Karlsruhe Institute of Technology

Particle Physics 1

Lecture 16: Deep inelastic scattering

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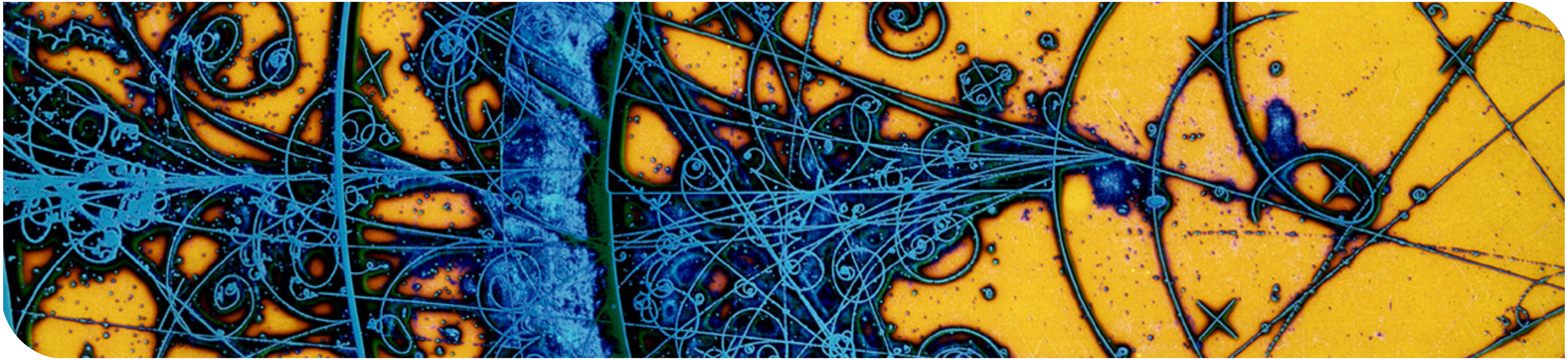
Wintersemester 2023/2024

