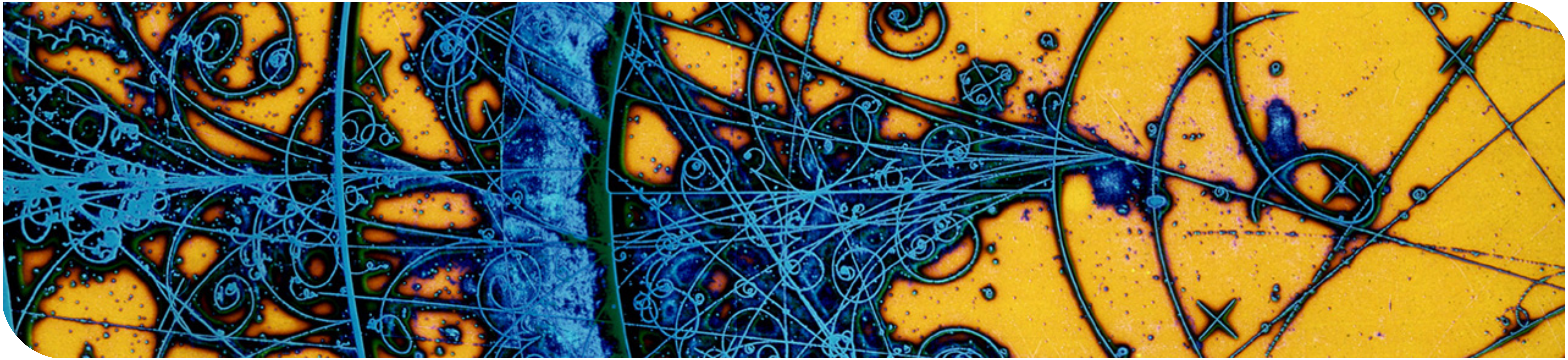
Wintersemester 2023/2024


Credit: CERN

Hardware exercise

Evaluation: Lecture

- Please only evaluate the lecture here, not the exercises. We will share an evaluation link for the exercises during the exercises as well.

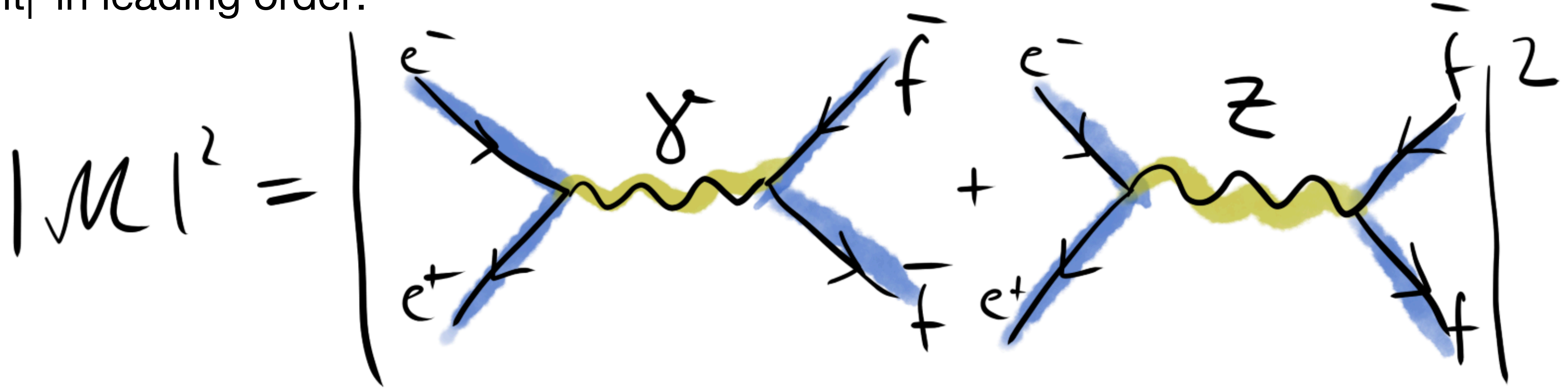
- Please use "general questions" to provide constructive feedback, point out good and bad things

Learning goals

- Understand differential cross section of $e^+e^- \to Z \to f^+f^-$
- Understand Z mass and width
- Understand concept of radiative corrections
- Understand event display and event categorisation
- Understand how to measure the number of light neutrinos
- Understand how to measure the weak mixing angle
- Understand the concept of global fits

"THE" observable: Fermion pair production

- **Resonant (s-channel) production** of Z bosons in e^+e^- scattering
  - Photon and Z boson: same quantum numbers → **interference**
  - |matrix element|² in leading order:



- Cross section $\sigma(e^+e^- \to \bar{f}f) = \sigma_\gamma + \sigma_{\gamma/Z} + \sigma_Z$:
  - $\sqrt{s} \ll m_Z$: photon exchange dominates
  - $\sqrt{s} \approx m_Z$: Z boson exchange dominates
  - However: Careful if the precision of low energy experiments (like Belle II) reaches per-mille level and all such effects have to be corrected for!

Fermion pair production: QED total cross section

- For small collision energy: $\sqrt{s} \ll m_Z$: photon exchange dominates
- Total cross section **decreases with increasing collision energy like** $1/s$ (i.e. inverse to the squared collision energy!) and neglecting all fermion masses and ignoring all radiative corrections:

$$\boxed{\sigma_{\mathrm{QED}} = N_C Q^2 \frac{4\pi\alpha^2}{3s}}$$

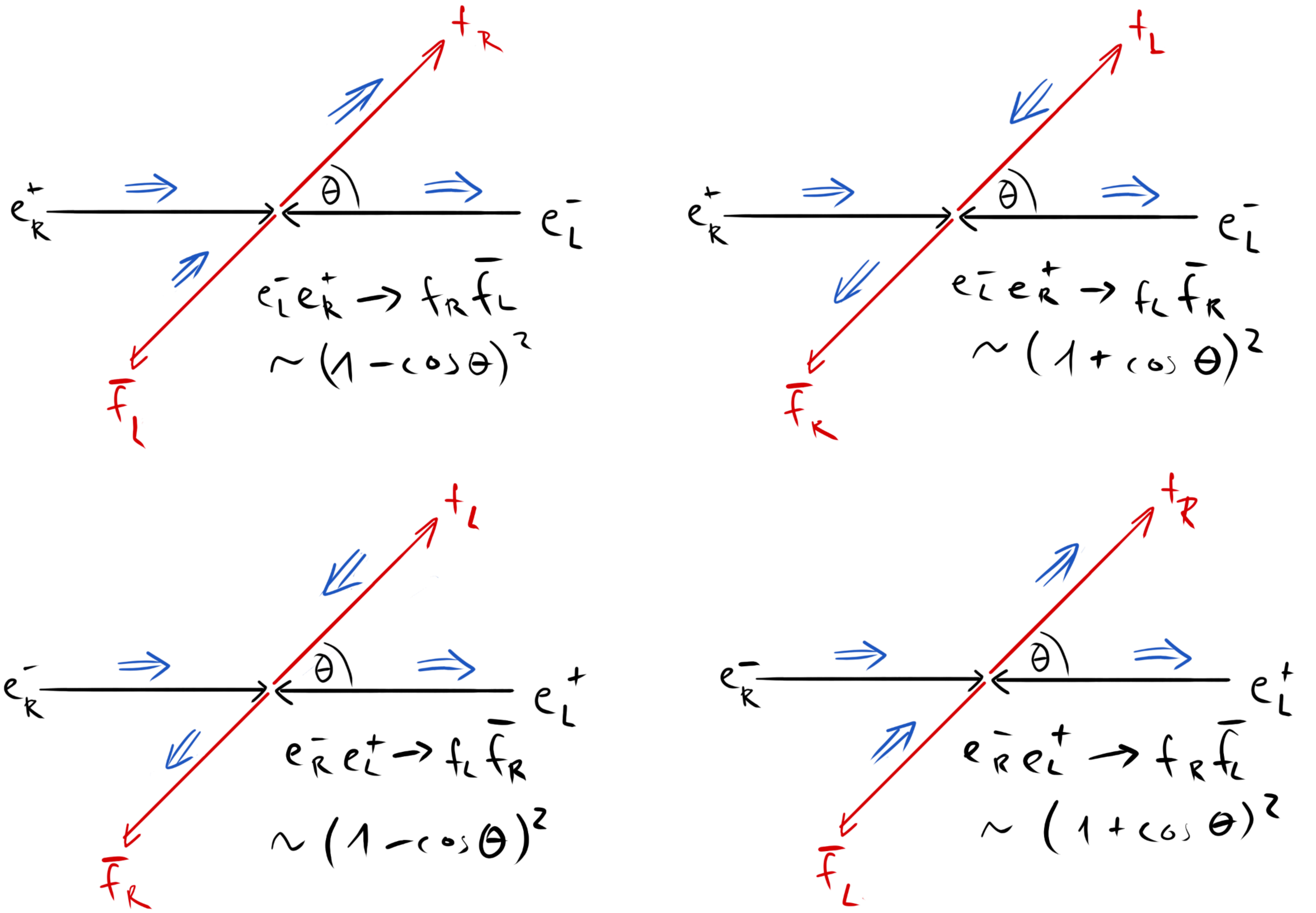
  - N_C: number of colour degrees of freedom (3 for quarks, 1 for leptons)
  - Q : electric charge in units of elementary charge e
  - α: fine structure constant* $\alpha(0) = \dfrac{e^2}{4\pi} \approx \dfrac{1}{137}$

* In SU(2)×U(1) this will be replaced by couplings g, g', the weak boson masses and the weak mixing angle

Fermion pair production: QED differential section

- The differential cross section can be obtained by using the QED Feynman rules or using helicity amplitudes:
  - Remember that
    - chirality is conserved at every vertex
    - for ultra-relativistic particles helicity and chirality are equal
    - a Photon is a spin=1 particle
    - you can decompose QED current in left- and right-handed component: $\bar{u}\gamma^\mu u = \bar{u}_L\gamma^\mu u_L + \bar{u}_R\gamma^\mu u_R$

Fermion pair production: QED differential section



Fermion pair production: QED differential section

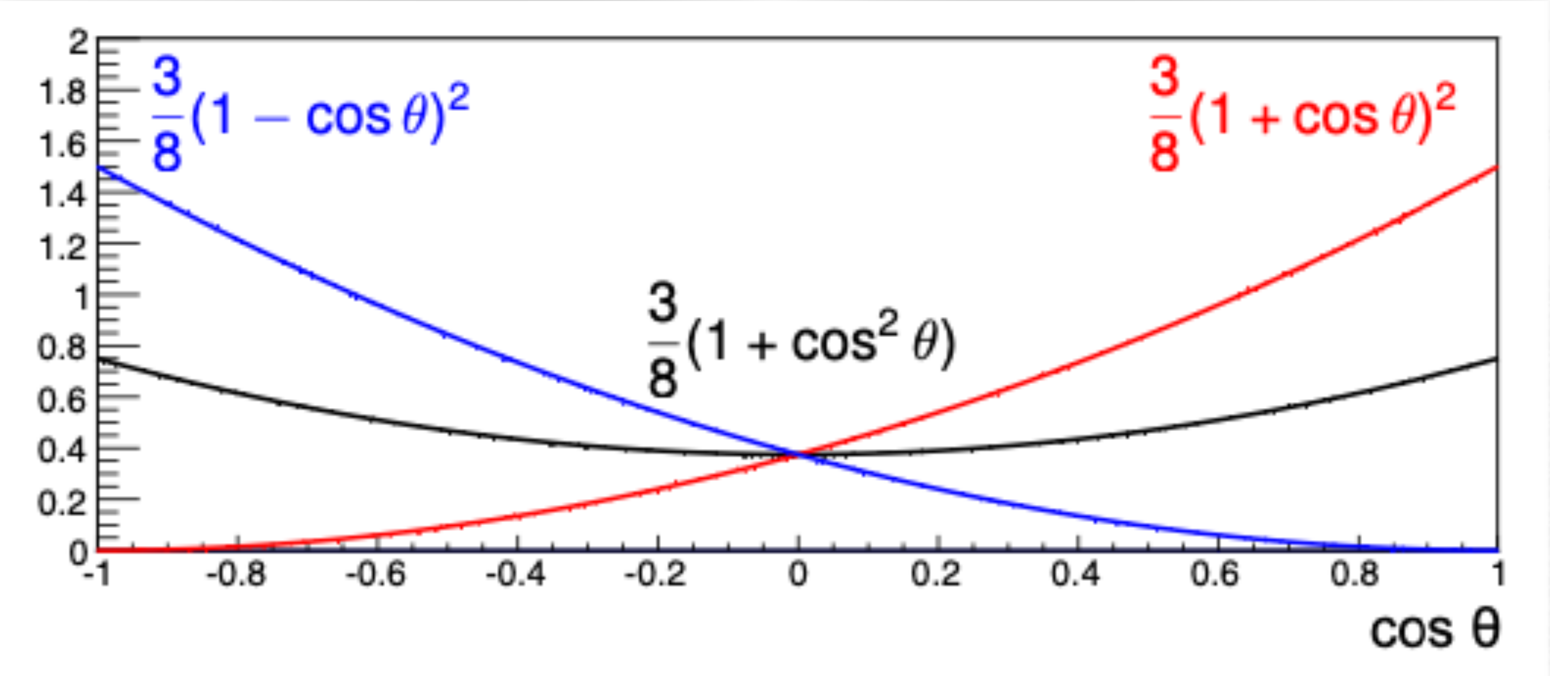
- The scattering angle is not arbitrary but must conserve angular momentum
  - Can be calculated using Wigner's d-matrices $d^1_{1,-1} = d^1_{-1,1} = \frac{1}{2}(1-\cos\theta) \approx -\frac{t}{s}$ and $d^1_{1,1} = d^1_{-1,-1} = \frac{1}{2}(1+\cos\theta) \approx -\frac{u}{s}$ with the Mandelstam variables s, t, and u

$$\begin{aligned} s &\approx 4E_1^* E_2^* \\ t &\approx -\frac{s}{2}(1-\cos\theta^*) \\ u &\approx -\frac{s}{2}(1+\cos\theta^*) \end{aligned}$$

  - In most e^+e^- colliders (like LEP or SuperKEKB) the beams are not polarized and the L and R components are equally present:

$$|\mathcal{M}|^2 \sim \left(-\frac{u}{s}\right)^2 + \left(-\frac{t}{s}\right)^2 = \frac{u^2+t^2}{s^2} \propto (1+\cos^2\theta)$$

- $$\boxed{\frac{d\sigma}{d\cos\theta} = N_C Q^2 \frac{\pi\alpha^2}{2s}(1+\cos^2\theta))}$$

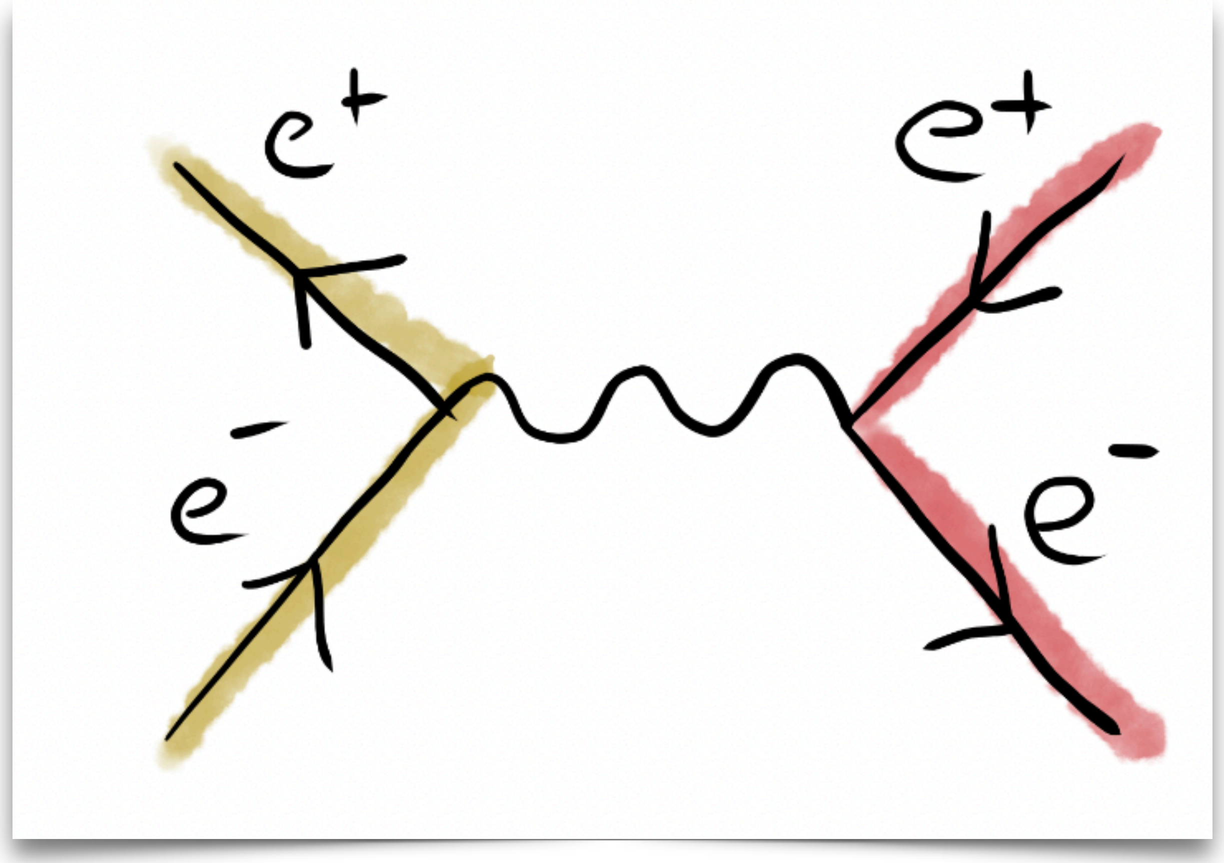


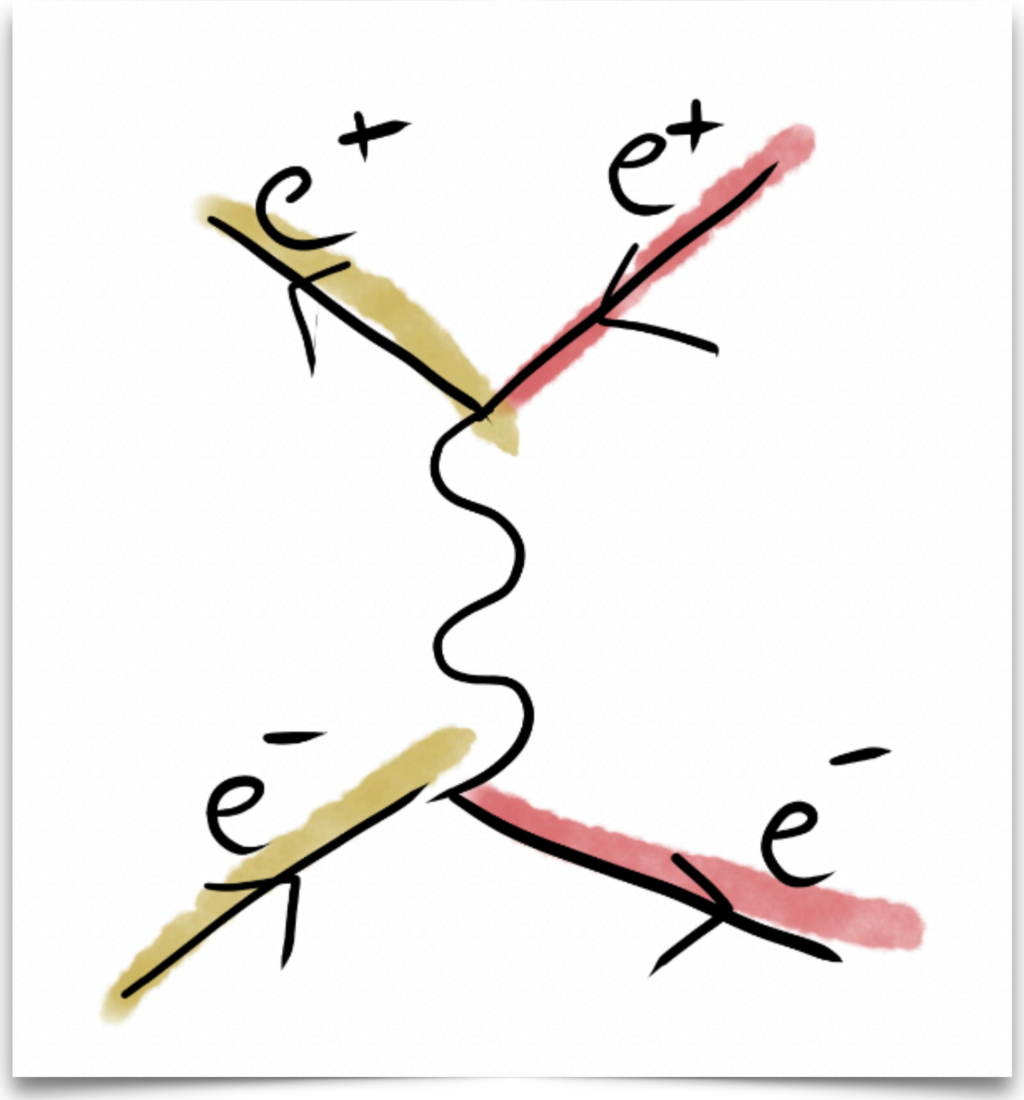
Fermion pair production: Bhabha scattering

- Special case Bhabha scattering $e^+e^- \to e^+e^-$
- Identical particles in initial and final state: t-channel contribution

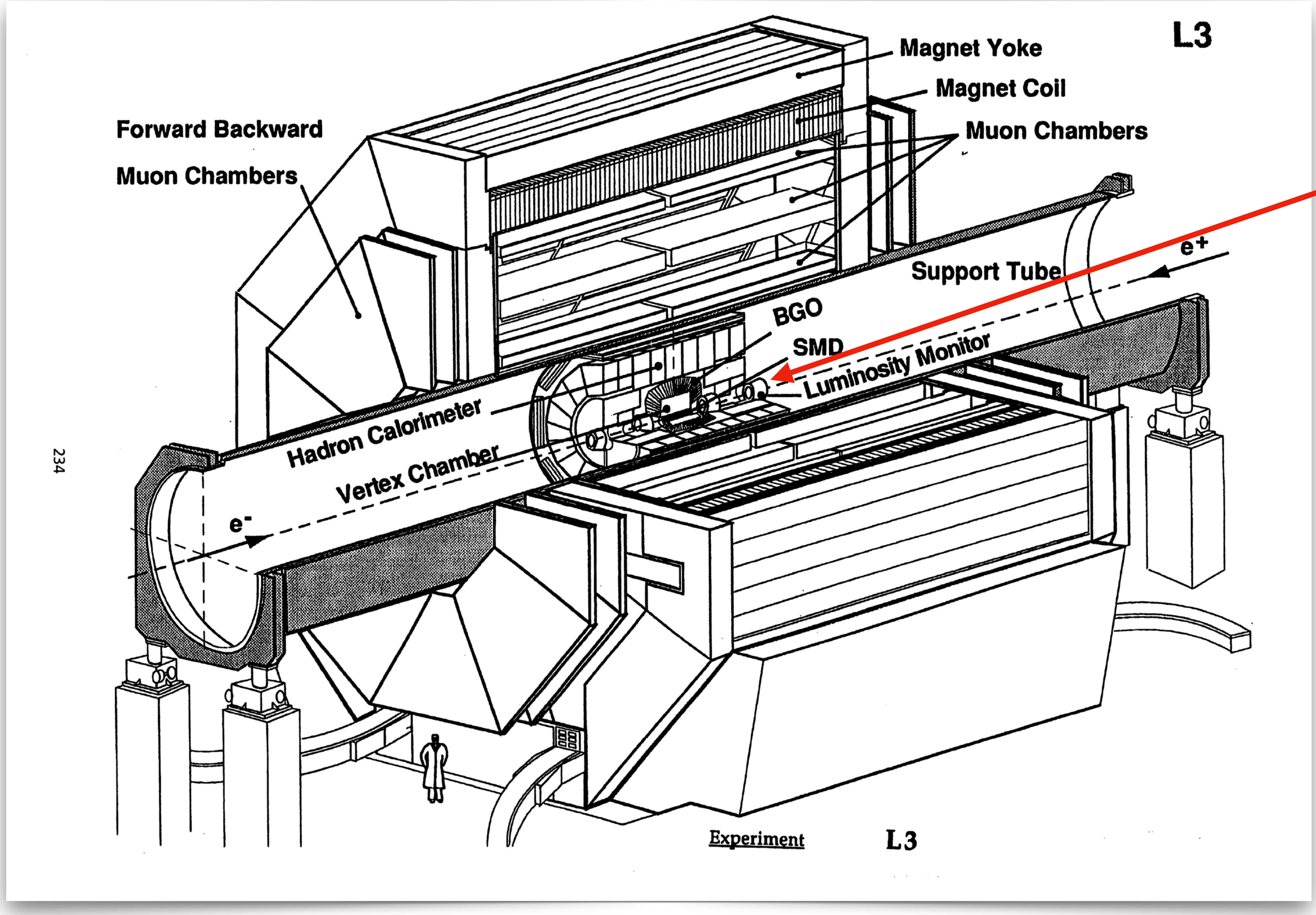
$$\frac{d\sigma}{d\cos\theta} \sim \frac{1}{\sin^4(\theta/2)}$$