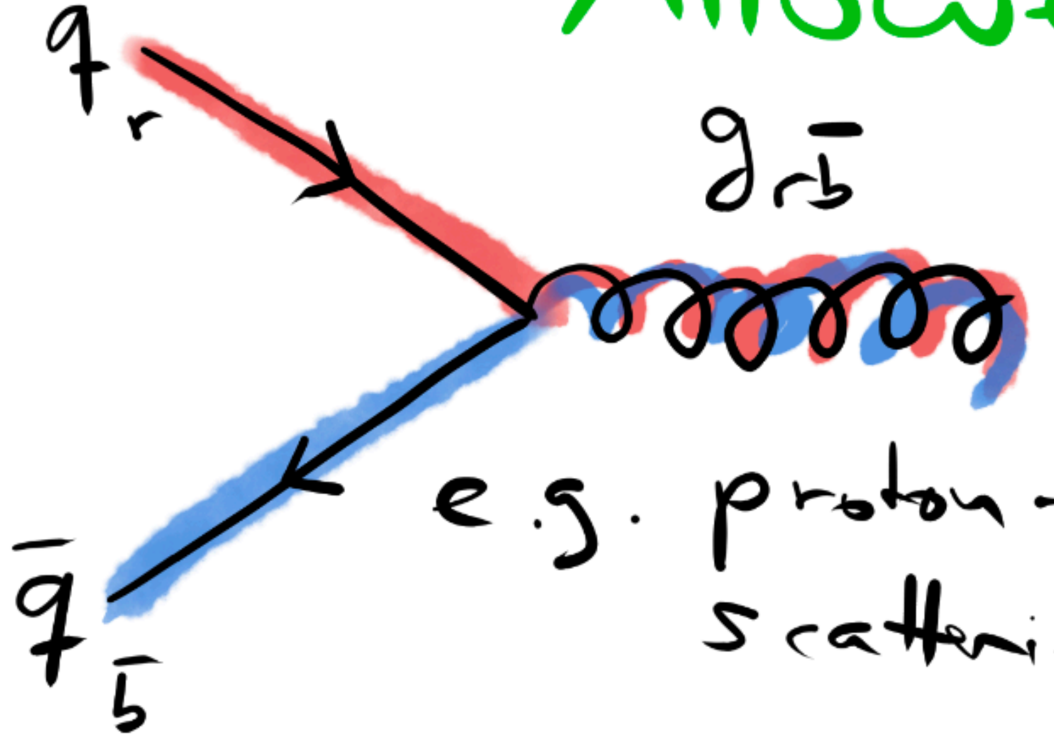


Credit: CERN

Questions from past lectures

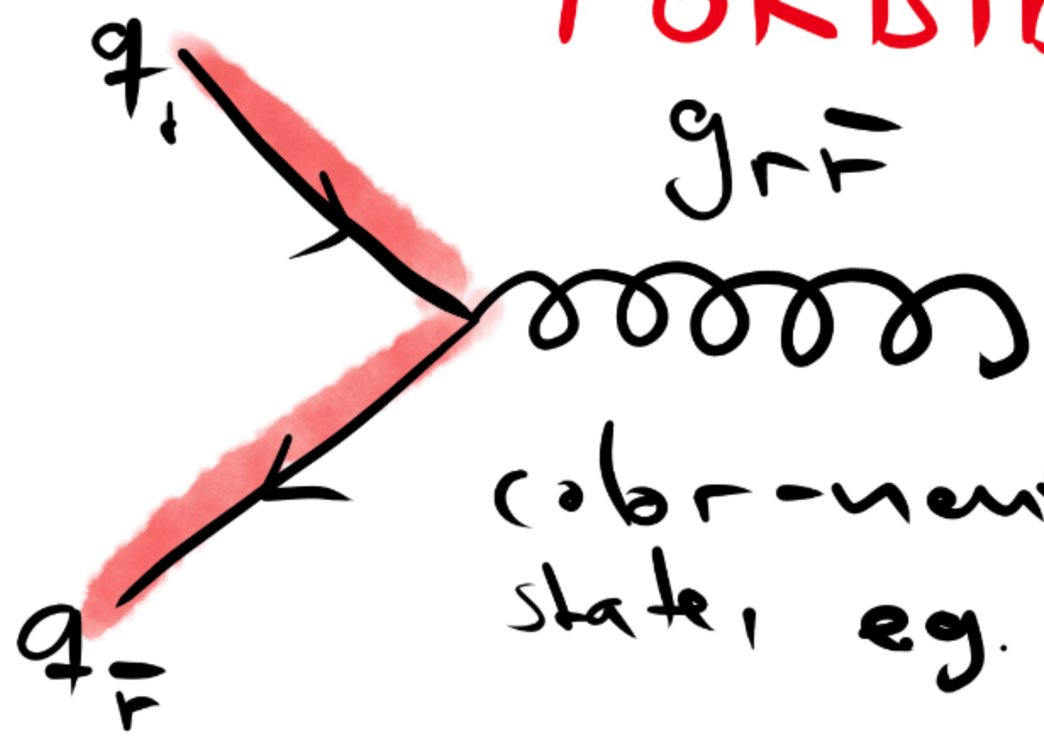
How can two quarks annihilate into a gluon?

Allowed ✓



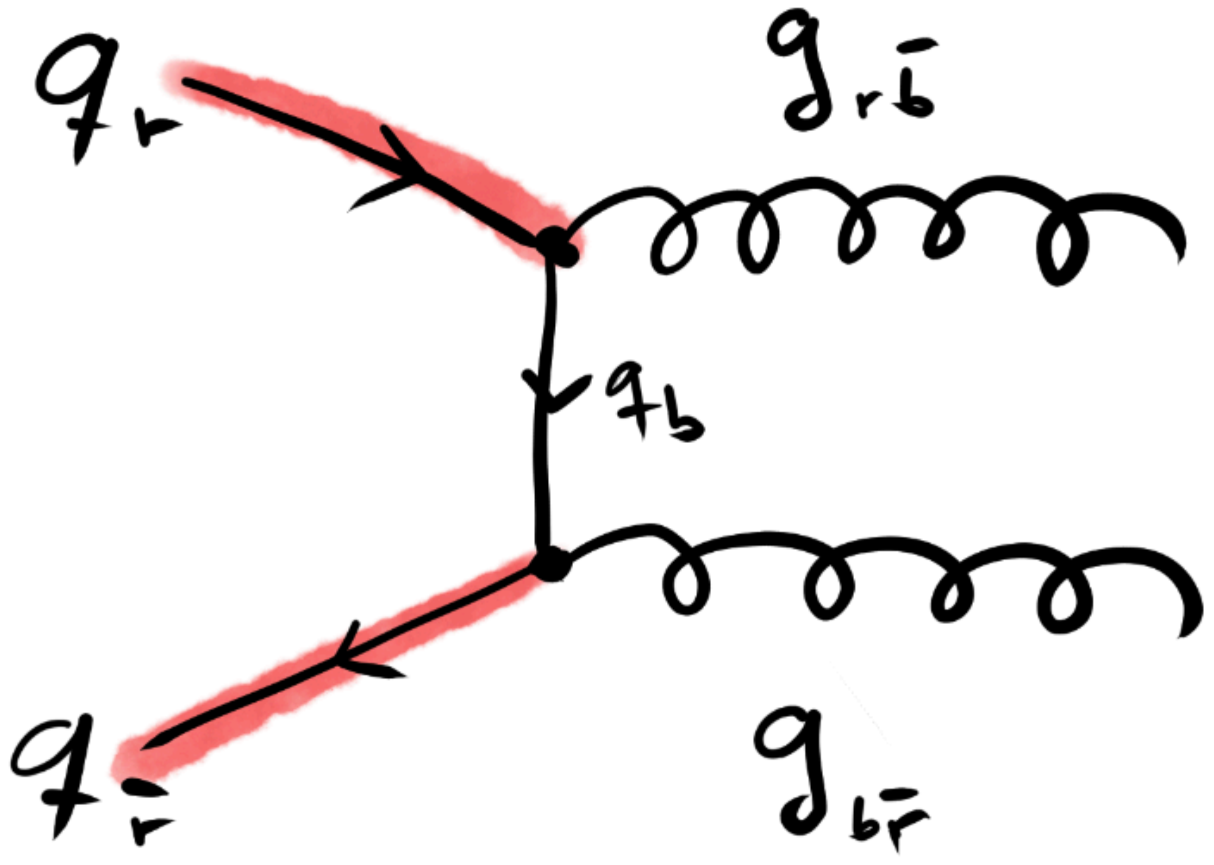
e.g. proton-antiproton scattering at Tevatron

FORBIDDEN!!!



color-neutral bound-state, e.g. ϕ, $\Upsilon(4s)$, ...

Color*-allowed ✓



Learning goals

- Towards W, top quark, and Higgs boson measurements: Understanding the proton with very high precision
- Understand deep inelastic scattering variables
- Understand key experiments: H1 and ZEUS at HERA

How many free parameters does the standard model have?

- 12 fermion masses (if $m_\nu \neq 0$) or 12 (if $m_\nu \neq 0$) Yukawa couplings
  - As of 2022, the only not measured particle of the Standard Model is the anti-tau-neutrino!
- 3 couplings constants (g, g', g_s)
- m_H
- m_Z or m_W
- **3 rotation angles and one CP violating phase (CKM matrix) ✓**
- 3 rotation angles and one CP violating phase (PMNS matrix) (if $m_\nu \neq 0$) (→ later)
- **1 CP violating angle θ ✓**