- Extremely asymmetric distribution (scattered e^- in the same direction as incoming e^-)
- Very large total cross section, dominated by small angle scattering
- Used for **luminosity monitoring**





Luminosity monitoring



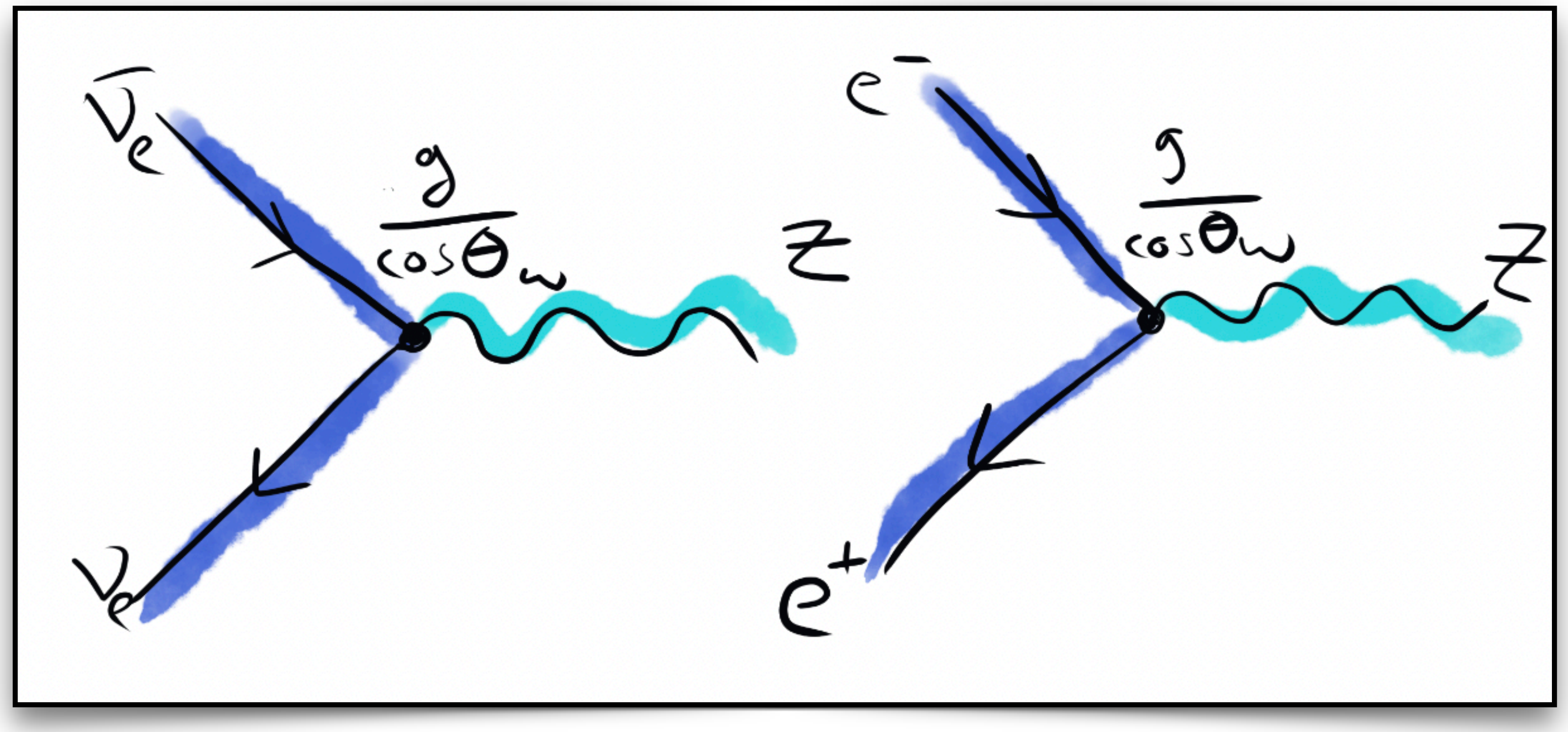
Luminosity monitor:
Very forward
calorimeter
(extremely high rate)

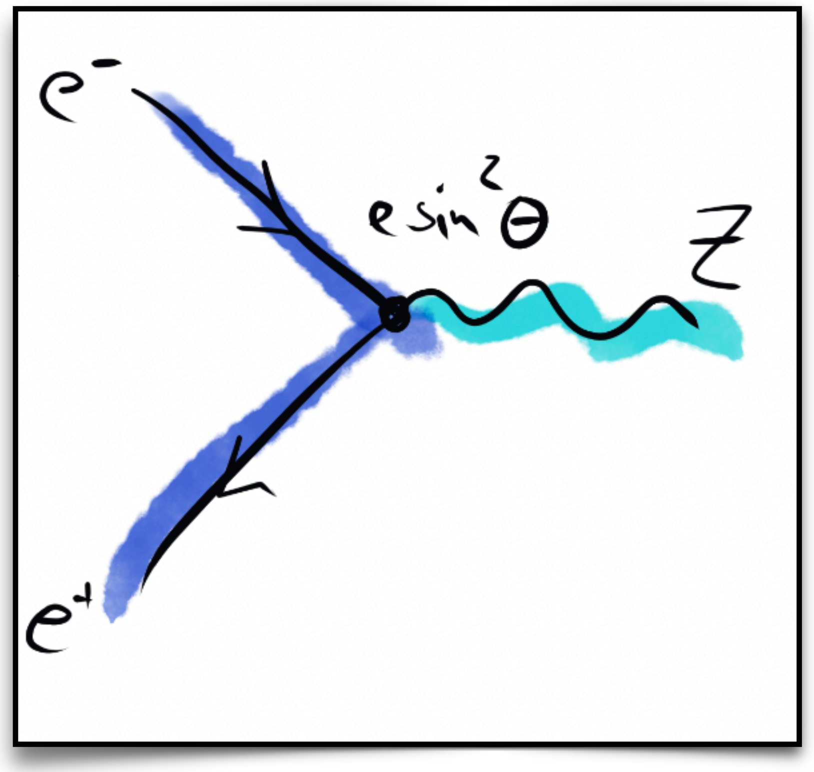
Fermion pair production: Z differential cross section

- Electroweak interactions are parity violating:

$$\mathscr{L}_{\mathrm{NC}} = -\frac{\sqrt{g^2 + g'^2}}{2} j^{\mu}_{\mathrm{NC}} Z_{\mu}$$

$$= -\frac{g}{2\cos\theta_W}\left[\left(\bar{\nu}_e\left(\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\right)\nu_e\right) - \left(\bar{e}\left(\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\right)e\right) - 2\sin^2\theta_W(\bar{e}\gamma^{\mu}e)\right] Z_{\mu}$$





Fermion pair production: Z differential cross section

- Using $g_L = g_V - g_A$, $g_R = g_V + g_A$:

Left-right: $$\frac{d\sigma(e^-_L e^+_R \to f_R \bar{f}_L)}{d\cos\theta} \sim (g_L{}^{\ell})^2 (g_R{}^{f})^2 \,(1 - \cos\theta)^2$$

Right-left: $$\frac{d\sigma(e^-_R e^+_L \to f_L \bar{f}_R)}{d\cos\theta} \sim (g_R{}^{\ell})^2 (g_L{}^{f})^2 \,(1 - \cos\theta)^2$$

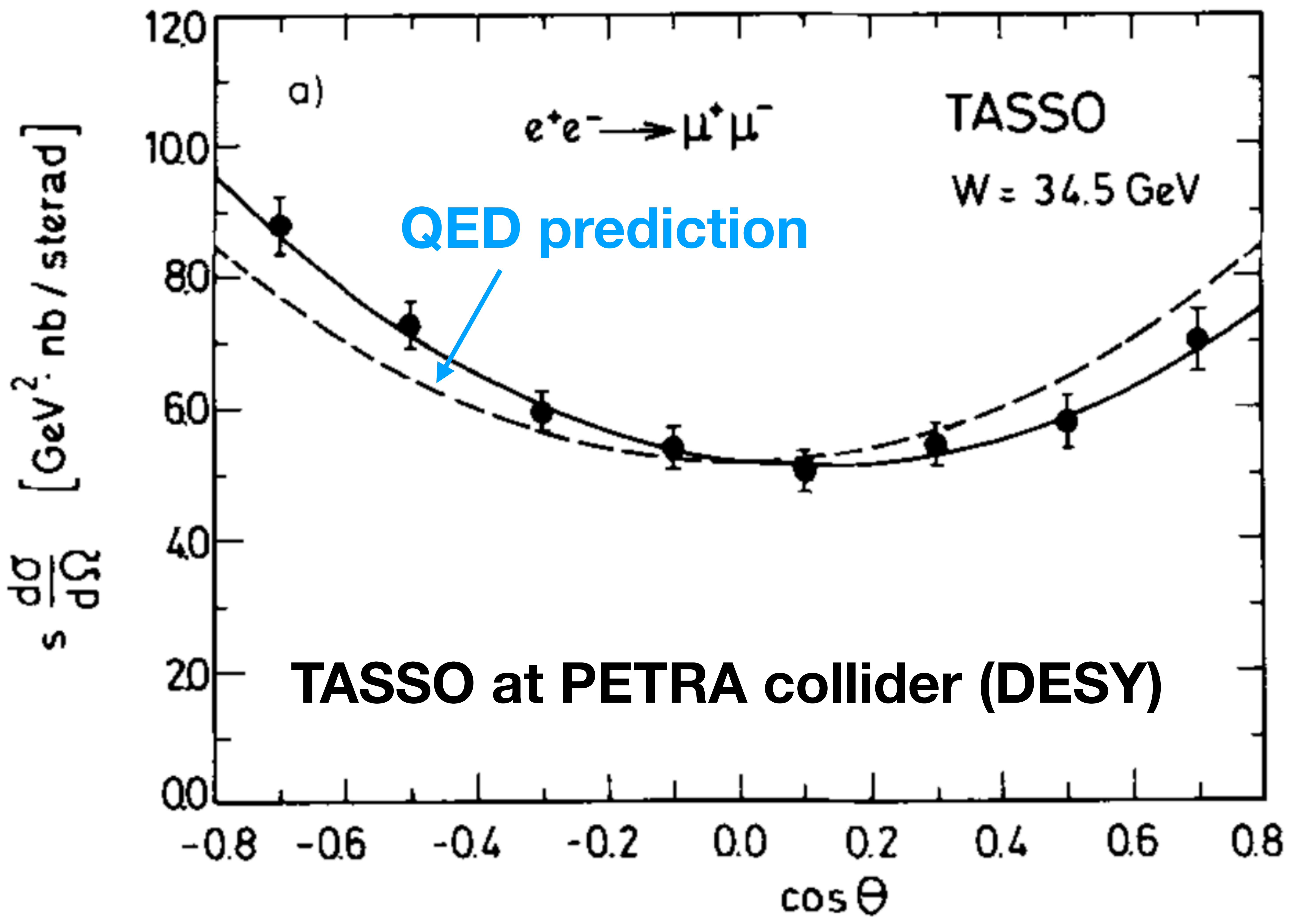
Left-left: $$\frac{d\sigma(e^-_L e^+_R \to f_L \bar{f}_R)}{d\cos\theta} \sim (g_L{}^{\ell})^2 (g_L{}^{f})^2 \,(1 + \cos\theta)^2$$

Right-right: $$\frac{d\sigma(e^-_R e^+_L \to f_R \bar{f}_L)}{d\cos\theta} \sim (g_R{}^{\ell})^2 (g_R{}^{f})^2 \,(1 + \cos\theta)^2$$

- Differential cross section for Z-boson exchange:

$$\frac{d\sigma}{d\cos\theta} \sim 1 + \cos^2\theta + A_{fb}\cos\theta$$

Angular distributions for $\sqrt{s} \ll m_Z$



Althoff, M., Braunschweig, W. *et al.* An improved measurement of electroweak couplings from $e^+e^- \to e^+e^-$ and $e^+e^- \to \mu^+\mu^-$. *Z. Phys. C - Particles and Fields* **22**, 13–21 (1984). https://doi.org/10.1007/BF01577559

γZ interference

$$\frac{2s}{\pi}\frac{1}{N_c^{\mathrm{f}}}\frac{d\sigma_{\mathrm{ew}}}{d\cos\theta}(\mathrm{e^+e^-}\to\mathrm{f\bar{f}}) \;=$$

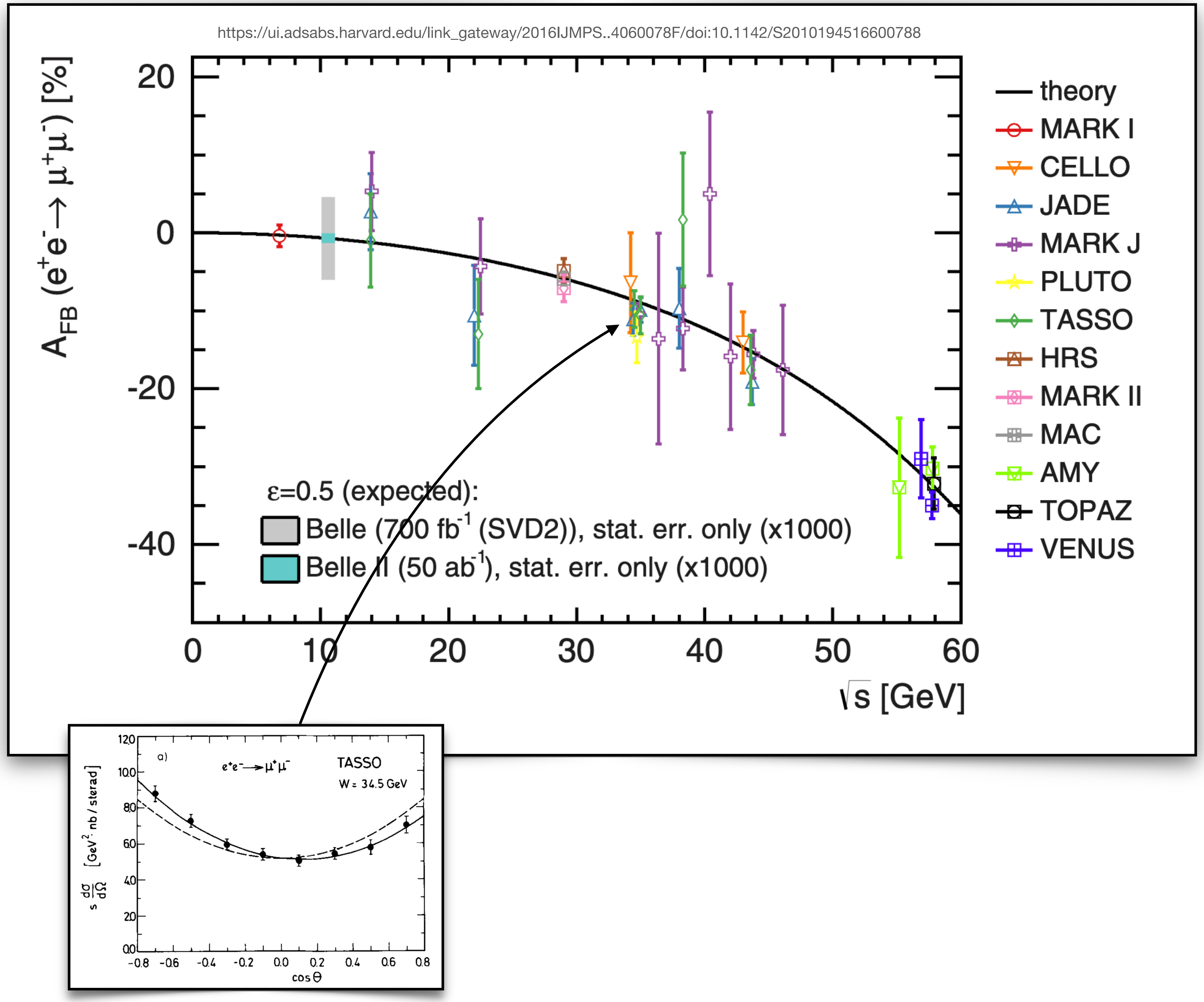
$$\underbrace{|\alpha(s)Q_{\mathrm{f}}|^2\,(1+\cos^2\theta)}_{\sigma^\gamma}$$

$$\underbrace{-8\Re\left\{\alpha^*(s)Q_{\mathrm{f}}\chi(s)\left[\mathcal{G}_{\mathrm{Ve}}\mathcal{G}_{\mathrm{Vf}}(1+\cos^2\theta)+2\mathcal{G}_{\mathrm{Ae}}\mathcal{G}_{\mathrm{Af}}\cos\theta\right]\right\}}_{\gamma\text{–Z interference}}$$

$$\underbrace{\begin{array}{c}+16|\chi(s)|^2\left[(|\mathcal{G}_{\mathrm{Ve}}|^2+|\mathcal{G}_{\mathrm{Ae}}|^2)(|\mathcal{G}_{\mathrm{Vf}}|^2+|\mathcal{G}_{\mathrm{Af}}|^2)(1+\cos^2\theta)\right.\\ \left.+8\Re\left\{\mathcal{G}_{\mathrm{Ve}}\mathcal{G}_{\mathrm{Ae}}{}^*\right\}\Re\left\{\mathcal{G}_{\mathrm{Vf}}\mathcal{G}_{\mathrm{Af}}{}^*\right\}\cos\theta\right]\end{array}}_{\sigma^{\mathrm{Z}}}$$

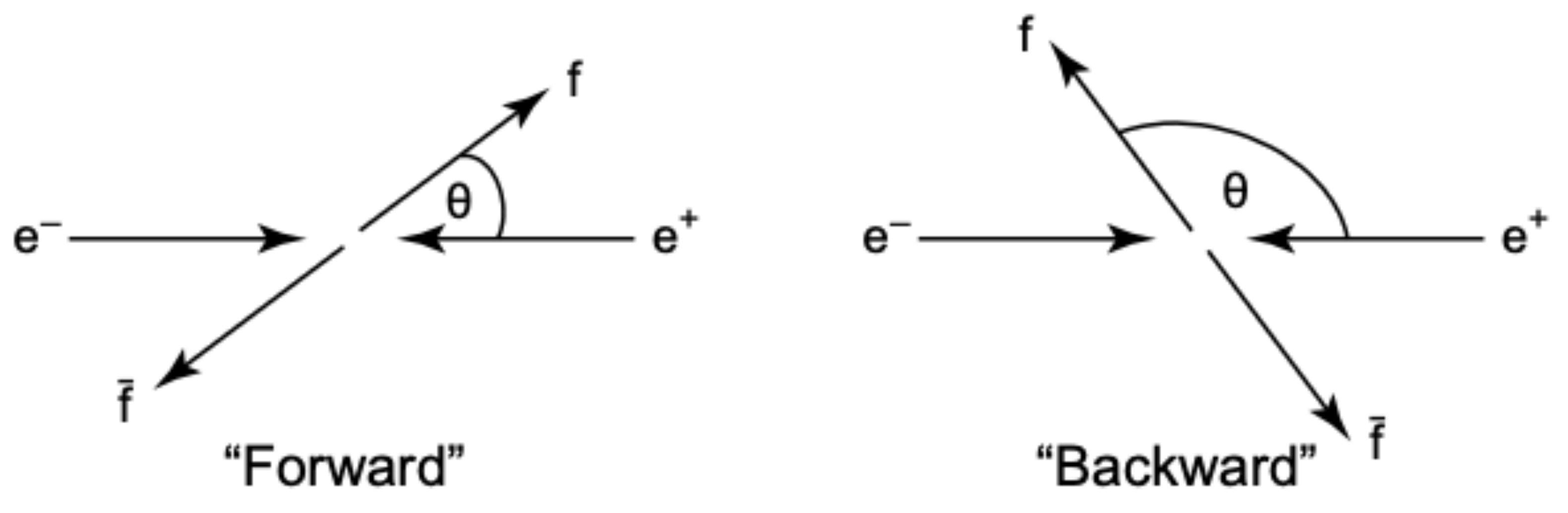
with:

$$\chi(s) \;=\; \frac{G_{\mathrm{F}}m_{\mathrm{Z}}^2}{8\pi\sqrt{2}}\,\frac{s}{s-m_{\mathrm{Z}}^2+is\Gamma_{\mathrm{Z}}/m_{\mathrm{Z}}}\,,$$



Angular distributions for $\sqrt{s} \approx m_Z$

- “forward” and “backward” are defined with respect to the scattering angle of the outgoing fermion

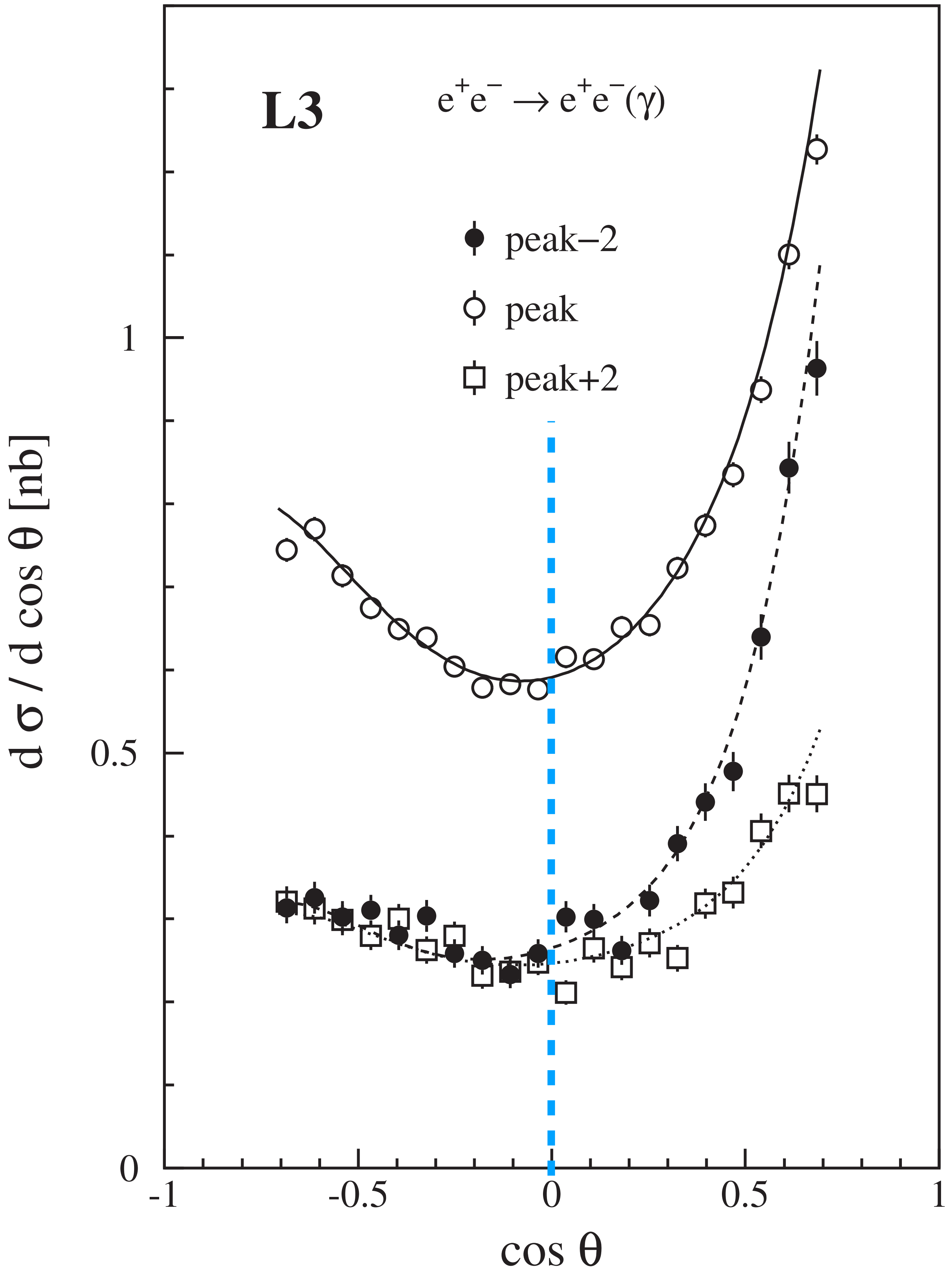


$$\sigma_F = \int_0^{\pi/2} \frac{d\sigma}{d\cos\theta}\, d\theta, \quad \sigma_B = \int_{\pi/2}^{\pi} \frac{d\sigma}{d\cos\theta}\, d\theta$$