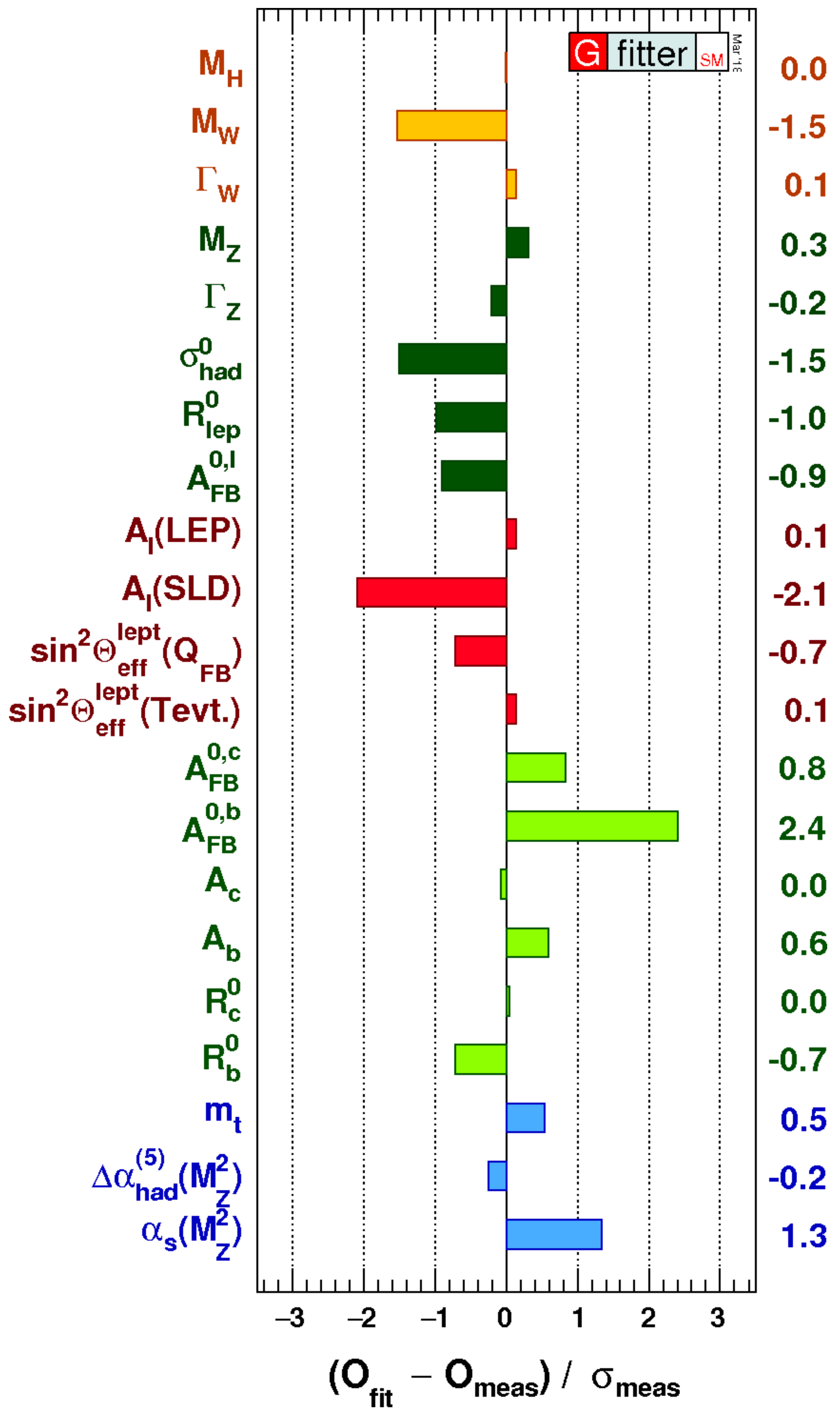
→ **26 free parameters (19 if $m_\nu = 0$)**, the representation can be chosen differently

Having determined this finite number of parameters from experiment, any Standard Model observable can in principle be predicted to any desired accuracy!

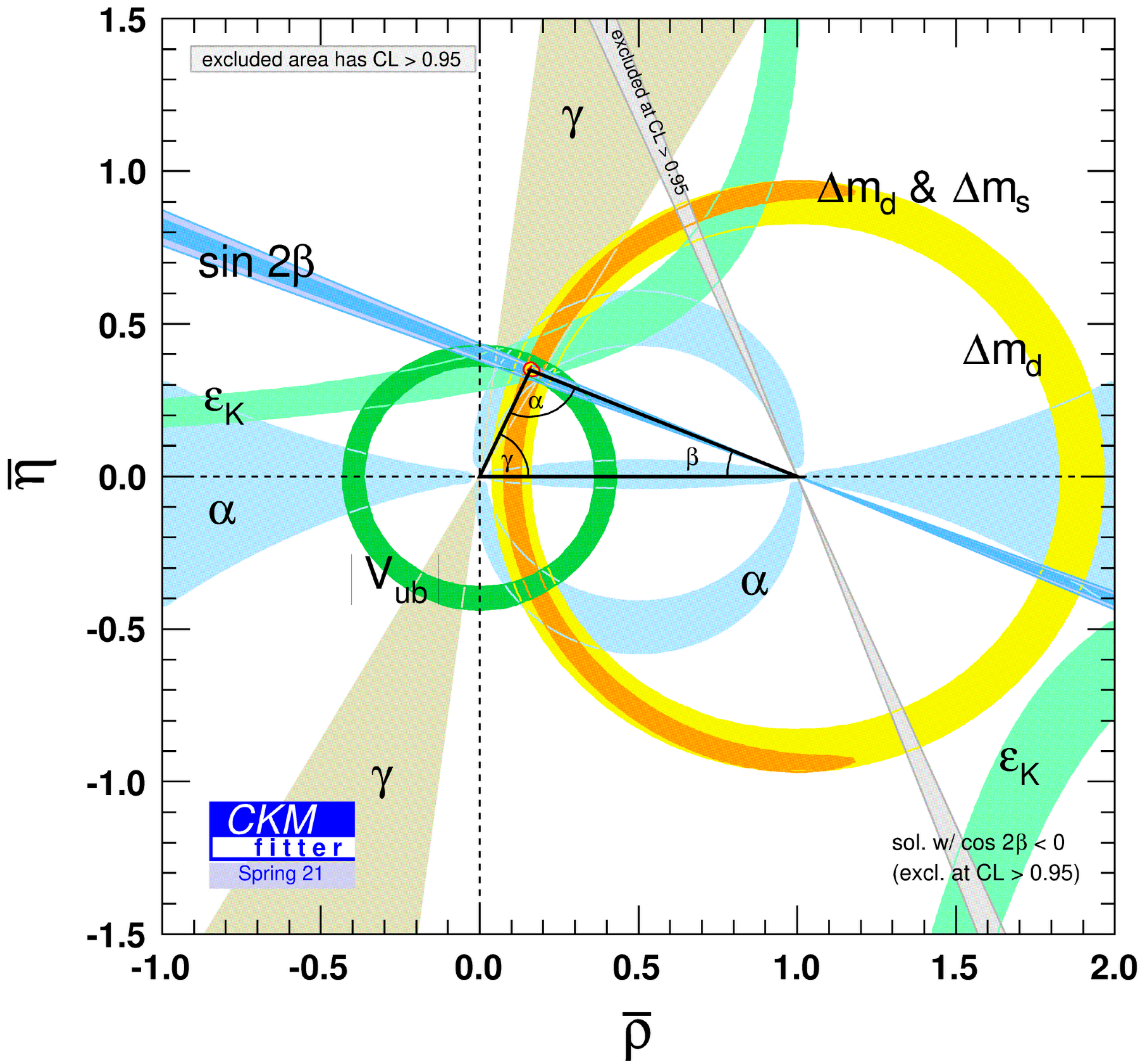
How many free parameters does the standard model have?

- Covered in TP1 so far:
  - 3 couplings constants (g, g', g_s) or ($\alpha_{QED}(0)$, $\alpha_S(m_Z)$, G_F) and running of couplings
  - m_Z
  - 3 rotation angles and one CP violating phase (CKM matrix)
  - CP violating phase θ in QCD (strongly limited by not observing a neutron EDM)
- Missing so far:
  - Higgs mass
  - Precision W mass
  - Precision top quark mass
  - Neutrino masses
  - Neutrino 3 rotation angles and one CP violating phase (PMNS matrix)