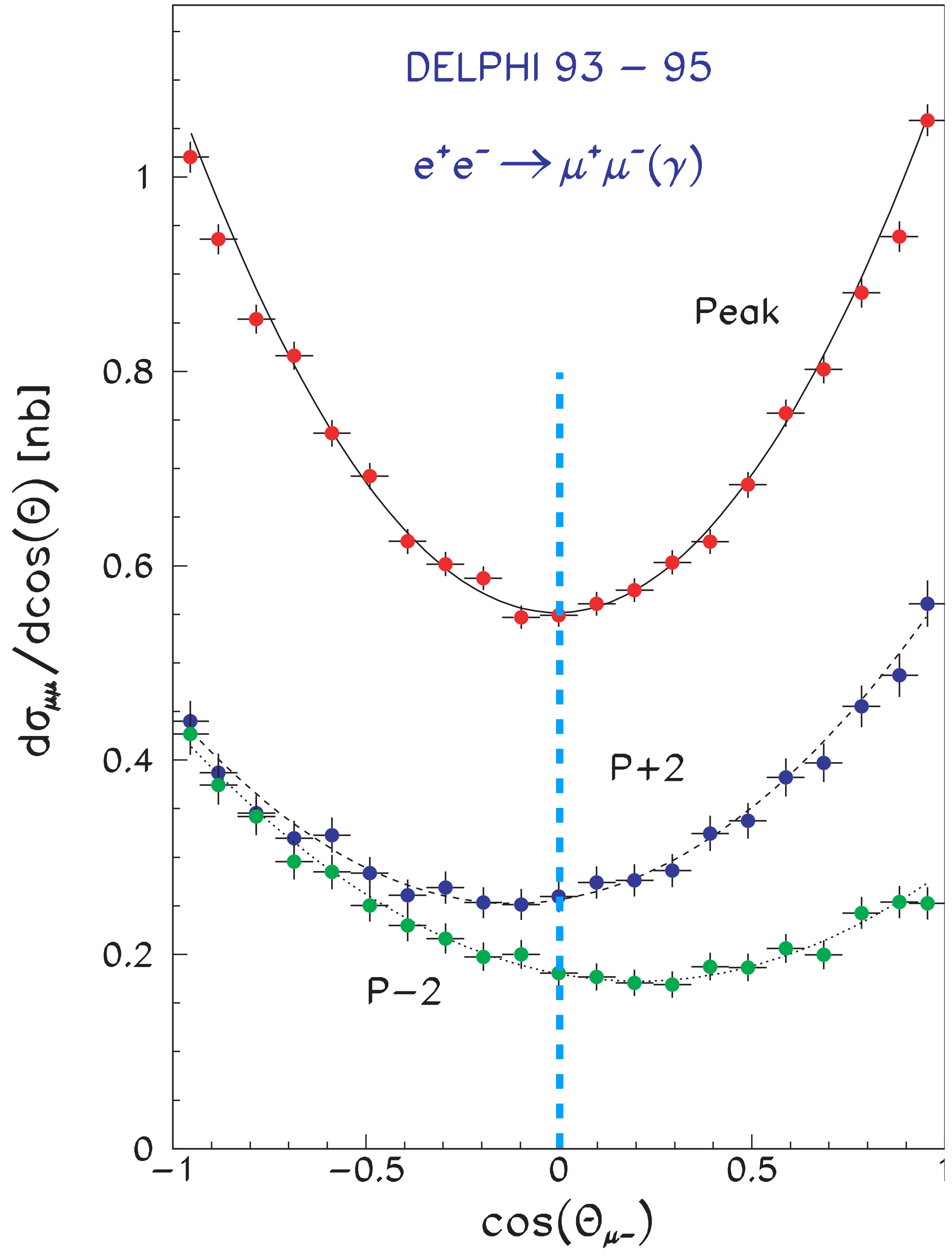
- Forward-backward asymmetry:

$$A_{\mathrm{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$e^+e^- \to e^+e^-$



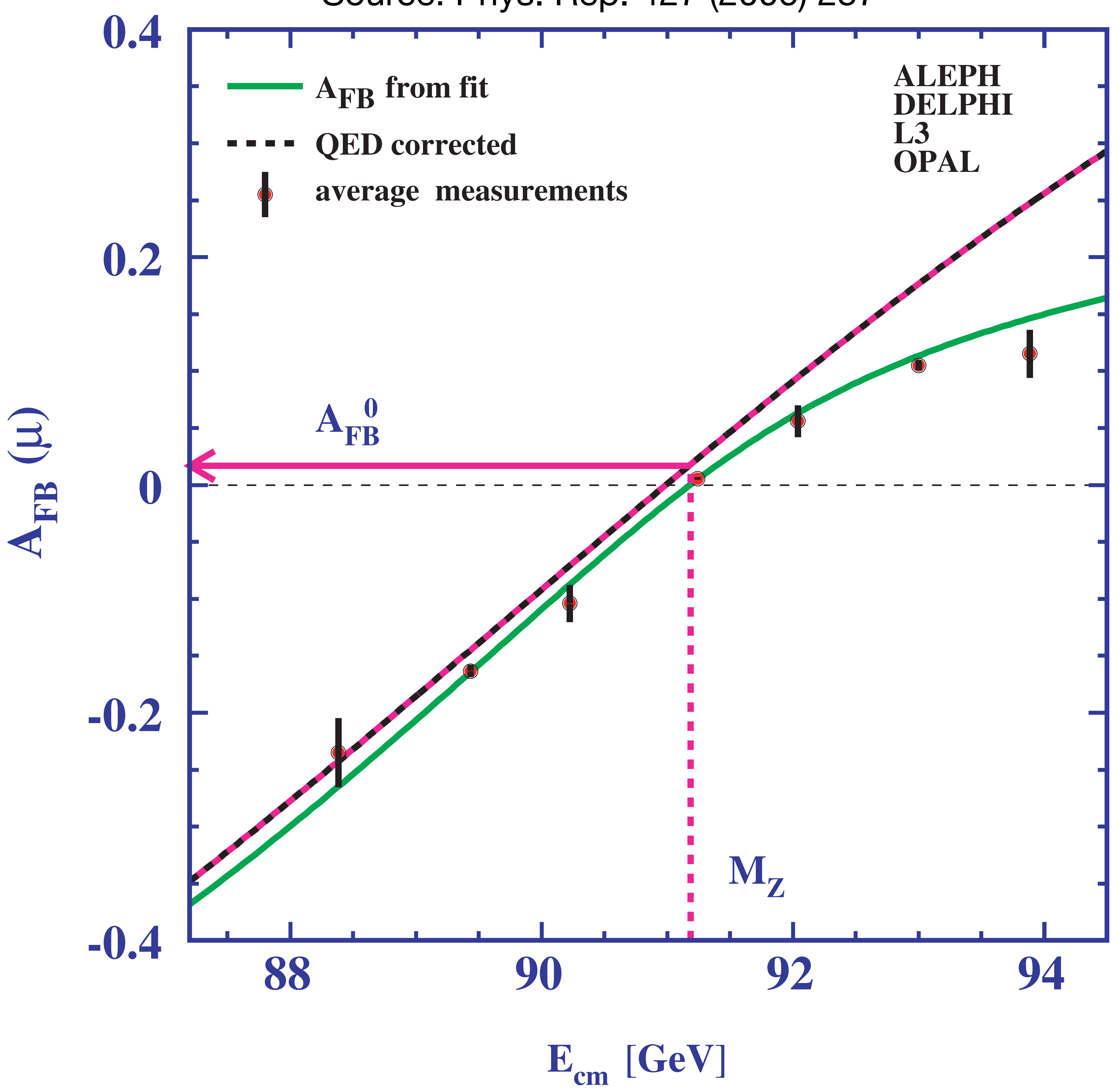
$e^+e^- \to \mu^+\mu^-$

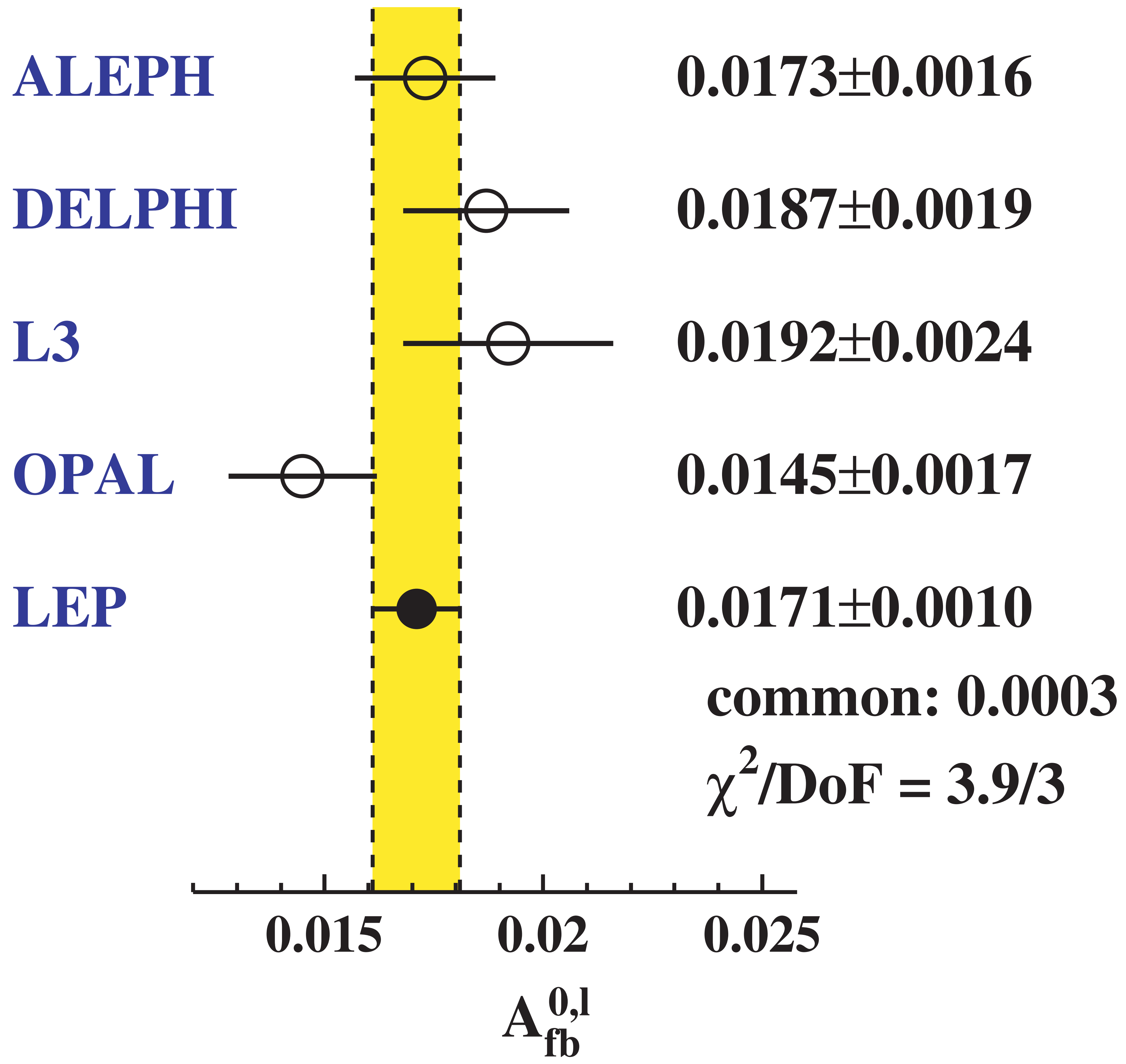


https://arxiv.org/abs/hep-ex/0509008

Angular distributions for $\sqrt{s} \approx m_Z$

Source: Phys. Rep. 427 (2006) 257



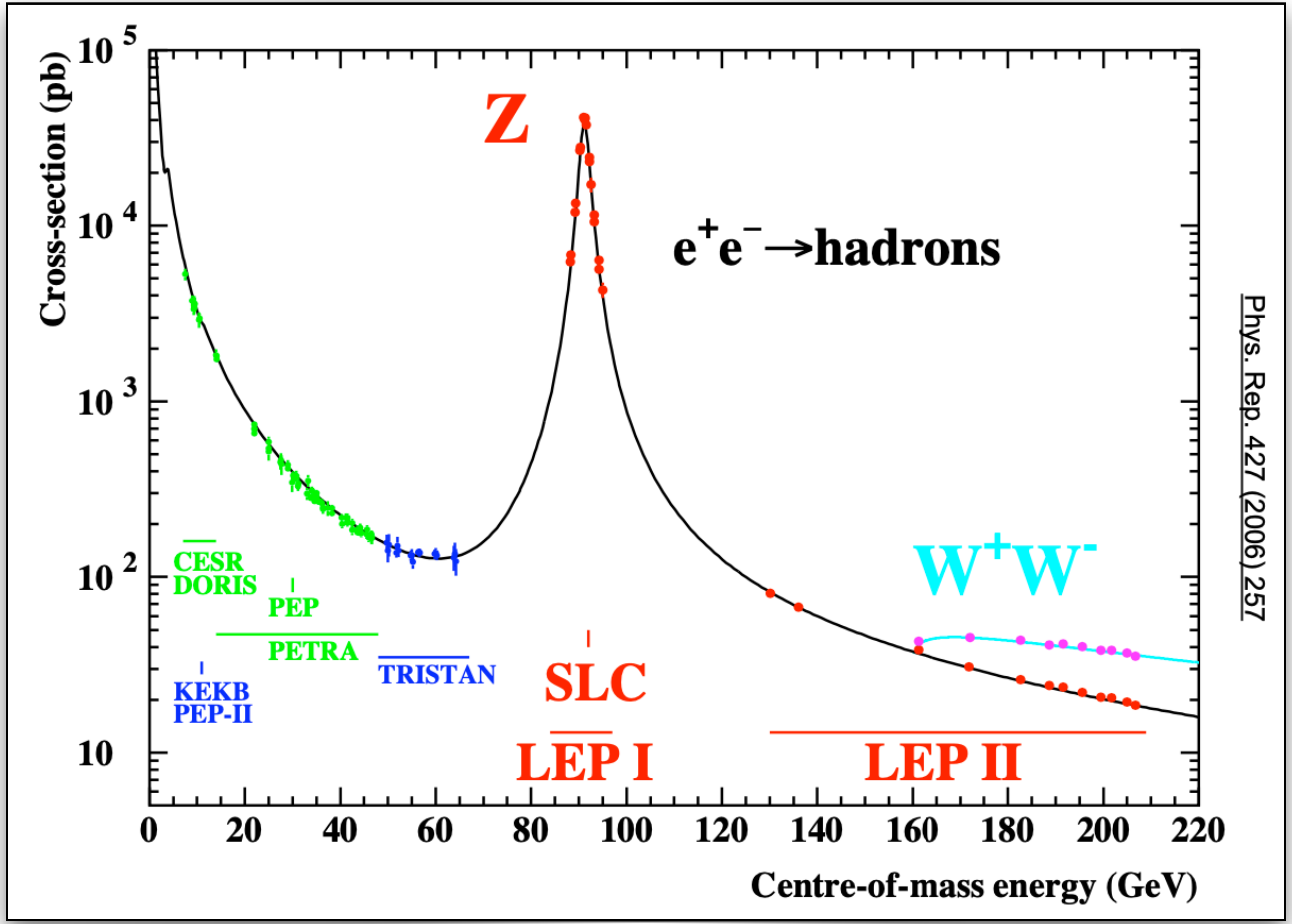


https://arxiv.org/abs/hep-ex/0509008

Z-Pole

- Propagator: Z-boson unstable → resonance in scattering amplitude
  - Wave function for stable particle at rest: $\psi \sim \exp(-imt)$
  - Unstable particle $\psi^*\psi \sim \exp(-t/\tau) \rightarrow \psi \sim \exp(-imt)\exp(\Gamma t/2)$
  - Decay width is inverse of the lifetime: $\Gamma = 1/\tau$
  - Breit-Wigner prescription: Replace m by $m - i\Gamma/2$ in the propagator
    (QFT: scattering amplitude contains a pole on the complex plane)

Z-Pole



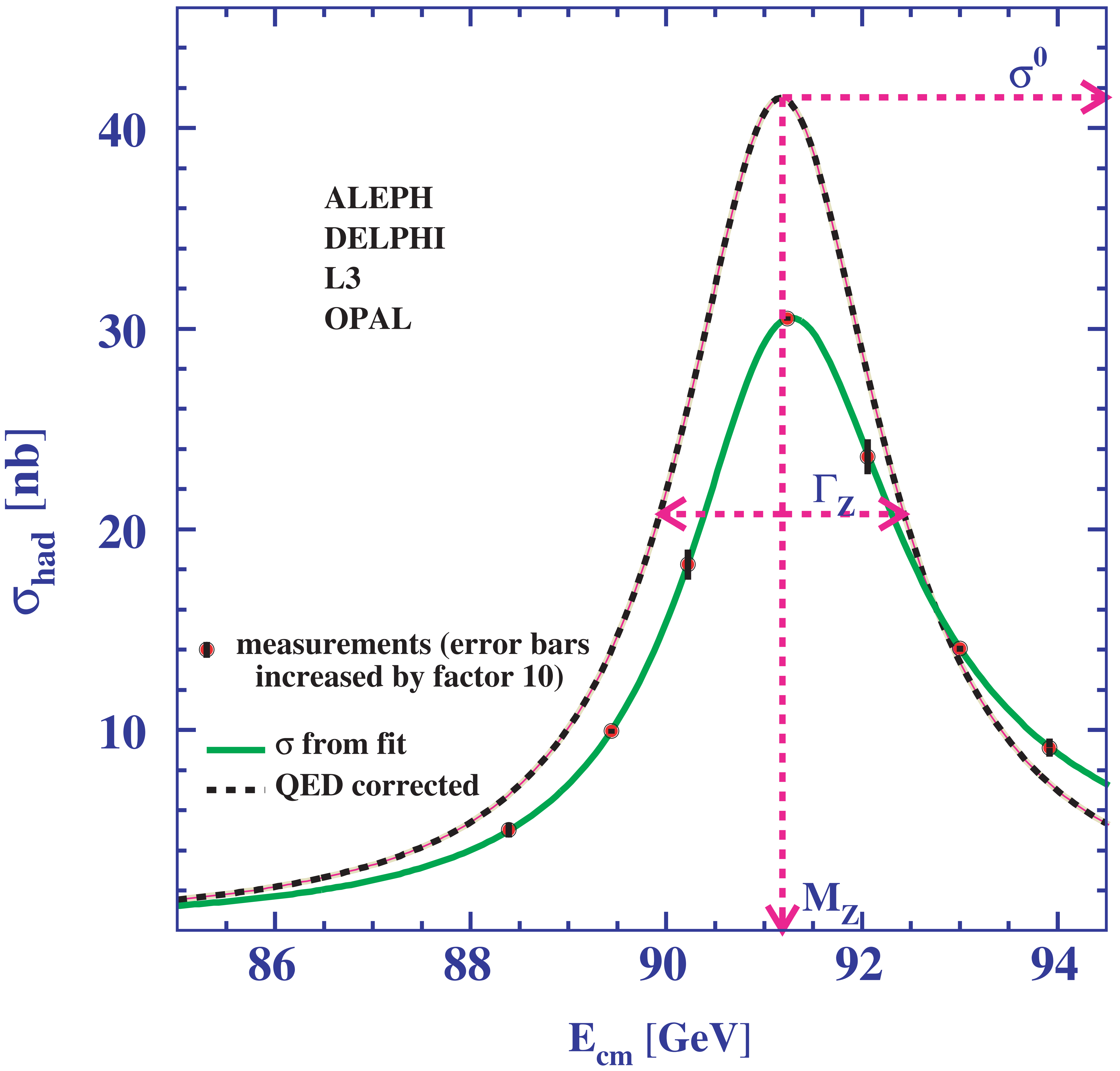
Z
$e^+e^- \rightarrow$ hadrons
W^+W^-
Cross-section (pb)
10^5
10^4
10^3
10^2
10
CESR
DORIS
PEP
PETRA
KEKB
PEP-II
TRISTAN
SLC
LEP I
LEP II
0
20
40
60
80
100
120
140
160
180
200
220
Centre-of-mass energy (GeV)
Phys. Rep. 427 (2006) 257