Overconstraining the Standard Model



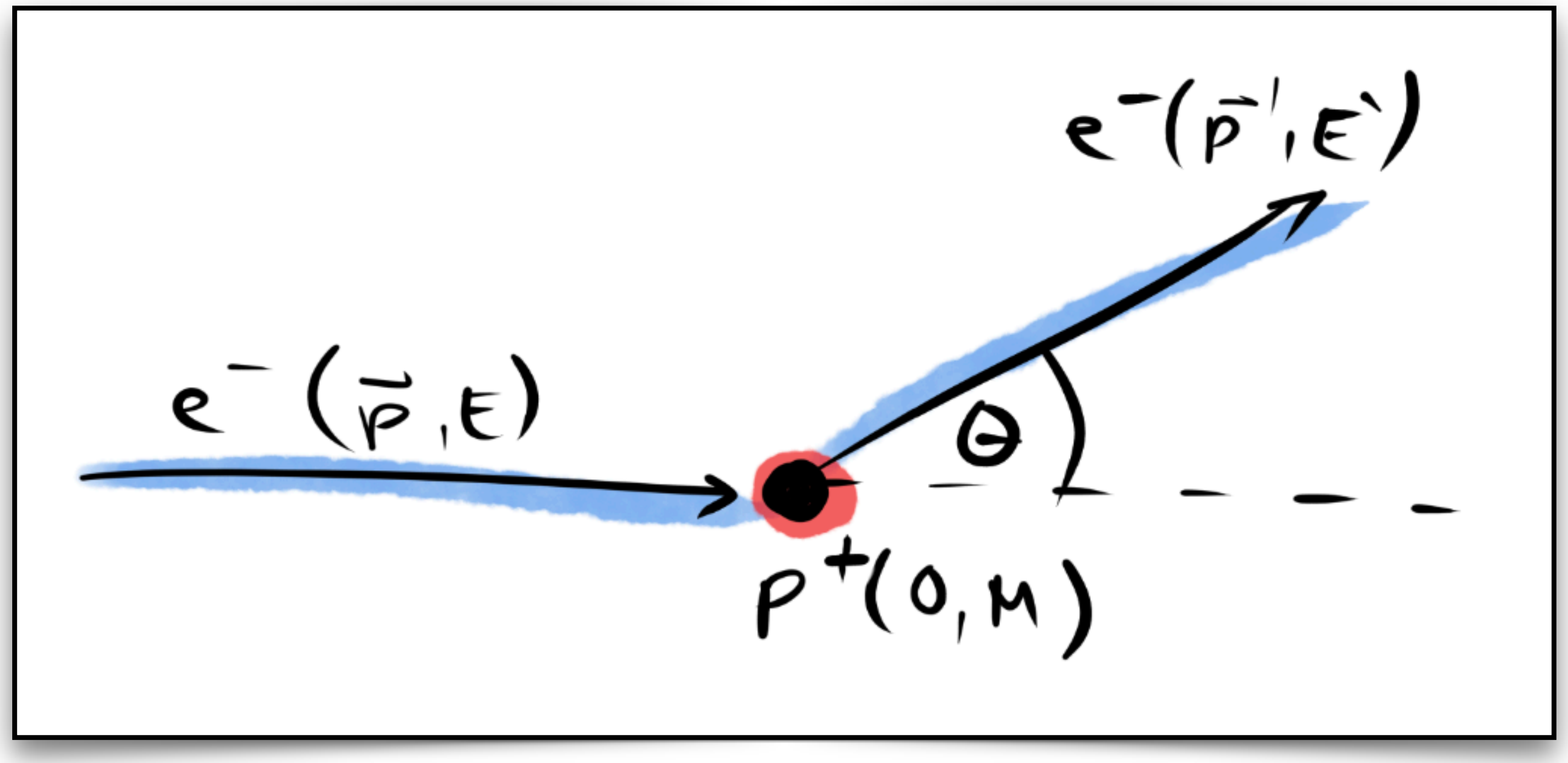
http://project-gfitter.web.cern.ch/



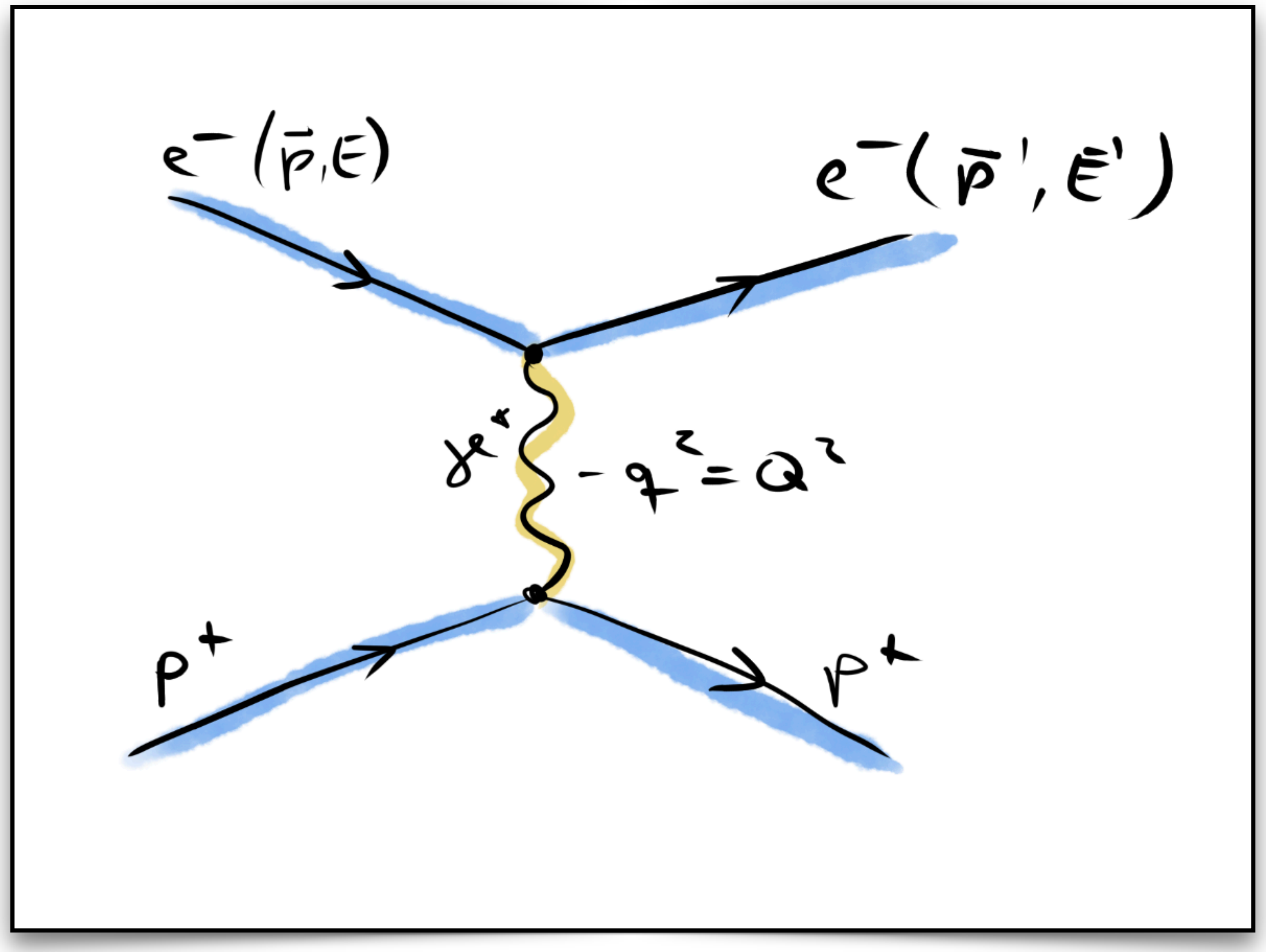
http://ckmfitter.in2p3.fr/www/results/plots_spring21/png/rhoeta_large.png

Electron-Proton Scattering: Classical

- Classical elastic point-like particle with charge z scattering off a point-like particle with charge Z, no momentum transfer ($E = E'$): **Rutherford scattering**