- Lecture end 05.12.2023

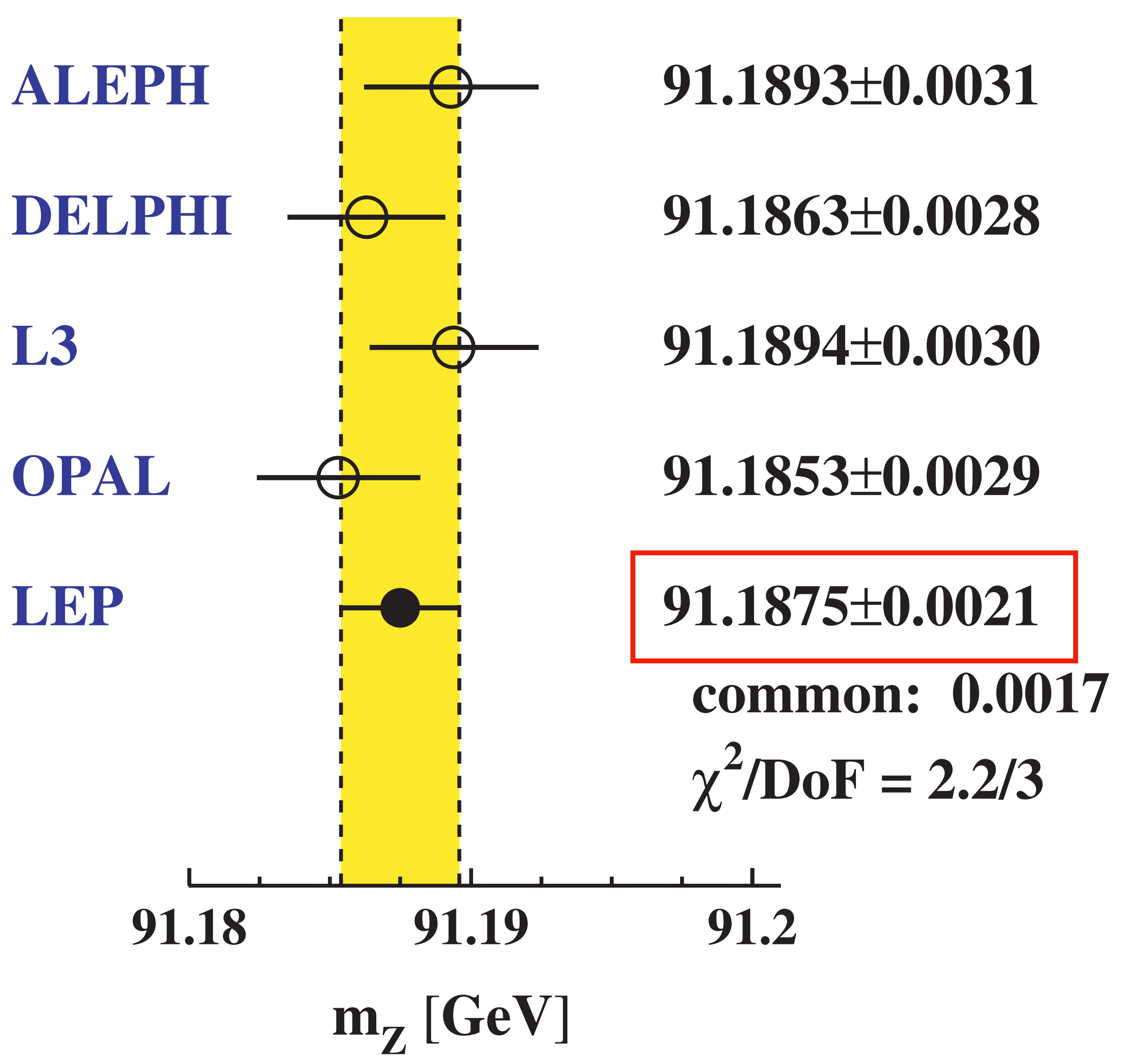
Hadronic cross section

- Precision measurement of Z-resonance in hadronic events:
  - Energy scan (varying collision energy in LEP)
  - Correction of data for radiative effects
- Observables:
  - position of peak → M_Z
  - height of the peak → σ^0_{had}
  - width of the peak (FWHM) → Γ_Z
- Many observables are correlated, complicated electroweak fitting frameworks developed: ZFITTER and others

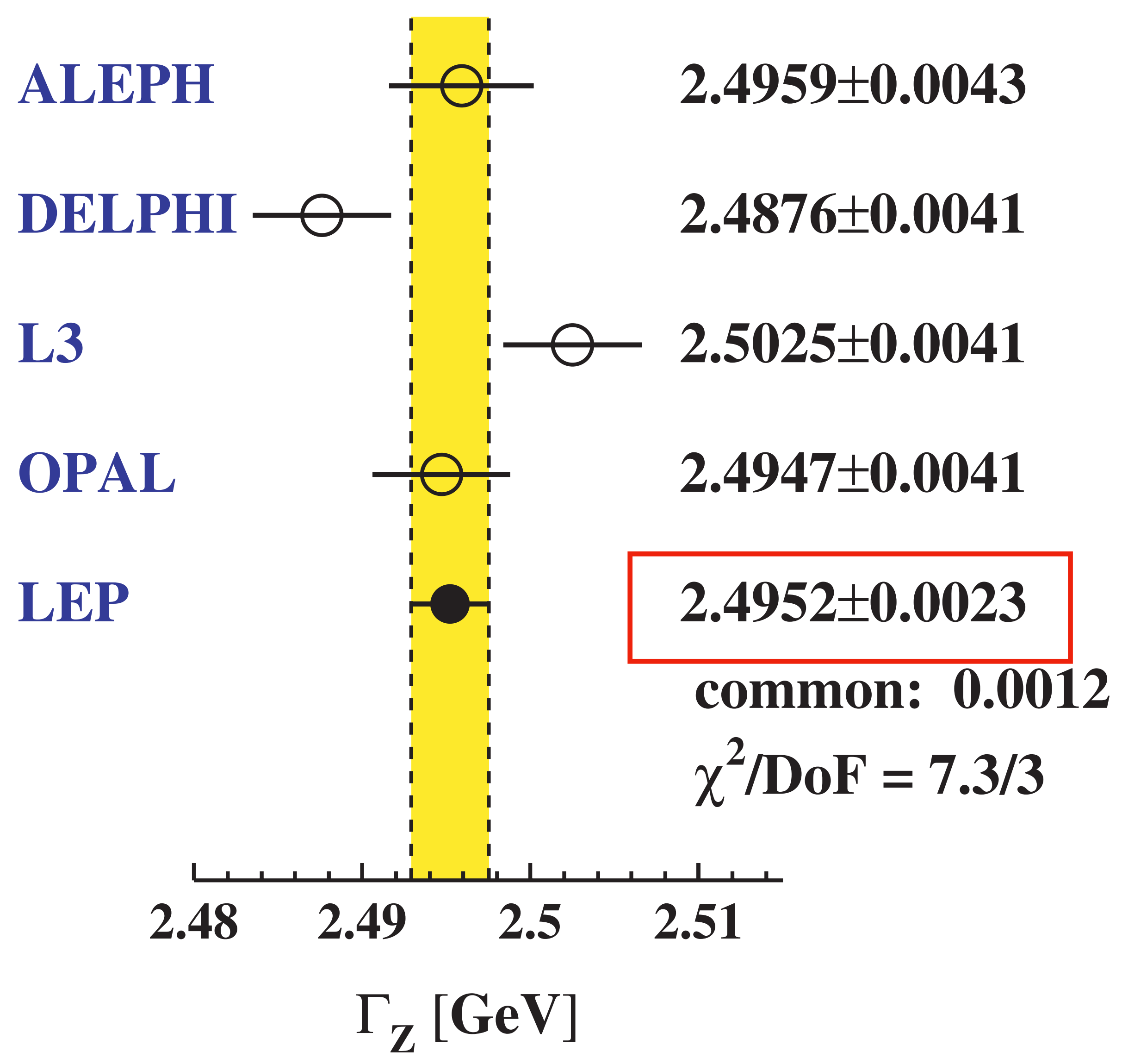
Source: Phys. Rep. 427 (2006) 257



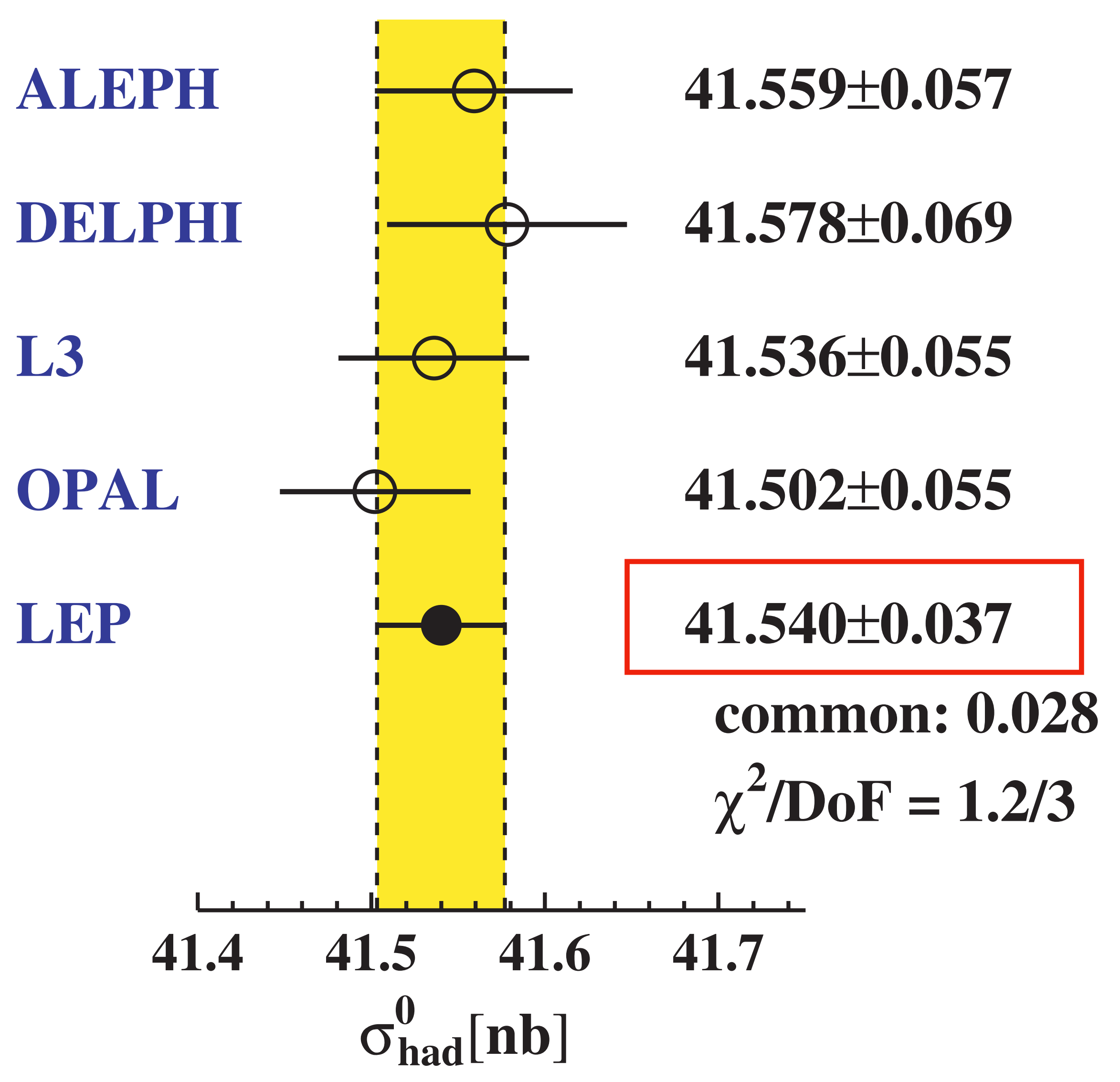
Z pole parameters



relative uncertainty: 2.3×10^{-5}



relative uncertainty: 9.2×10^{-4}



relative uncertainty: 8.9×10^{-4}

Source: Phys. Rep. 427 (2006) 257

Decay width

- Total width Γ_Z of the Z resonance:
  - Sum of partial decay widths
  - Consider all possible Z-boson decays in the standard model:
    - 5 quarks (u, d, s, c, b) - the top quark is too heavy, 3 charged leptons, 3 neutrinos

$$\Gamma_Z = \sum_f \Gamma_f = \sum_{q=u,d,s,c,b} \Gamma_q + \sum_{\ell=e,\mu,\tau} \Gamma_\ell + \sum_{\nu=\nu_e,\nu_\mu,\nu_\tau} \Gamma_\nu$$

    - Partial width in the standard model (without corrections):

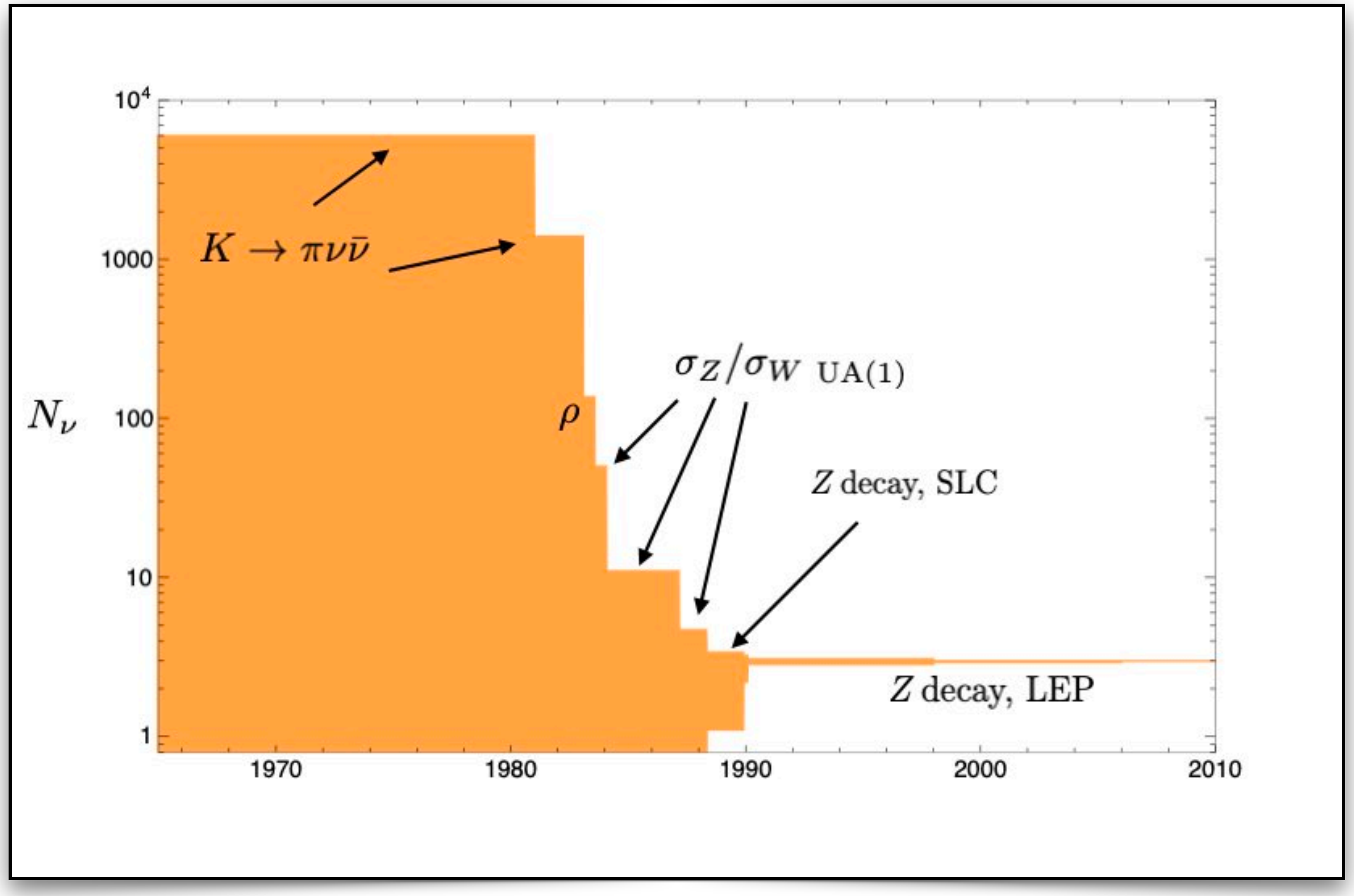
$$\Gamma(Z \to f\bar{f}) = N_C^f \frac{G_F m_Z^3}{6\sqrt{2}\pi} \left((g_V^f)^2 + (g_A^f)^2 \right) \text{ with } g_V^f = I_3^f - 2Q_f \sin^2\theta_W \text{ and } g_A^f = I_3^f$$

    - Standard model predicts lepton universality*: $\Gamma_e = \Gamma_\mu = \Gamma_\tau$ and $\Gamma_{\nu_e} = \Gamma_{\nu_\mu} = \Gamma_{\nu_\tau}$

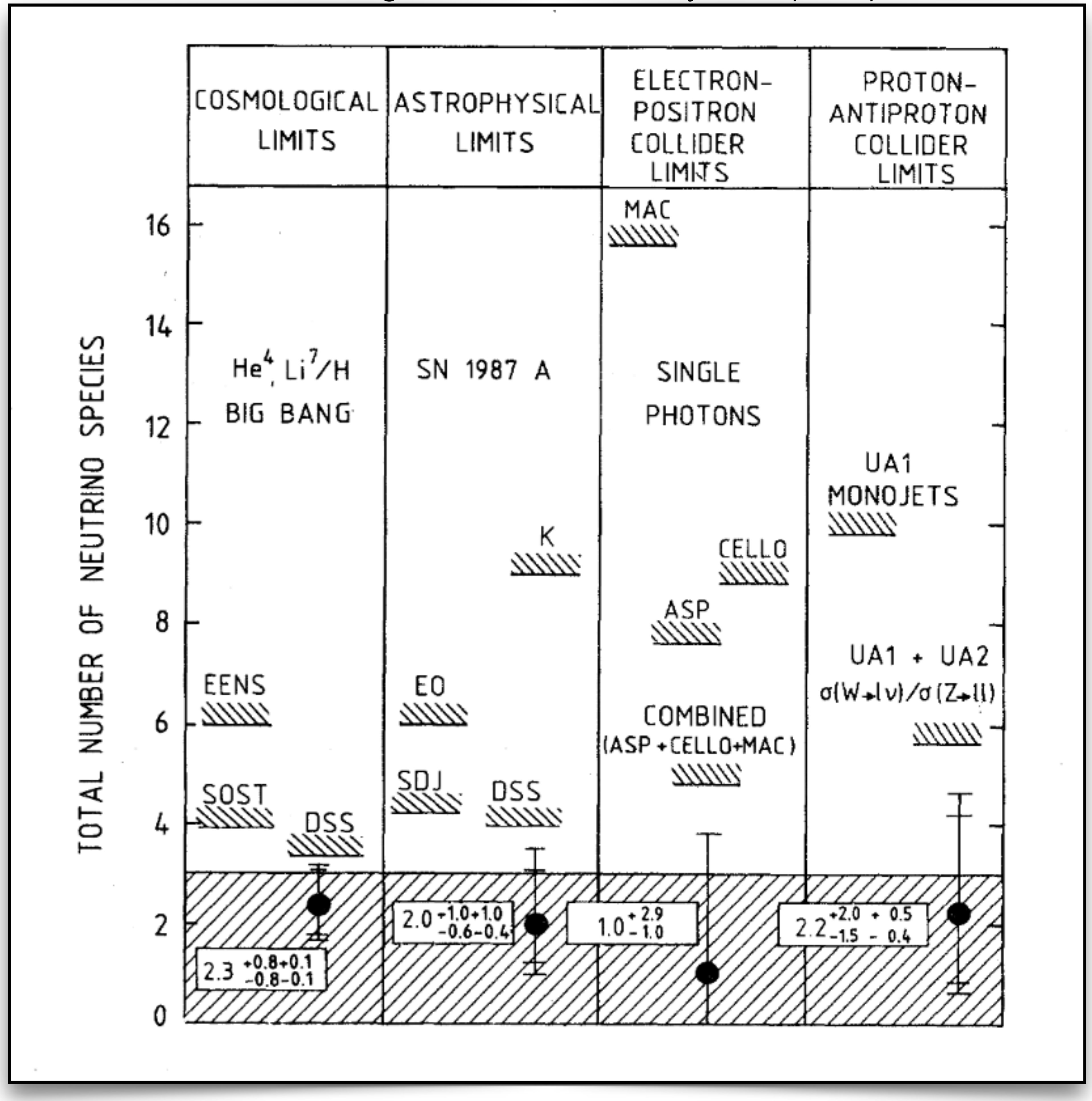
*Different lepton masses lead to small deviations from this universality.

Number of light neutrinos: History

Source: https://twitter.com/martinmbauer/status/1547987138112659456/photo/1



Source: D. Denegri et al, *Rev.Mod.Phys.* 62 (1990) 1-42



- Colliders and astrophysical observations in the 1980 narrowed the number of active neutrino species down → A large number of neutrino species would have made the Z peak invisible!

Number of light neutrinos

- Total cross section for $ee \rightarrow Z \rightarrow \bar{f}f$ at Z-peak (ignoring Breit-Wigner term):
  $\sigma = \frac{12\pi}{m_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}$ (with $\Gamma_e = \Gamma_\ell$ for lepton flavour universality)

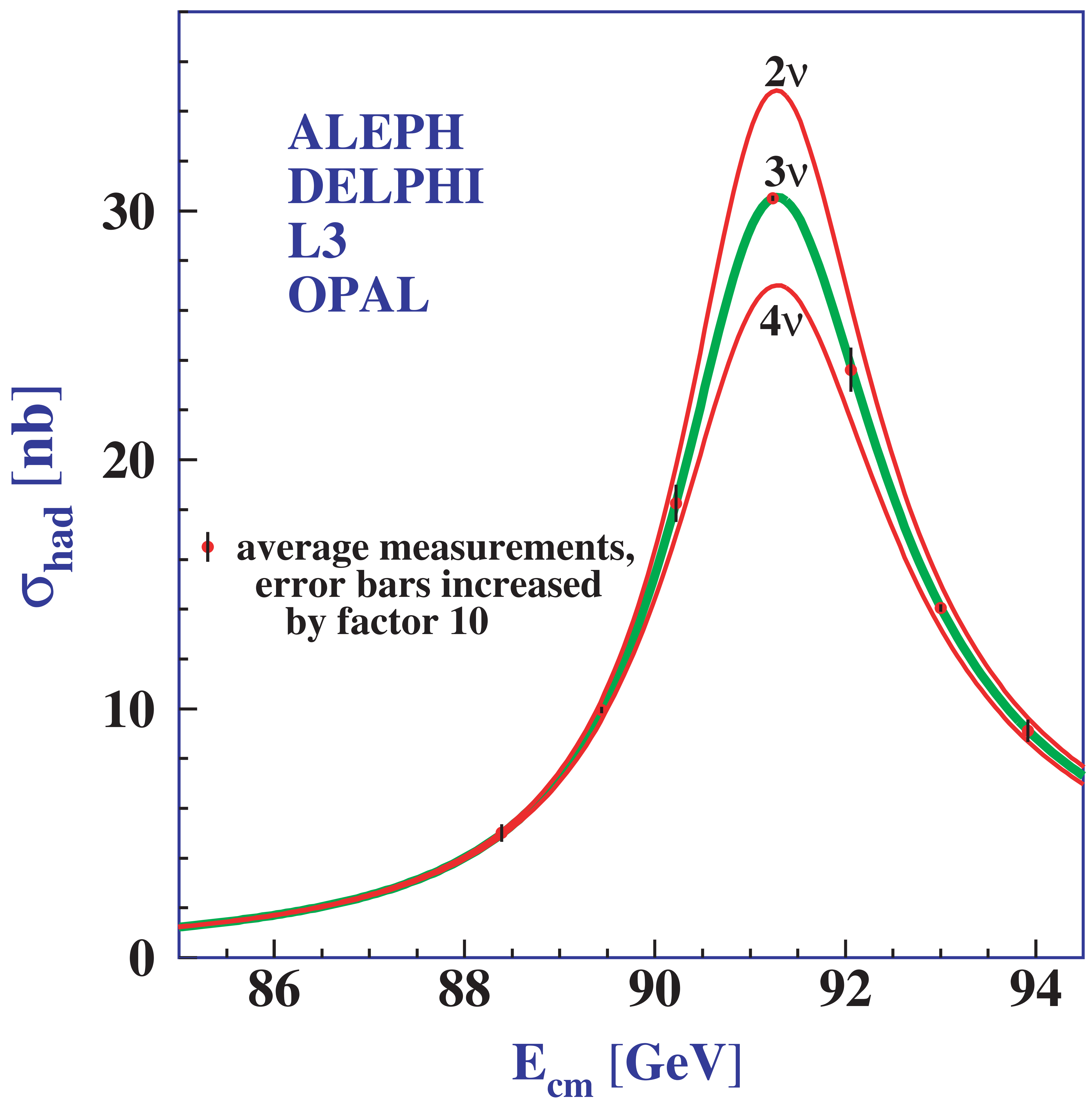
- Using $\Gamma_Z = \Gamma_{inv} + 3\Gamma_\ell + \Gamma_{had}$ and defining the ratio $R^0_{inv} = \frac{\Gamma_{inv}}{\Gamma_\ell}$:
  $R^0_{inv} = \frac{\Gamma_Z}{\Gamma_\ell} - 3 - R^0_\ell$ with $R^0_\ell = \frac{\Gamma_{had}}{\Gamma_\ell} = 20.767 \pm 0.025$

- $\sigma^0_{had} = \frac{12\pi}{m_Z^2}\frac{\Gamma_\ell\Gamma_{had}}{\Gamma_Z^2} \quad \leftrightarrow \quad \frac{\Gamma_Z}{\Gamma_\ell} = \sqrt{\frac{12\pi}{m_Z^2}\frac{R^0_\ell}{\sigma^0_{had}}} \quad \rightarrow R^0_{inv} = \sqrt{\frac{12\pi}{m_Z^2}\frac{R^0_\ell}{\sigma^0_{had}}} - 3 - R^0_\ell$

Invisible ratio can be determined by measuring charged leptonic and hadronic widths, Z-boson mass, and hadronic cross section!

Summary: Number of light neutrinos

Source: Phys. Rep. 427 (2006) 257

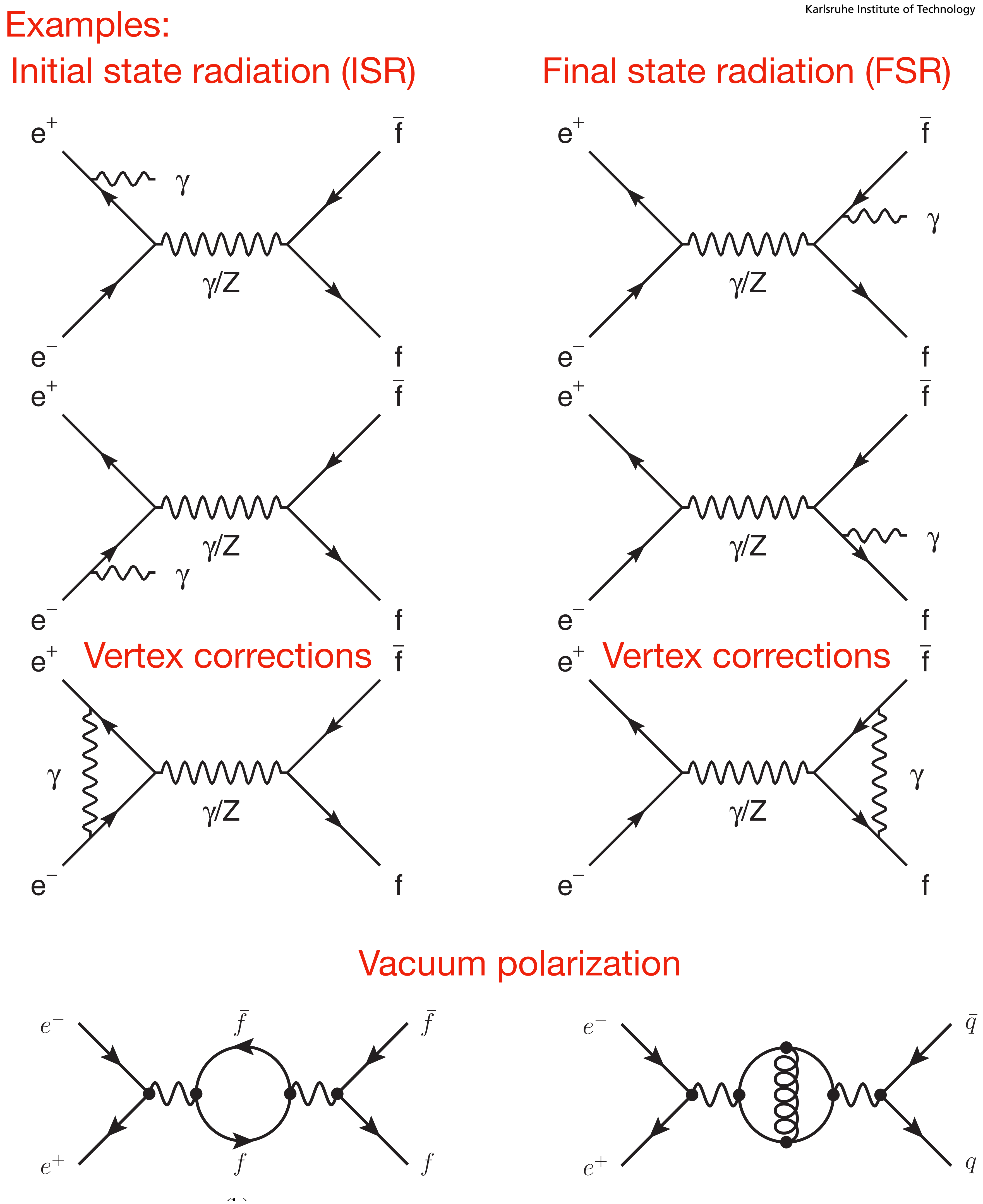


$$R^0_{inv} = \sqrt{\frac{12\pi}{m_Z^2}\frac{R^0_\ell}{\sigma^0_{had}}} - 3 - R^0_\ell = N_\nu \frac{\Gamma_{\nu\nu}}{\Gamma_{\ell\ell}}$$

$$N_\nu = 2.9840 \pm 0.0082$$

Radiative corrections

- Precision of LEP and SLC is sensitive to higher order corrections
  - Real emissions of photons
  - Loop corrections
  - Consequence (among others): Running of QED coupling constant $\alpha(m_Z^2) \approx \dfrac{1}{128} > \alpha(0) = \dfrac{1}{137}$
- LEP electroweak working group defined so called pseudo-observables to compare to theory predictions, often denoted “effective” in the variable names



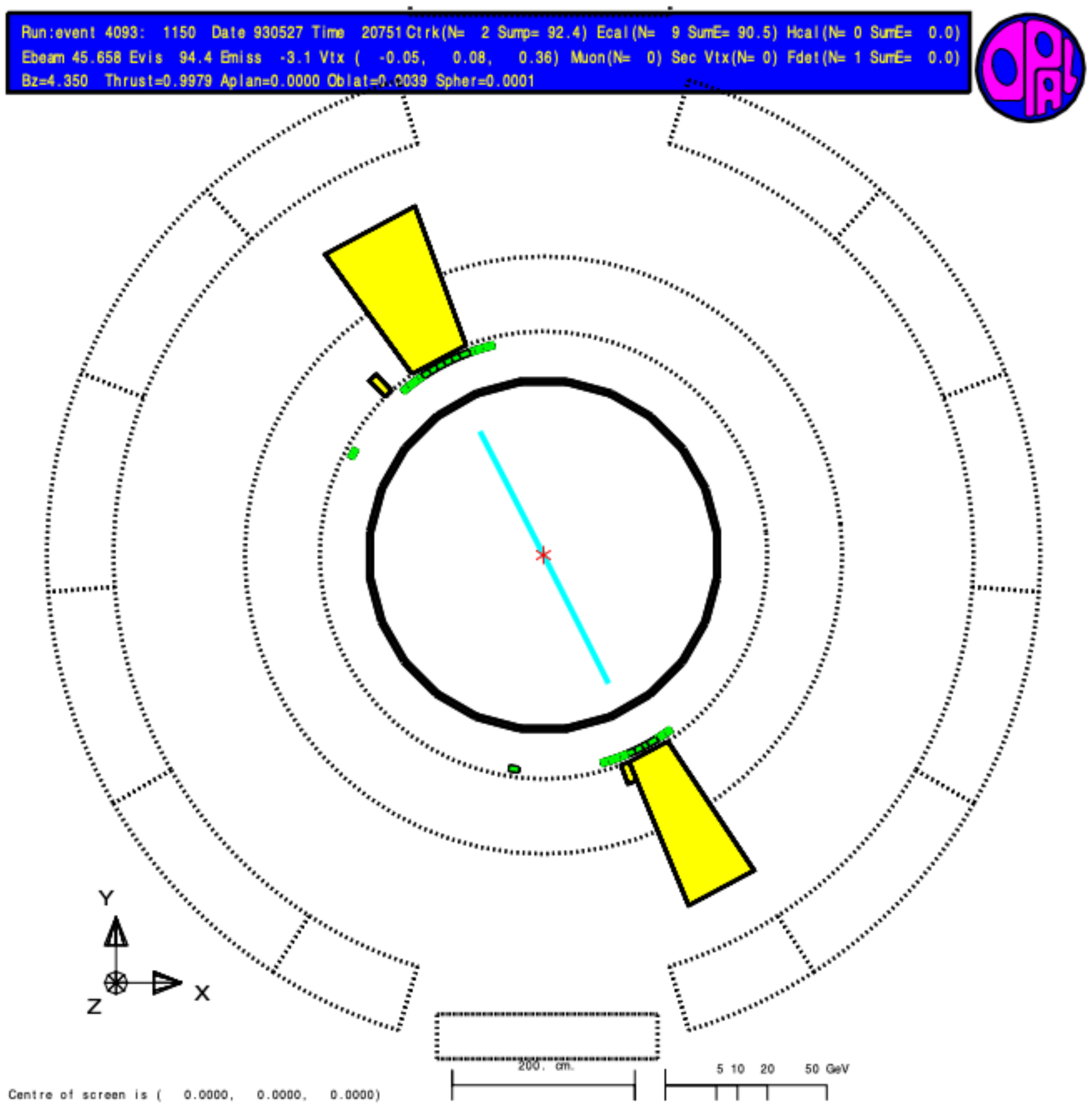
Summary: Decay width

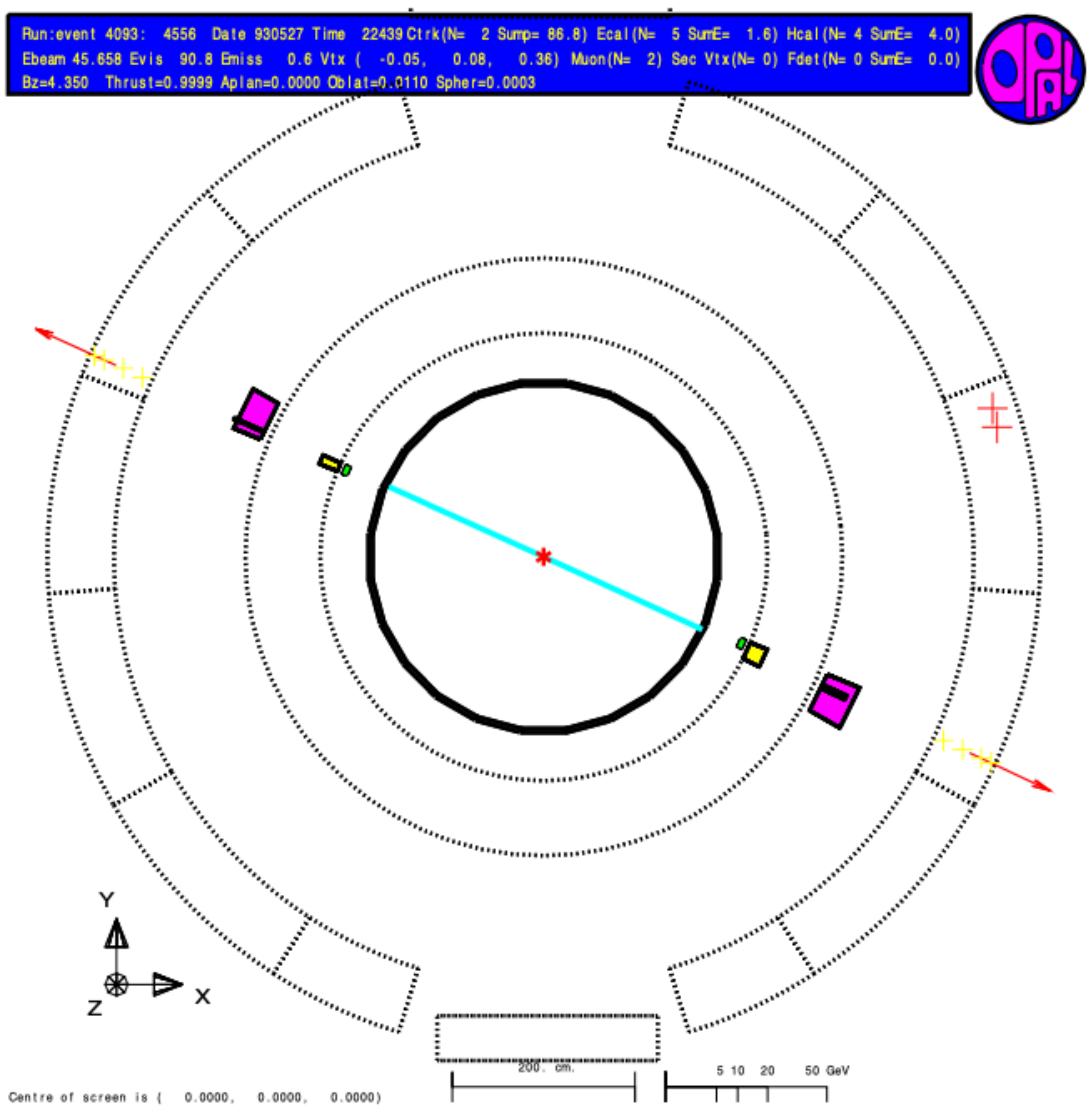
	Branching fraction (PDG22) [%]	Detection channel
left-handed neutrinos	20.00 ± 0.06	none (indirect)
charged leptons	3.3632 ± 0.0042 (e) 3.3662 ± 0.0066 (μ) 3.3696 ± 0.0083 (τ)	e and μ straight forward, τ in different decay modes
hadrons	69.911 ± 0.056	inclusive jets
up-type quarks (u, c) in three colors (top too heavy!)	11.6 ± 0.6 12.03 ± 0.21 (c)	jets (c with c-tagging)
down-type quarks (d, s, b) in three colors	15.6 ± 0.4 15.12 ± 0.05 (b)	jets (b with b-tagging)

Test of lepton-flavour universality: $\frac{\Gamma(\mu^+\mu^-)}{\Gamma(e^+e^-)} = 1.0001 \pm 0.0024$ $\quad \frac{\Gamma(\tau^+\tau^-)}{\Gamma(e^+e^-)} = 1.0020 \pm 0.0032$

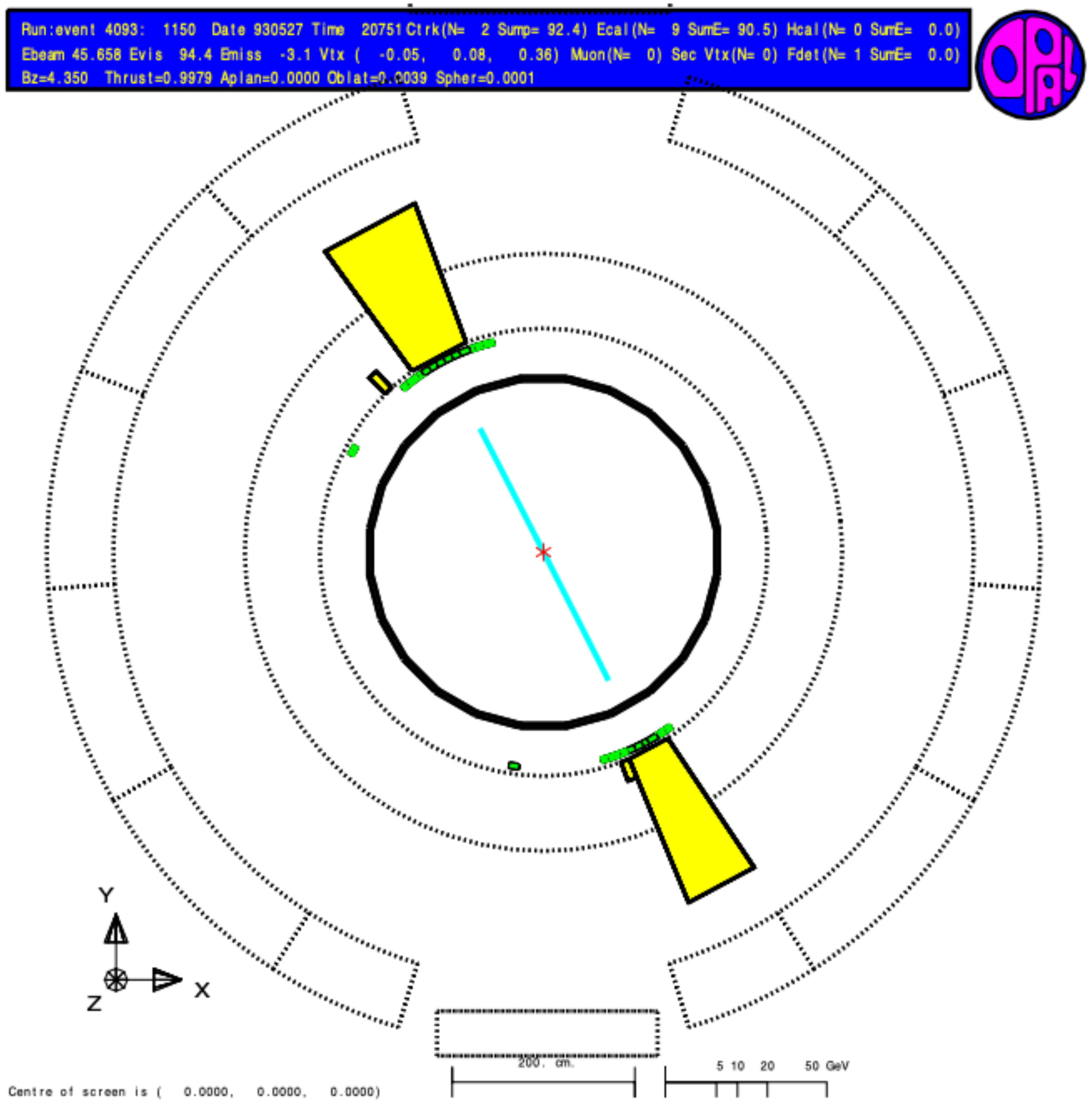
https://pdg.lbl.gov/2022/listings/rpp2022-list-z-boson.pdf

Event displays

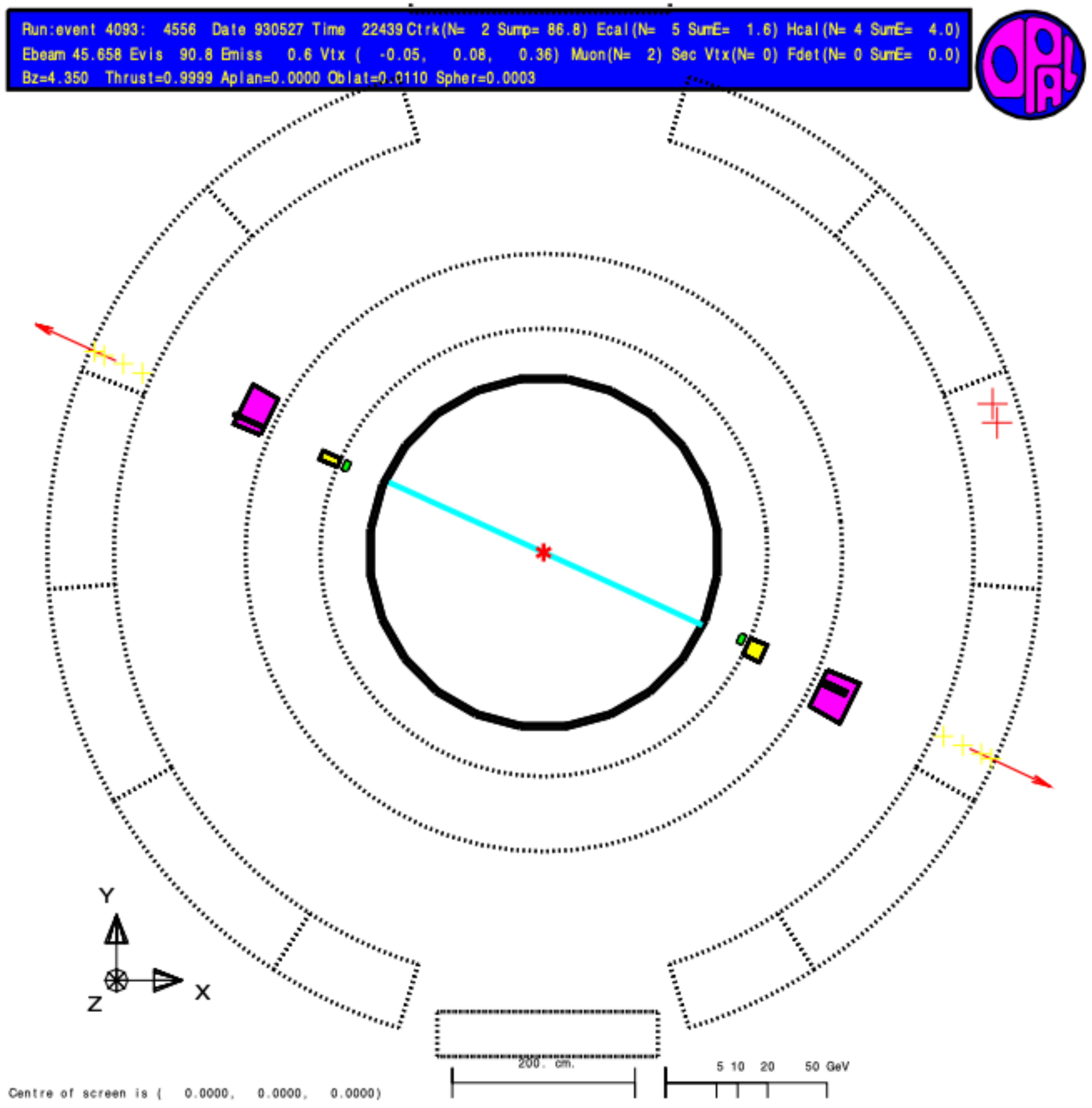




Solution: Event displays 1

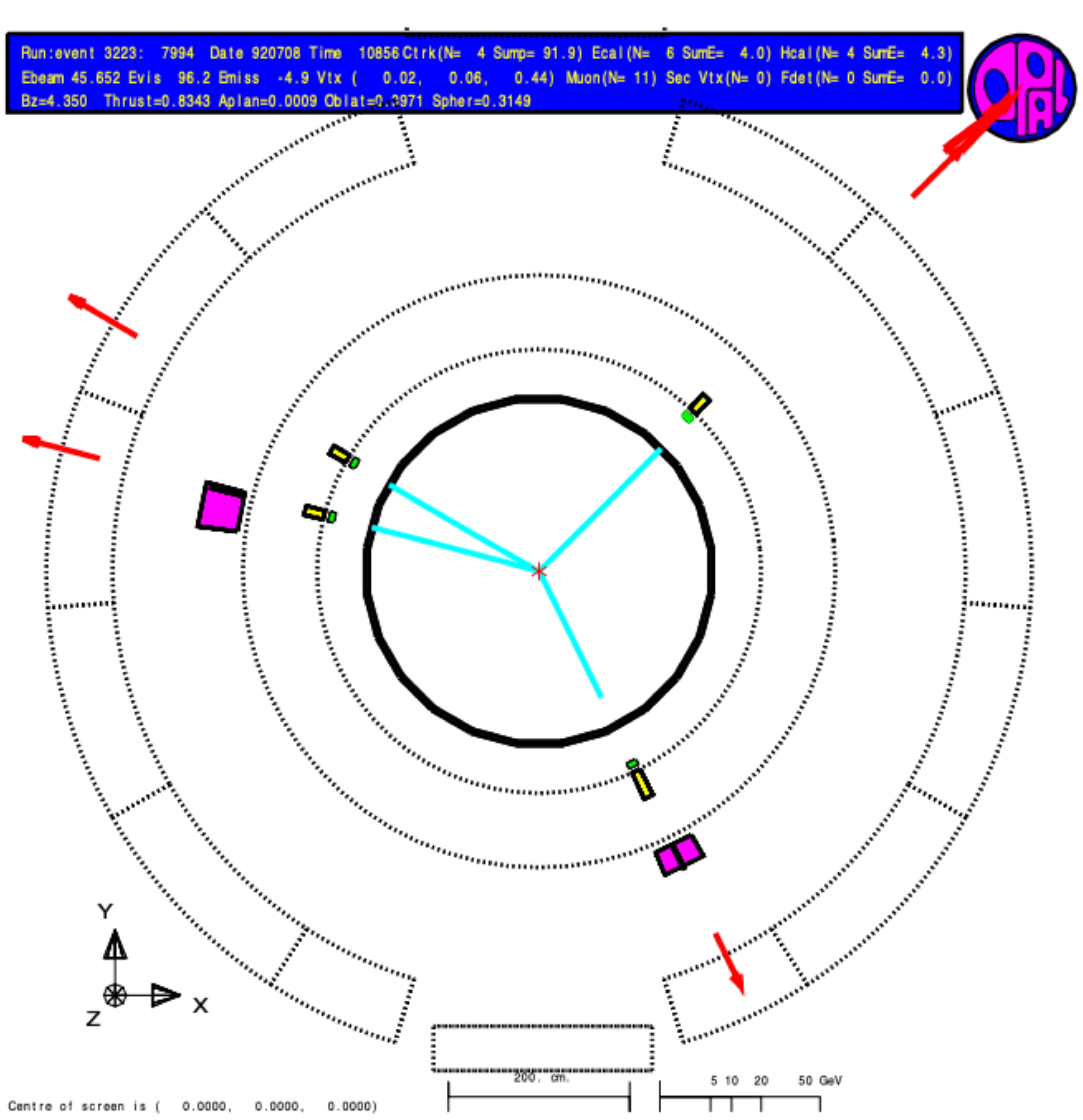


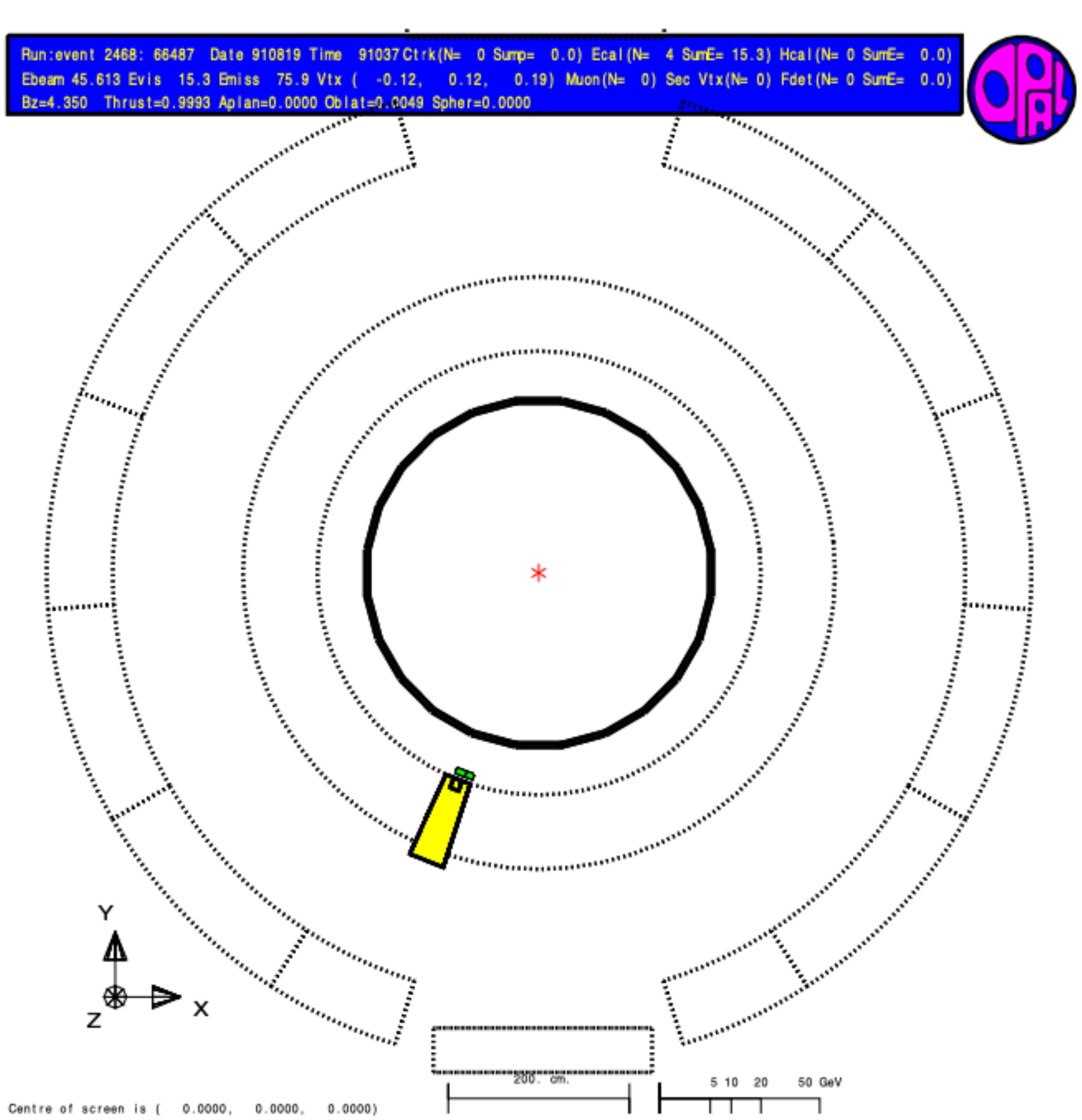
$e^+e^- \rightarrow Z \rightarrow e^+e^-$



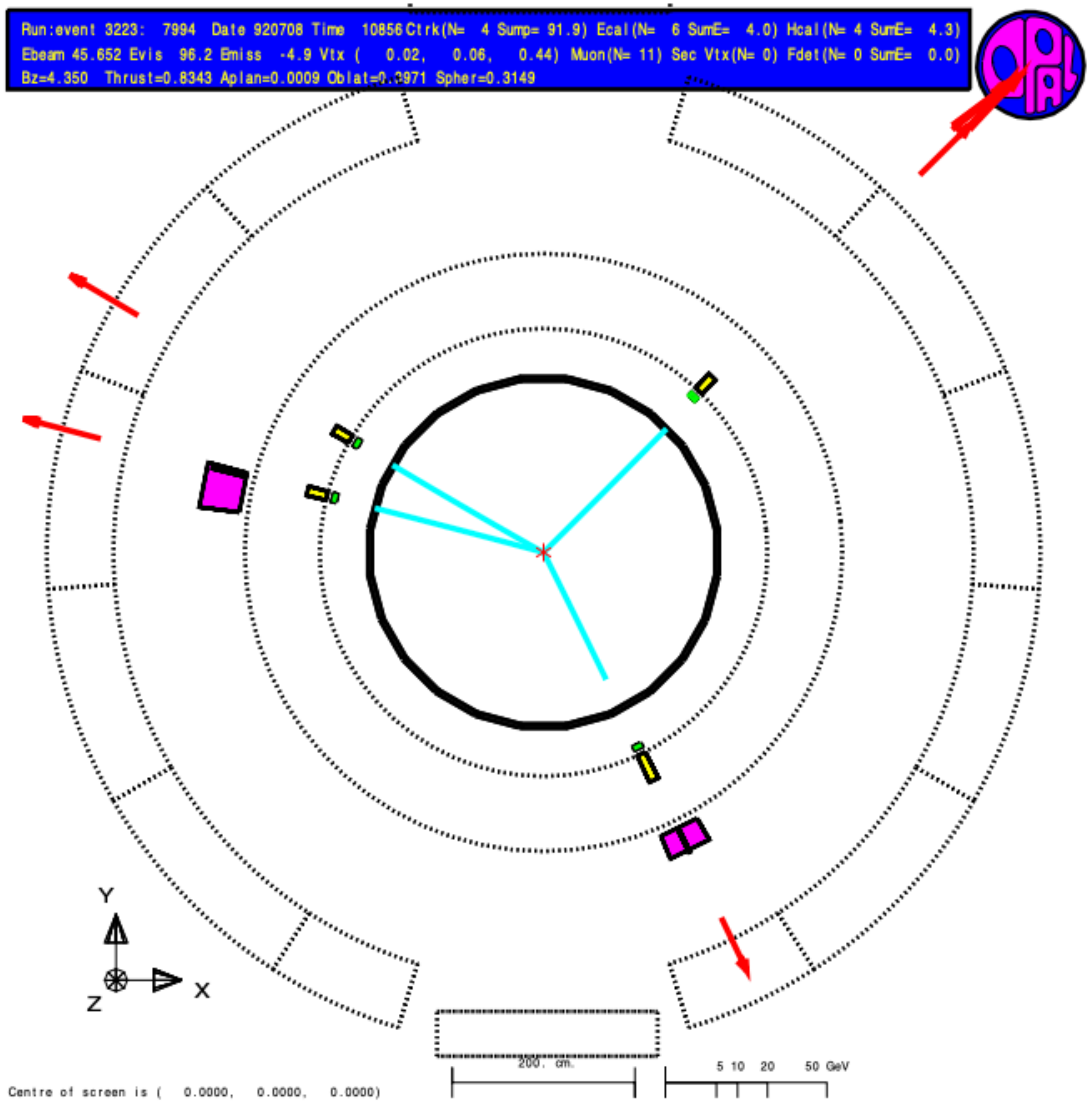
$e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$

Interactive: Event displays 2

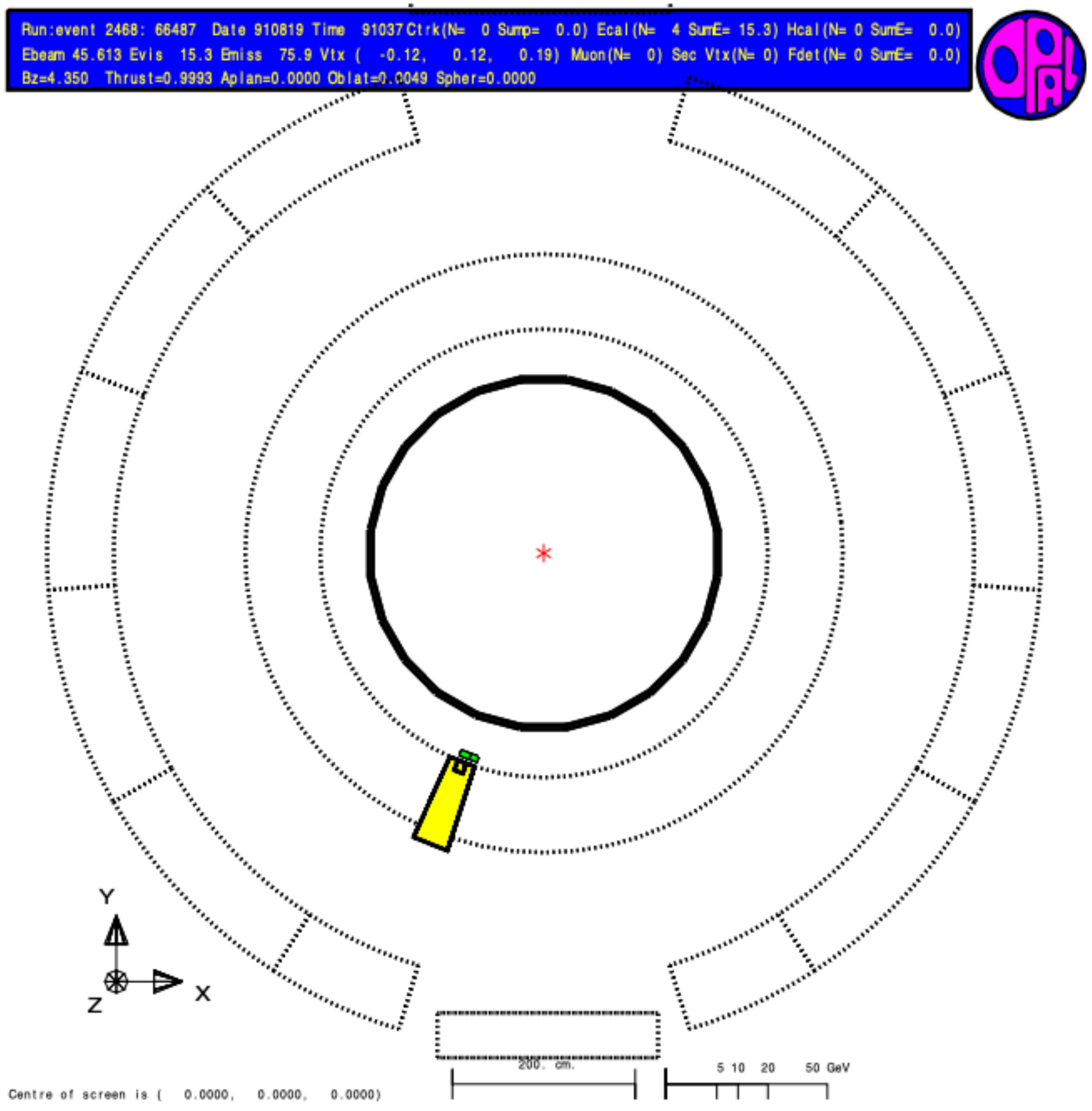




Solution: Event displays 2

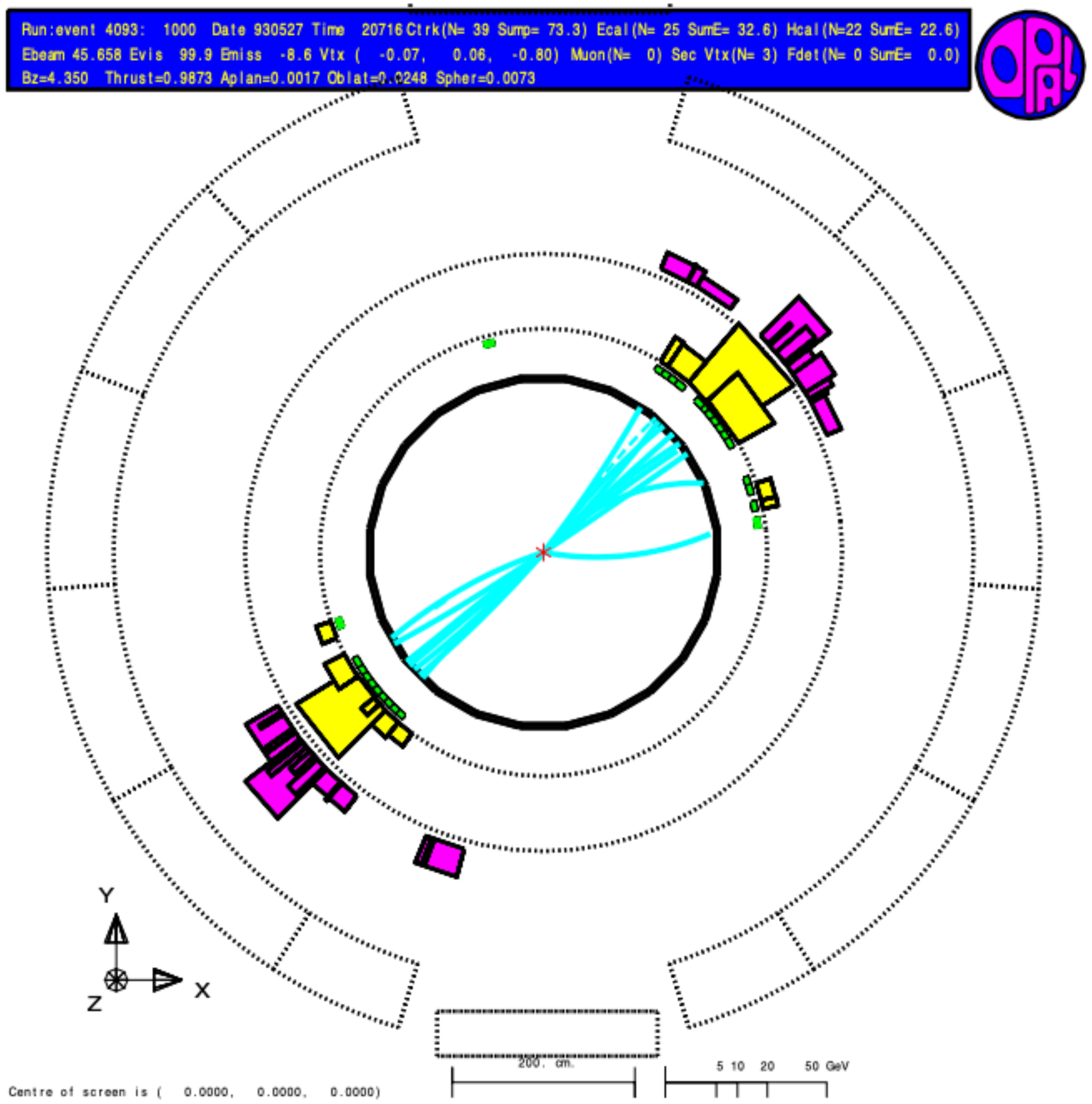


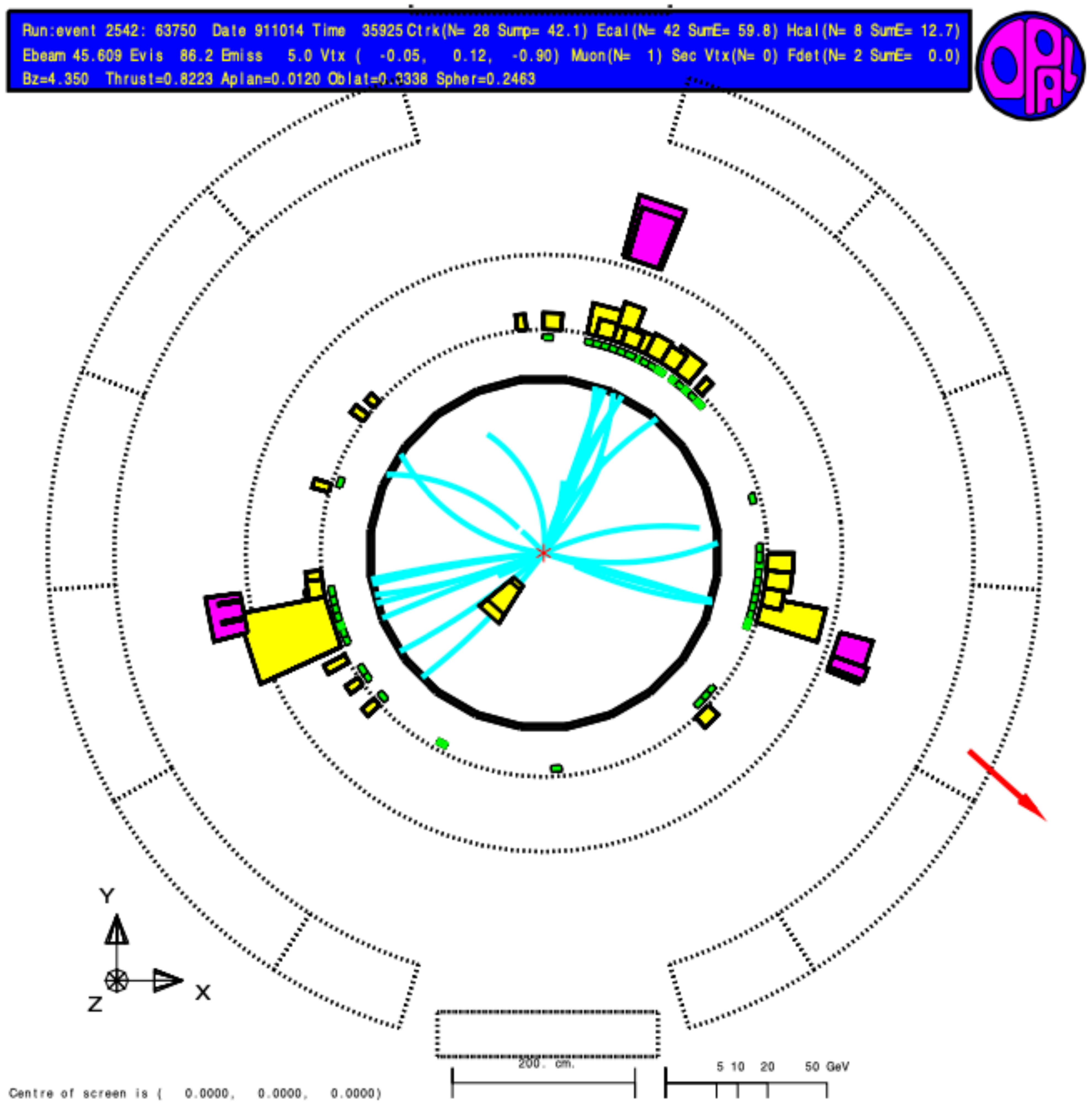
$e^+e^- \to Z \to \mu^+\mu^-\mu^+\mu^-$



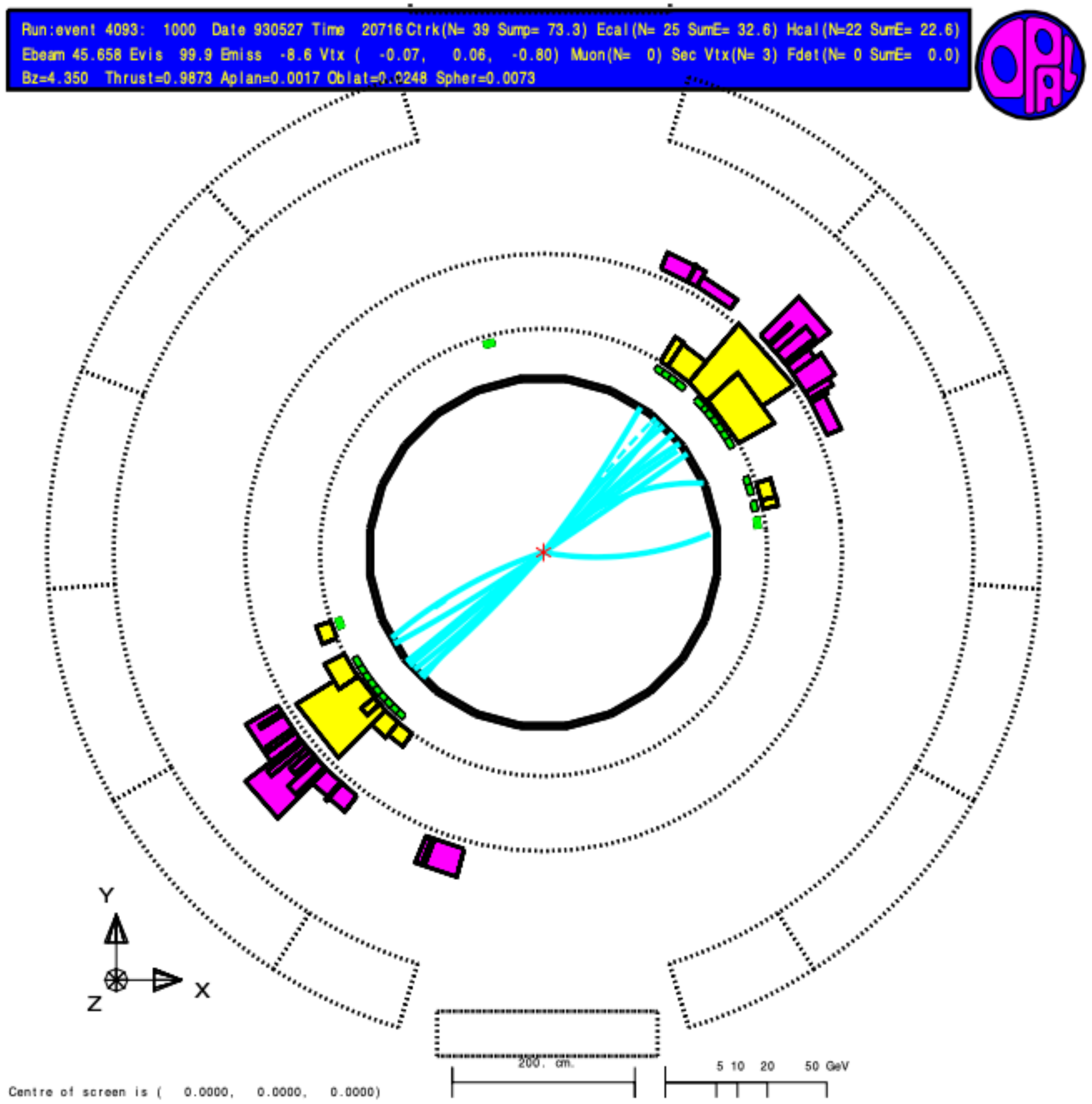
$e^+e^- \to Z\gamma \to \nu\nu\gamma$

Interactive: Event displays 3

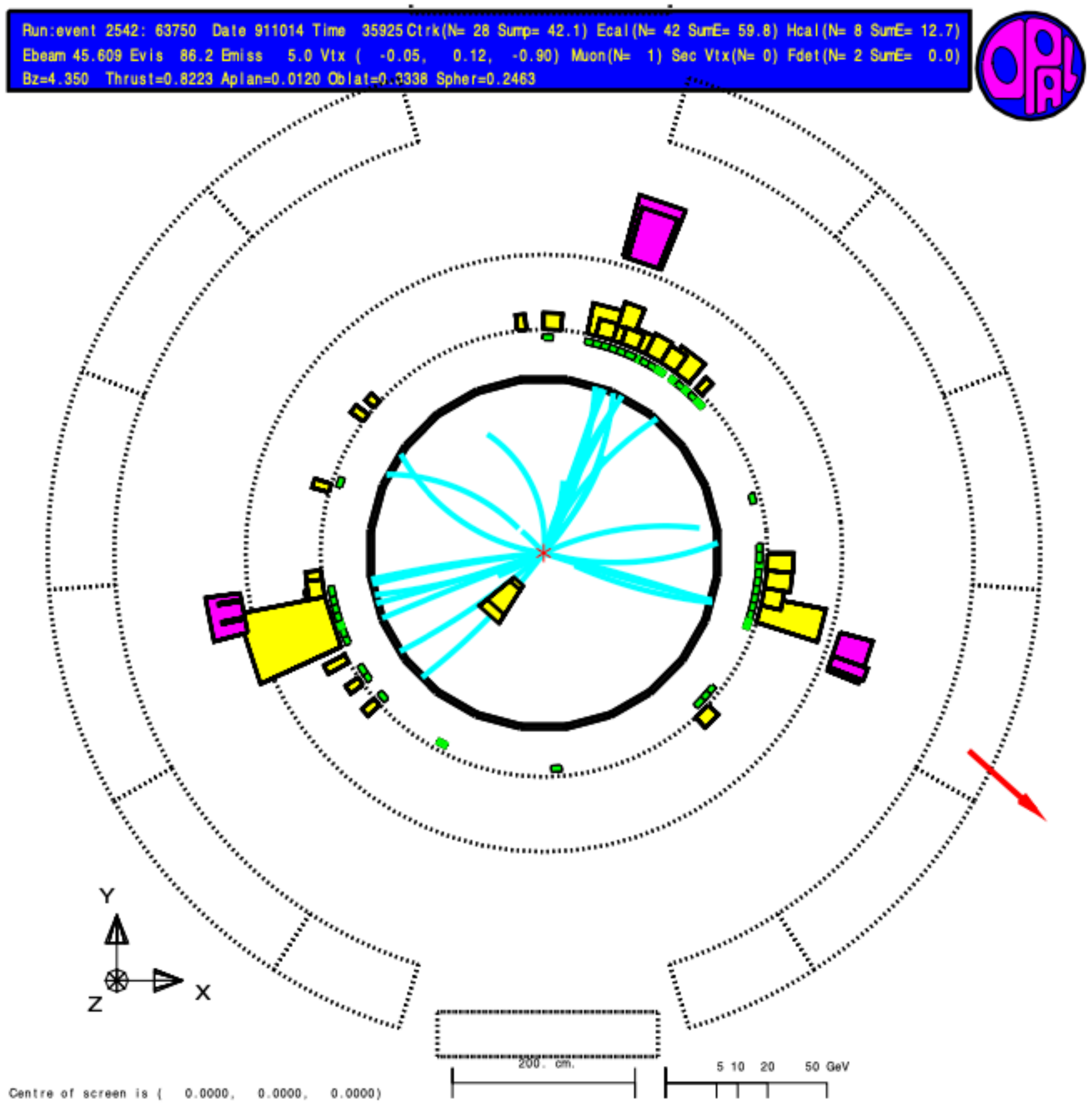




Solution: Event displays 3

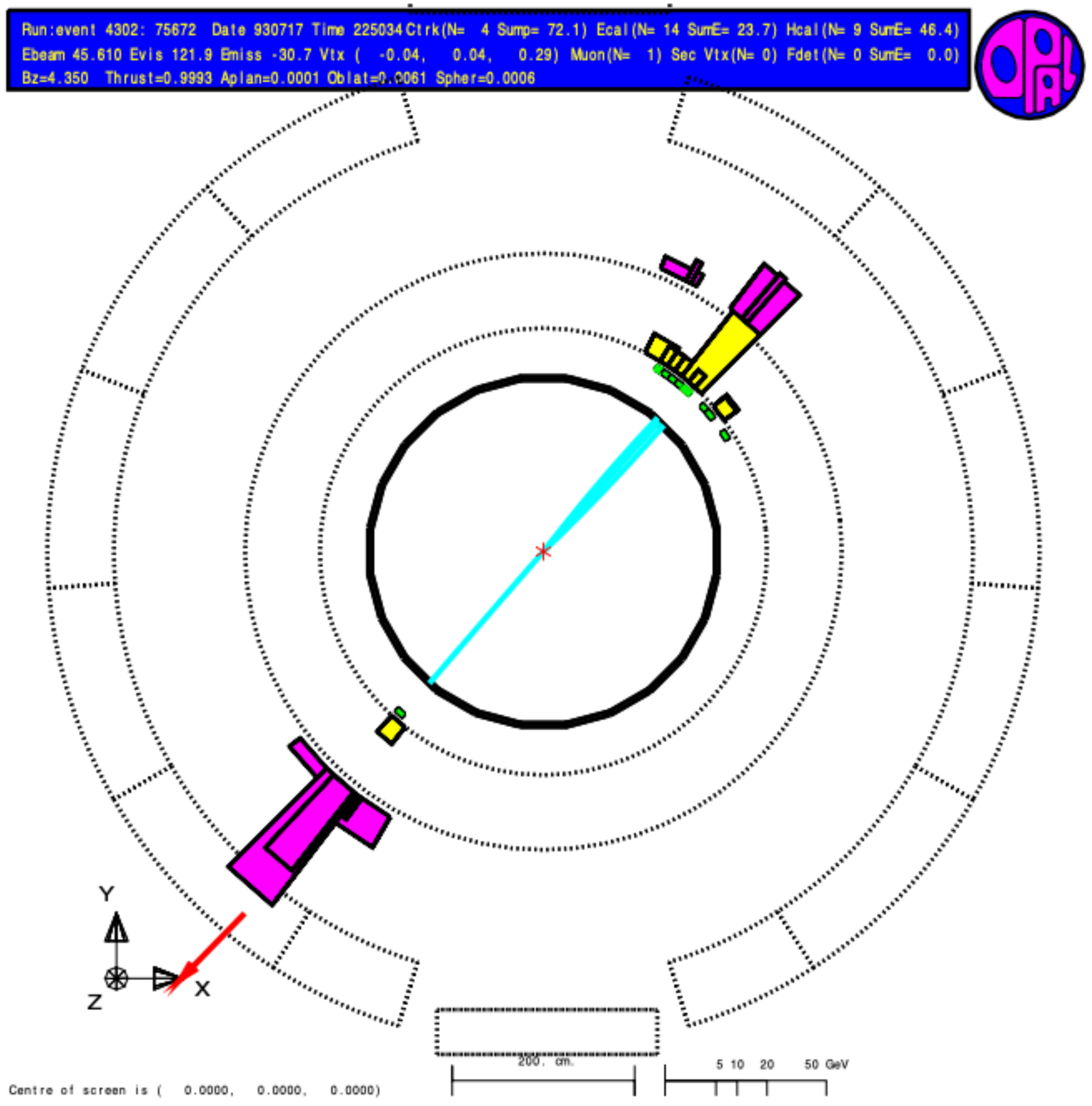


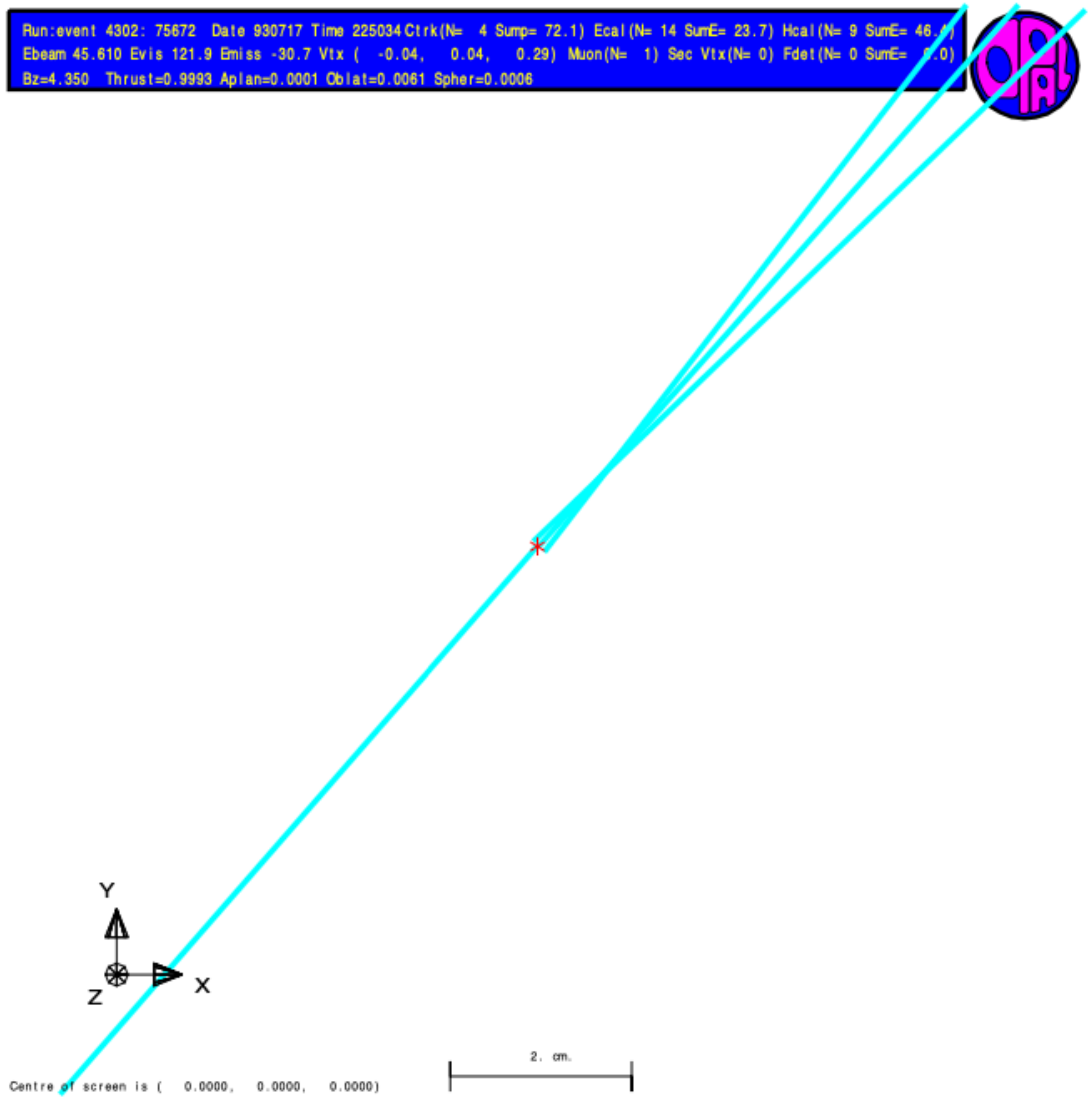
$e^+e^- \to Z \to \bar{q}q$



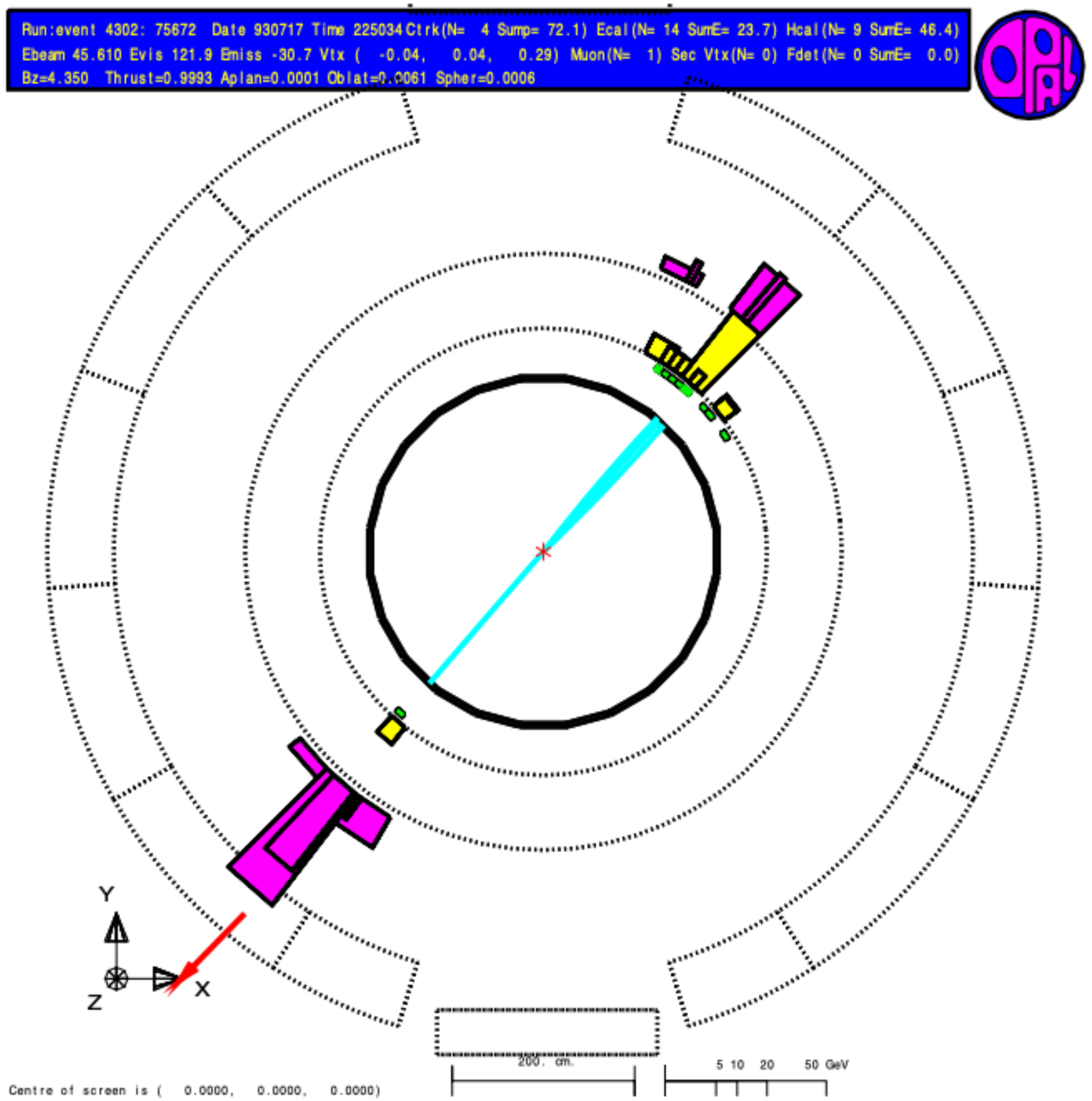
$e^+e^- \to Z \to \bar{q}qg$

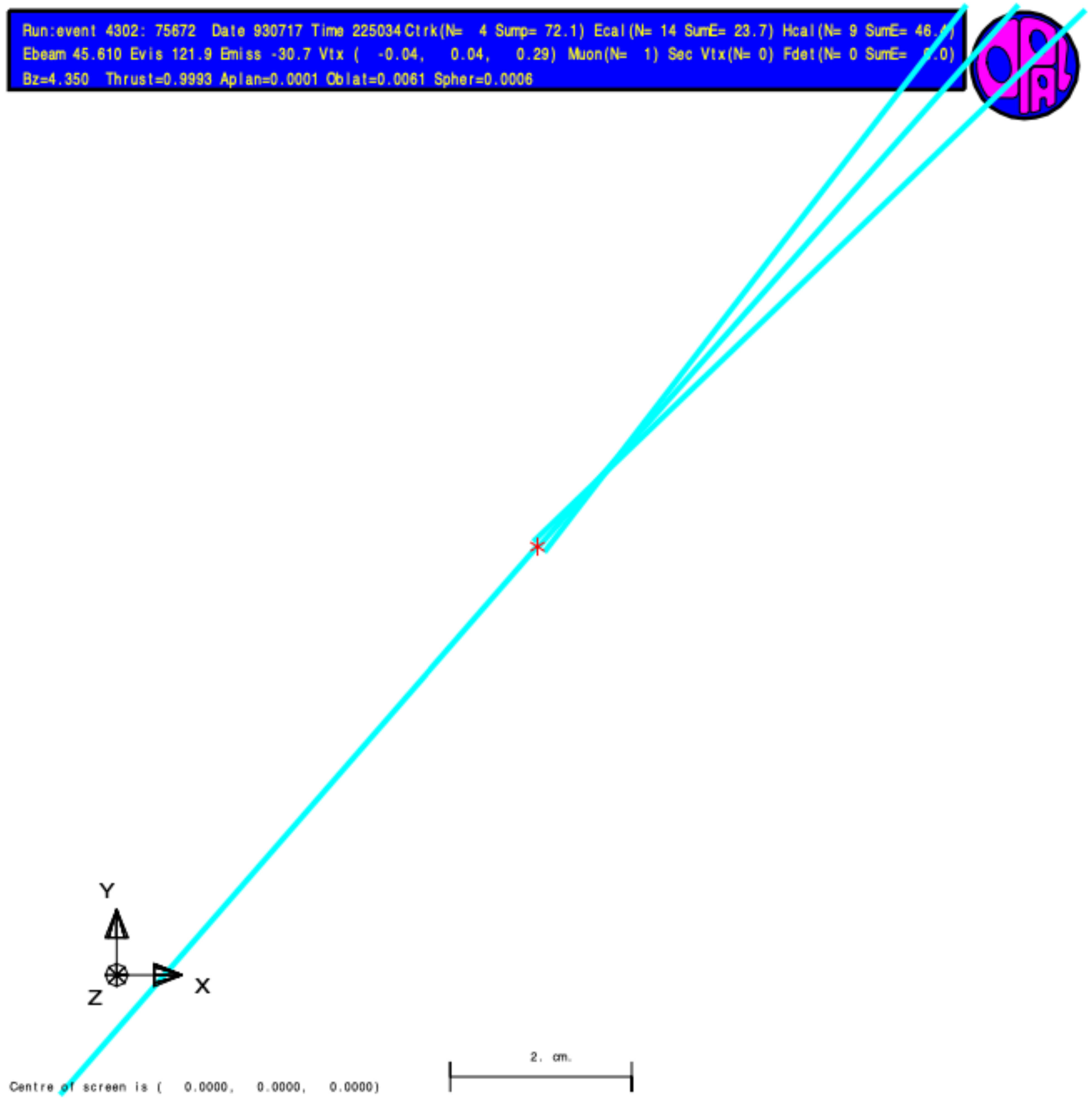
Interactive: Event displays 4





Solution: Event displays 4





$$e^+e^- \to Z \to \tau^+\tau^-$$

Neutrino-electron scattering

- Experiments at CERN SPS neutrino beam (1970s and 1980s)

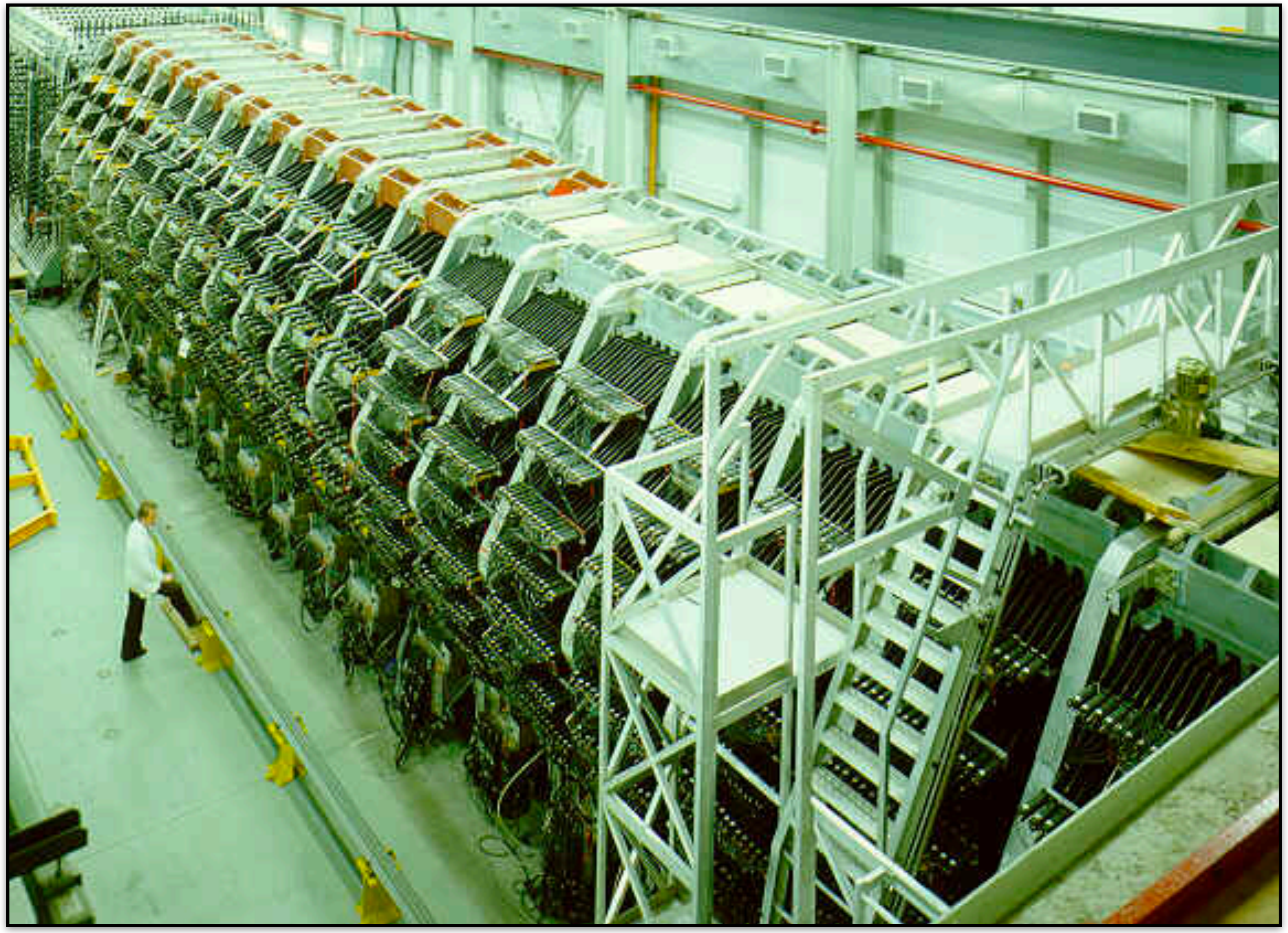
- Example: CDHS

  - CERN-Dortmund- Heidelberg-Saclay (Warsaw) collaboration

  - Iron-scintillator sampling calorimeter → hadron calorimeter and spectrometer at the same time

  - Calorimeter interleaved with drift chambers

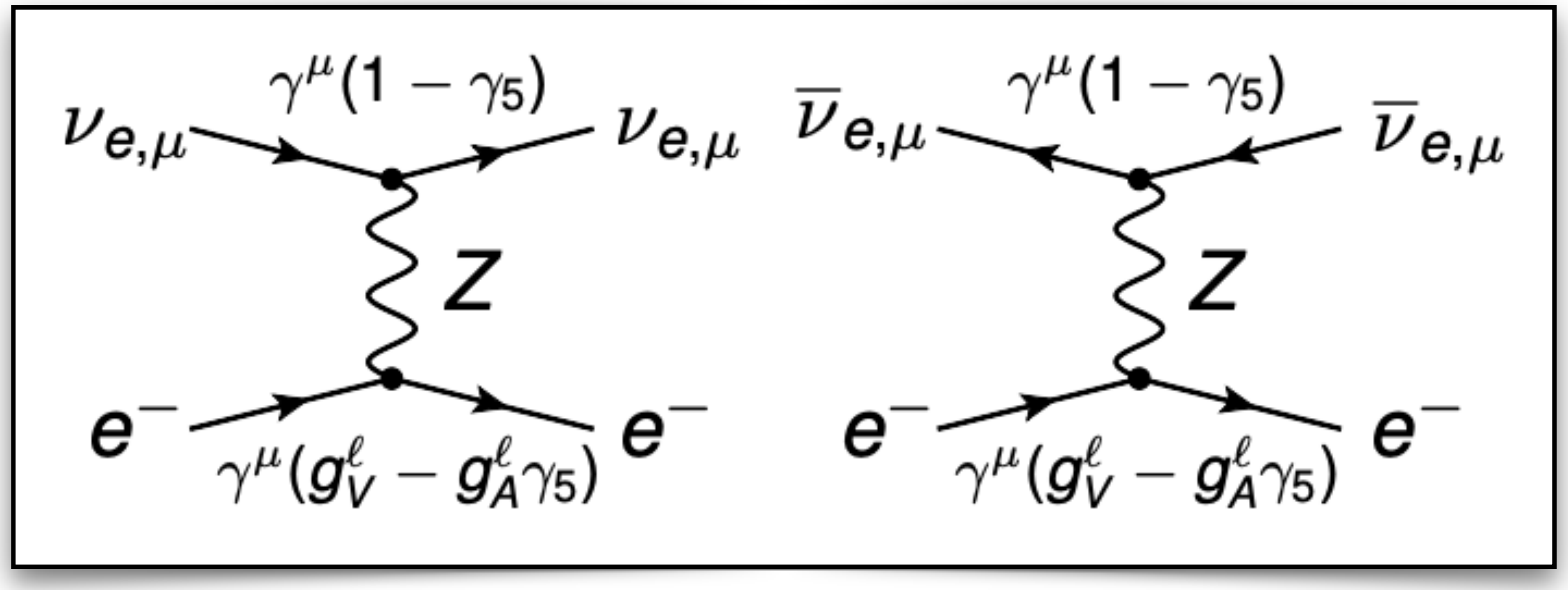
  - Later: Added 31m^3 hydrogen target in front of spectrometer to measure $\nu_\mu + H \to \mu^- + X$ in addition to $\nu_\mu + Fe \to \mu^- + X$



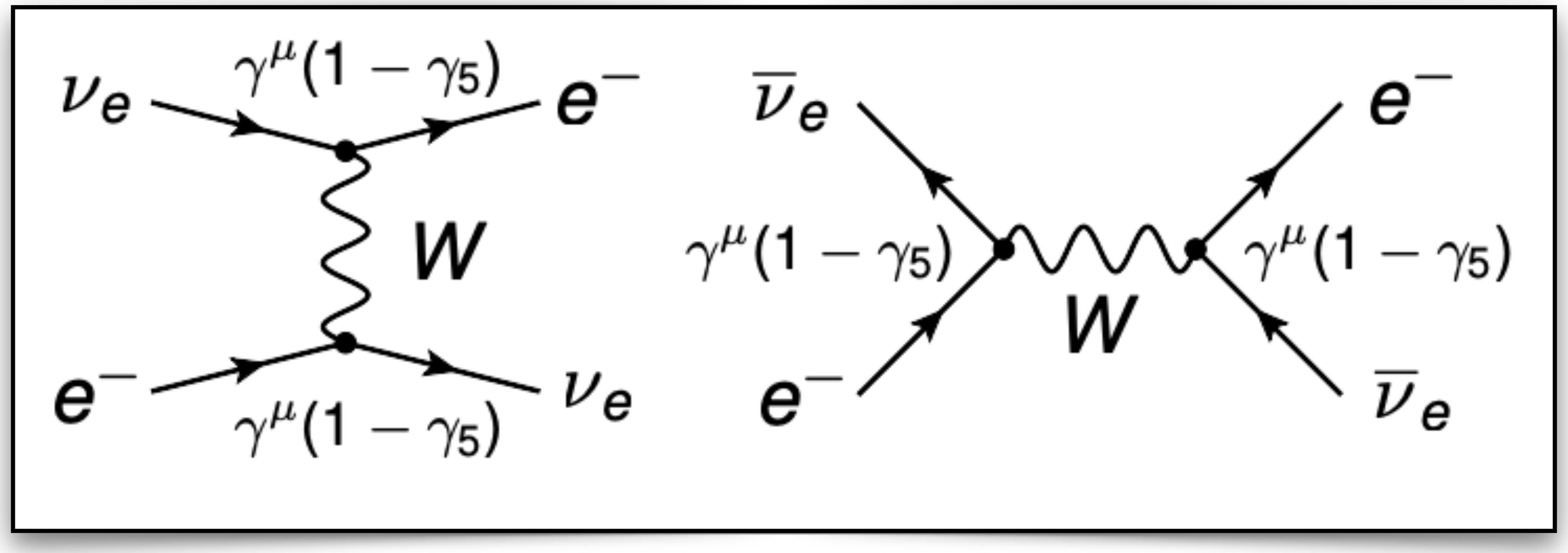
Source: https://knobloch.web.cern.ch/cdhs/cdhs1278.jpg

Neutrino-electron scattering

Neutral current (NC)



Charged current (CC)



- Difference between ν_e and ν_μ:
  - ν_μ only Z-exchange (NC)
  - ν_e on e: W-boson exchange (CC)
- Couplings:
  - Neutrinos pure V-A couplings ($g_V = g_A = 1$)
  - Electrons: $g_V \neq g_A \neq 1$

Neutrino-electron scattering

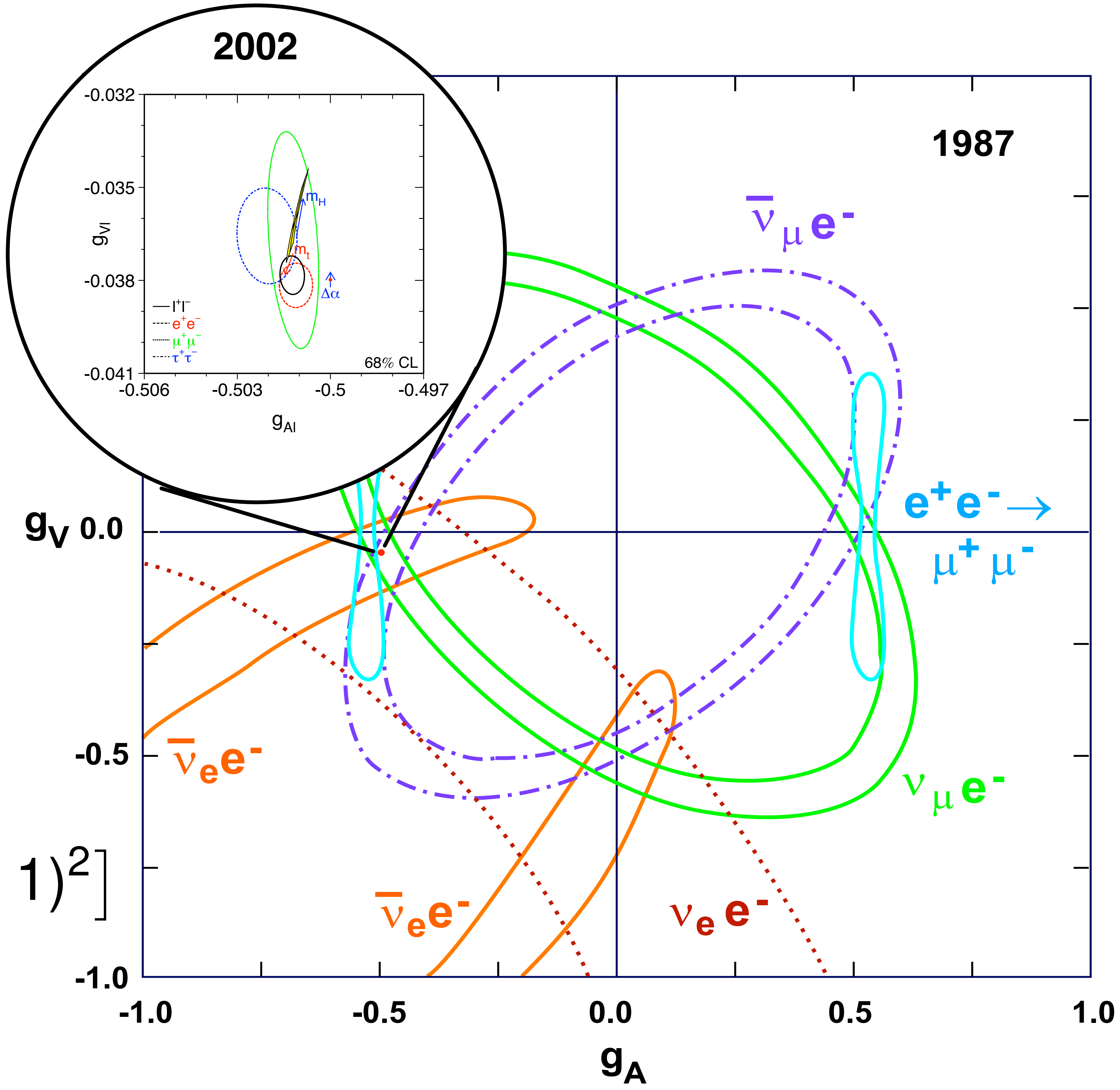
- Total cross sections:

$$\sigma(\nu_\mu e^- \to \nu_\mu e^-)$$
$$= \frac{G_F^2 s}{3\pi}\left[(g_V^\ell)^2 + g_V^\ell g_A^\ell + (g_A^\ell)^2\right]$$

$$\sigma(\overline{\nu}_\mu e^- \to \overline{\nu}_\mu e^-)$$
$$= \frac{G_F^2 s}{3\pi}\left[(g_V^\ell)^2 - g_V^\ell g_A^\ell + (g_A^\ell)^2\right]$$

$$\sigma(\nu_e e^- \to \nu_e e^-)$$
$$= \frac{G_F^2 s}{3\pi}\left[(g_V^\ell + 1)^2 + (g_V^\ell + 1)(g_A^\ell + 1) + (g_A^\ell + 1)^2\right]$$

$$\sigma(\overline{\nu}_e e^- \to \overline{\nu}_e e^-)$$
$$= \frac{G_F^2 s}{3\pi}\left[(g_V^\ell + 1)^2 - (g_V^\ell + 1)(g_A^\ell + 1) + (g_A^\ell + 1)^2\right]$$



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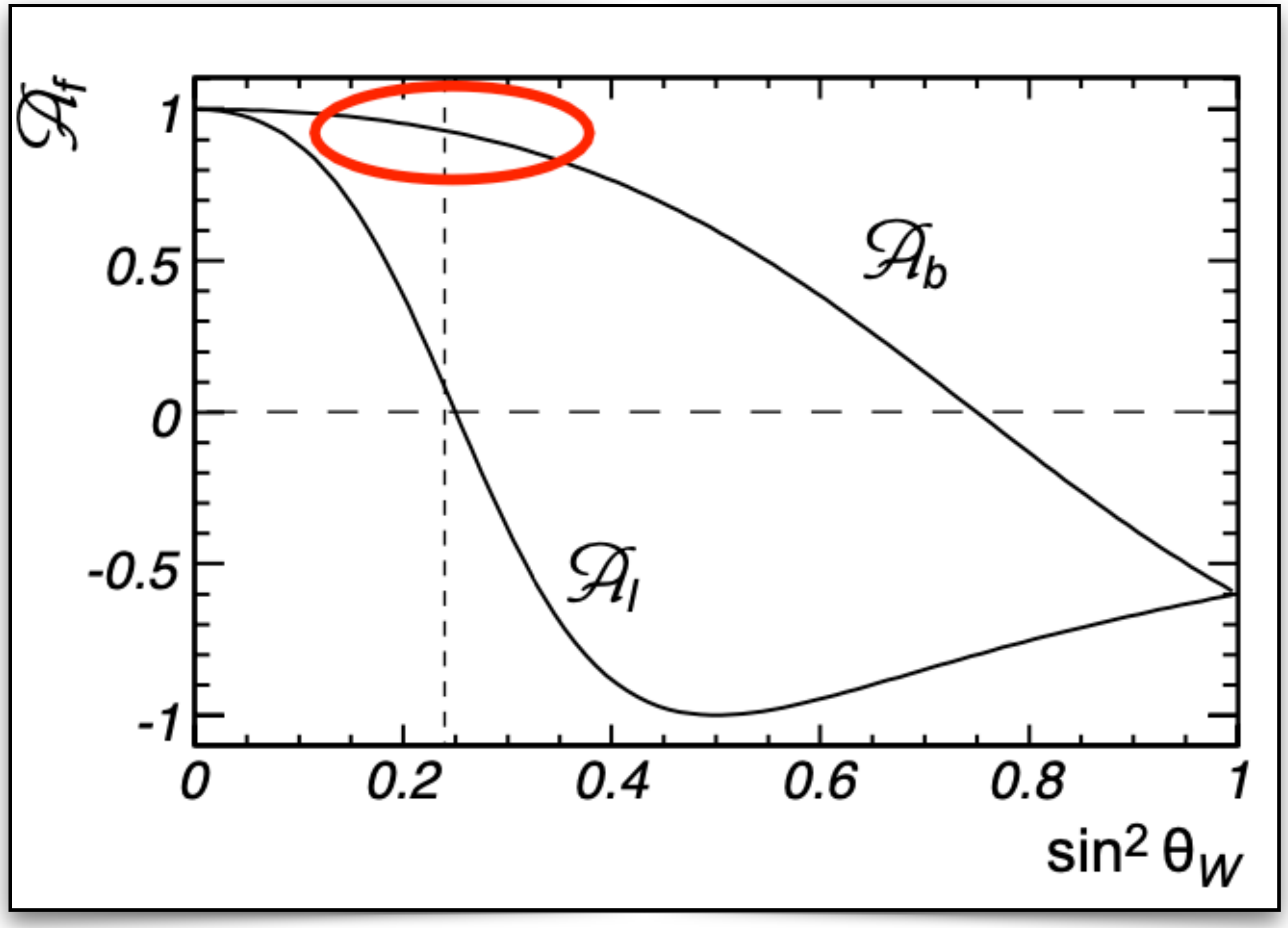
Weak mixing angle θ_W

- Given by electroweak theory:

$$\sin^2\theta^f_{W,eff} = \frac{I_{3,f}}{2Q_f}\left(1 - \frac{g_V^f}{g_A^f}\right) = 1 - \frac{m_W^2}{m_Z^2}$$

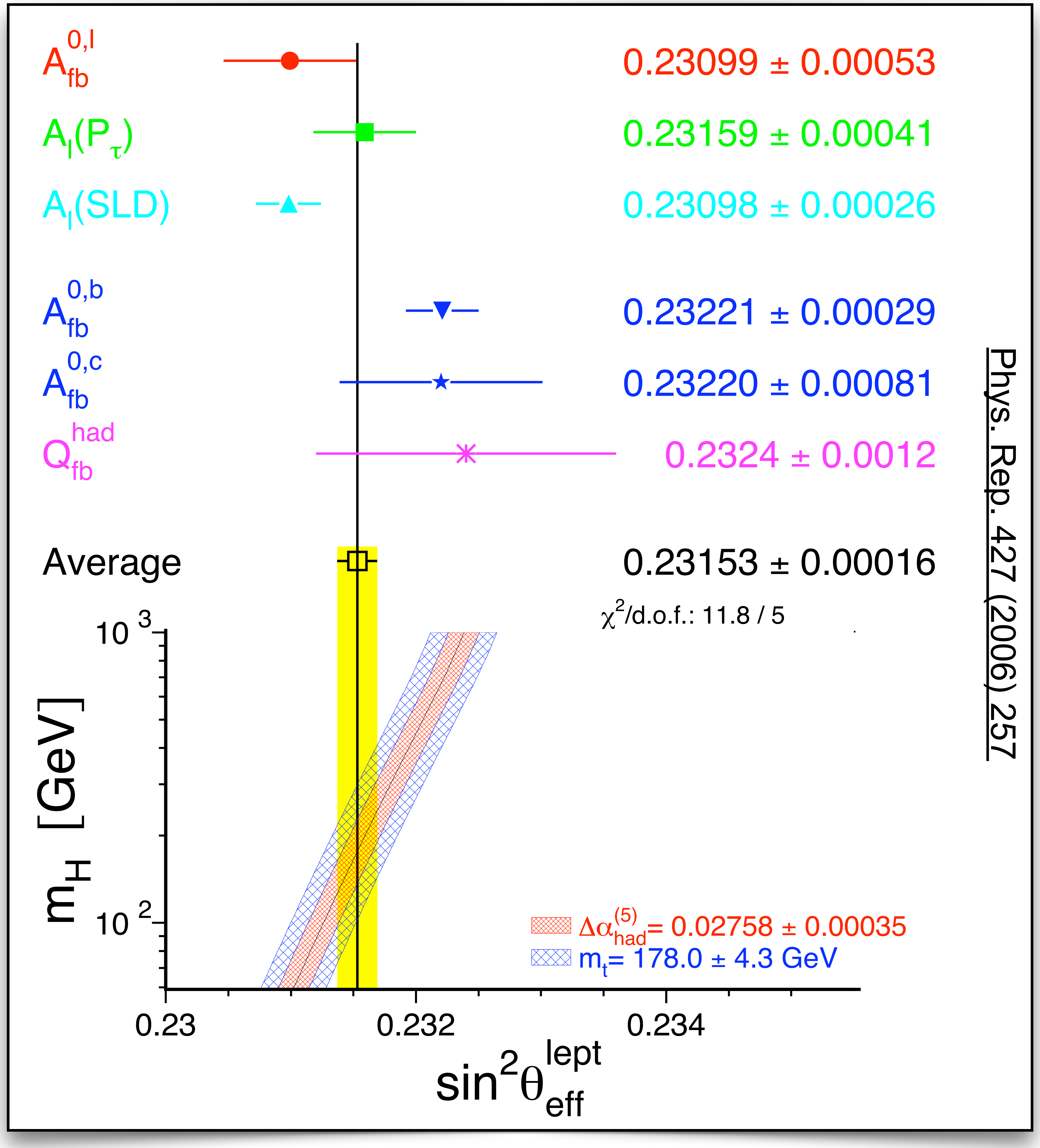
- At LEP and SLC, the highest precision comes from measurements of $A_{FB} = \frac{3}{4}\mathscr{A}_e\mathscr{A}_f$

with $\mathscr{A}_f = \dfrac{2g_V^f/g_A^f}{1+(g_V^f/g_A^f)^2}$



Summary: Weak mixing angle θ_W

- Most precise single measurement from A_{FB}^b
- Rather large discrepancy between leptonic and hadronic final states (~3σ)
- Yet another large discrepancy between NuTev (neutrino-nucleon scattering) (~3σ)



How many free parameters does the standard model have?

- 12 fermion masses (if $m_\nu \neq 0$, else 9 fermion masses) or 12 (9) Yukawa couplings
  - As of 2022, the only not measured particle of the Standard Model is the anti-tau-neutrino!
- 3 couplings constants (g, g’, g_s)
- m_H
- m_Z or m_W
- 3 rotation angles and one CP violating phase (CKM matrix) (→ later)
- 3 rotation angles and one CP violating phase (PMNS matrix) (if $m_\nu \neq 0$) (→ later)
- 1 CP violating angle θ (→ later)

→ **26 free parameters (19 if $m_\nu = 0$)**, the representation can be chosen differently

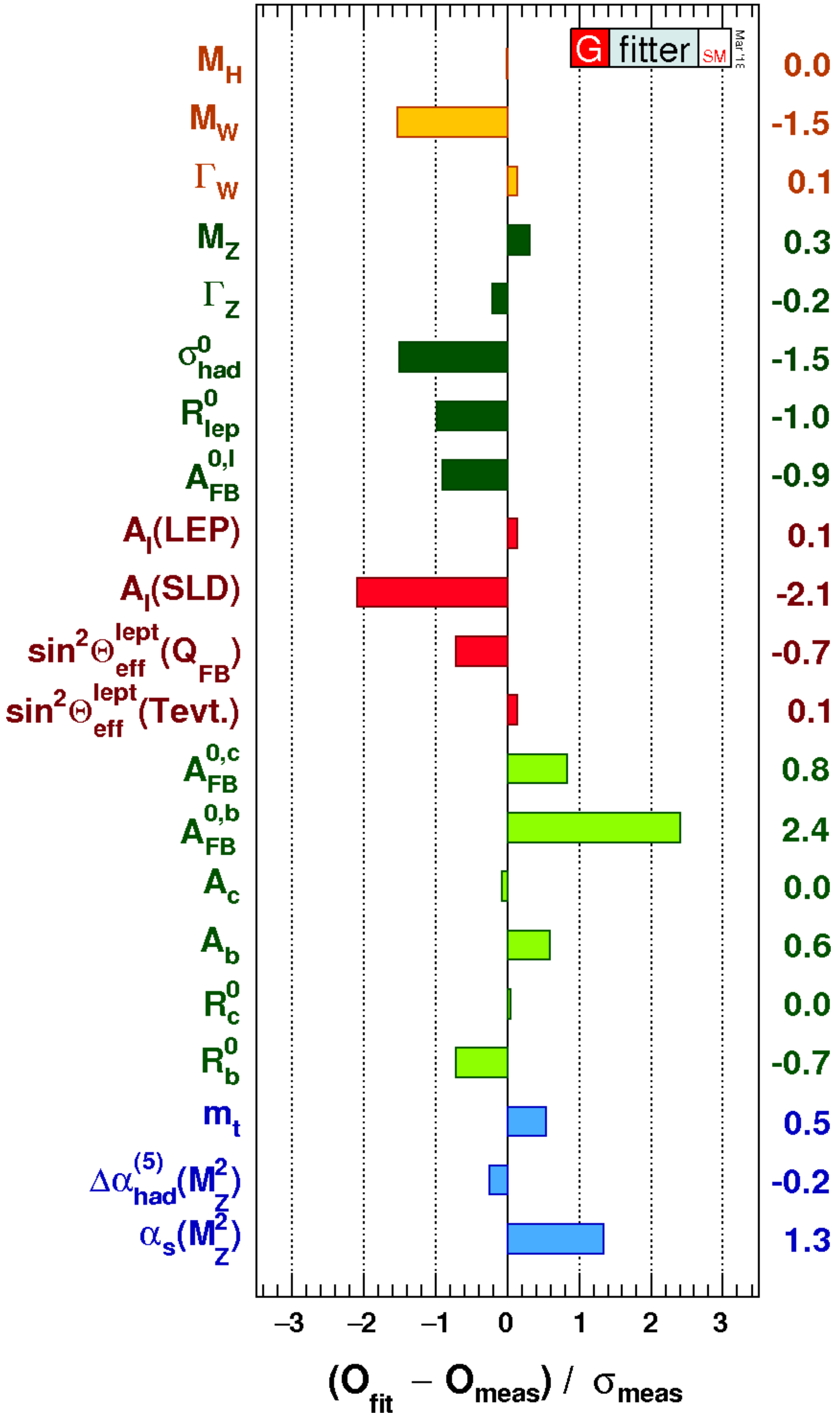
Having determined this finite number of parameters from experiments, any Standard Model observable can in principle be predicted to any desired accuracy!

Global fits

- The electroweak part of the Standard Model can be checked by performing a global fit, including all results from LEP, SLC, Tevatron, and the LHC

- Pulls: deviation of measurement from global fit, divided by uncertainty of measurement

- The SM is very consistent!



What questions do you have?