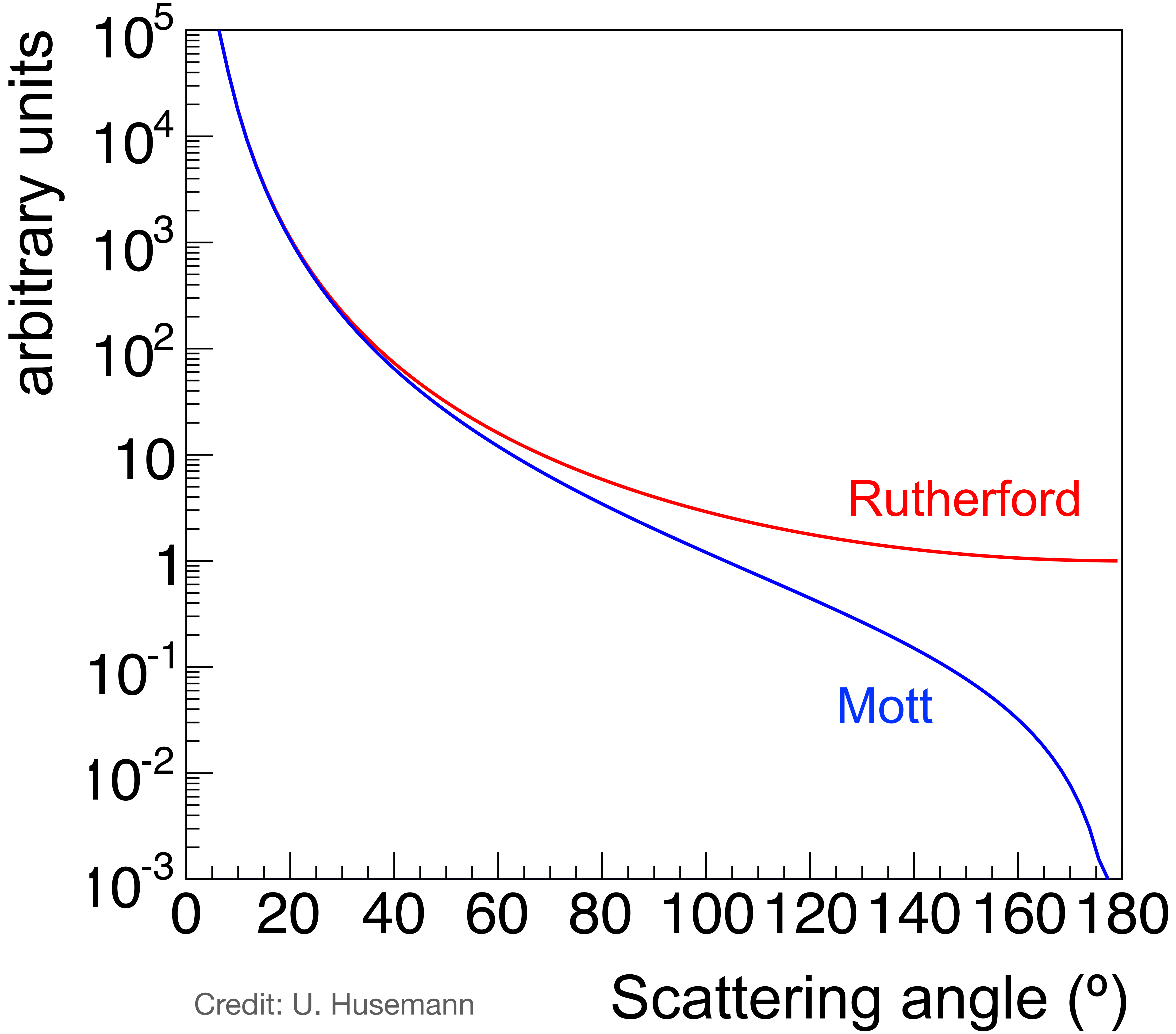
- $$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} = \left(\frac{zZ\alpha}{4E}\right)^2 \frac{1}{\sin^4 \theta/2}$$



Electron-Proton Scattering: Relativistic

- Relativistic elastic point-like spin-1/2 particle with charge z scattering off a point-like particle with charge Z, with momentum transfer (nuclear recoil) $E = E'$: **Mott scattering**
- $$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} \frac{E'}{E}(1 - \beta^2 \sin^2 \theta/2)$$
- Back-scattering (θ=180°) requires spin flip: strongly suppressed

Electron-Proton Scattering



Credit: U. Husemann

Electron-Proton Scattering: not point-like

- Two significant changes if scattering object is not point like: **Rosenbluth scattering**
  - Possible magnetic moment (allows spin-spin interactions)
  - Extended charge distribution
- $$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rosenbluth}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left(\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2) \tan^2\theta/2\right)$$

  with **electric and magnetic form-factors** $G_E(Q^2)$ and $G_M(Q^2)$ and $\tau = \frac{Q^2}{4M^2}$
- Scattering described by just one independent variable (e.g. Q^2)
- For small momentum transfers, the form factors are the Fourier-transforms of the charge and magnetic moment distributions

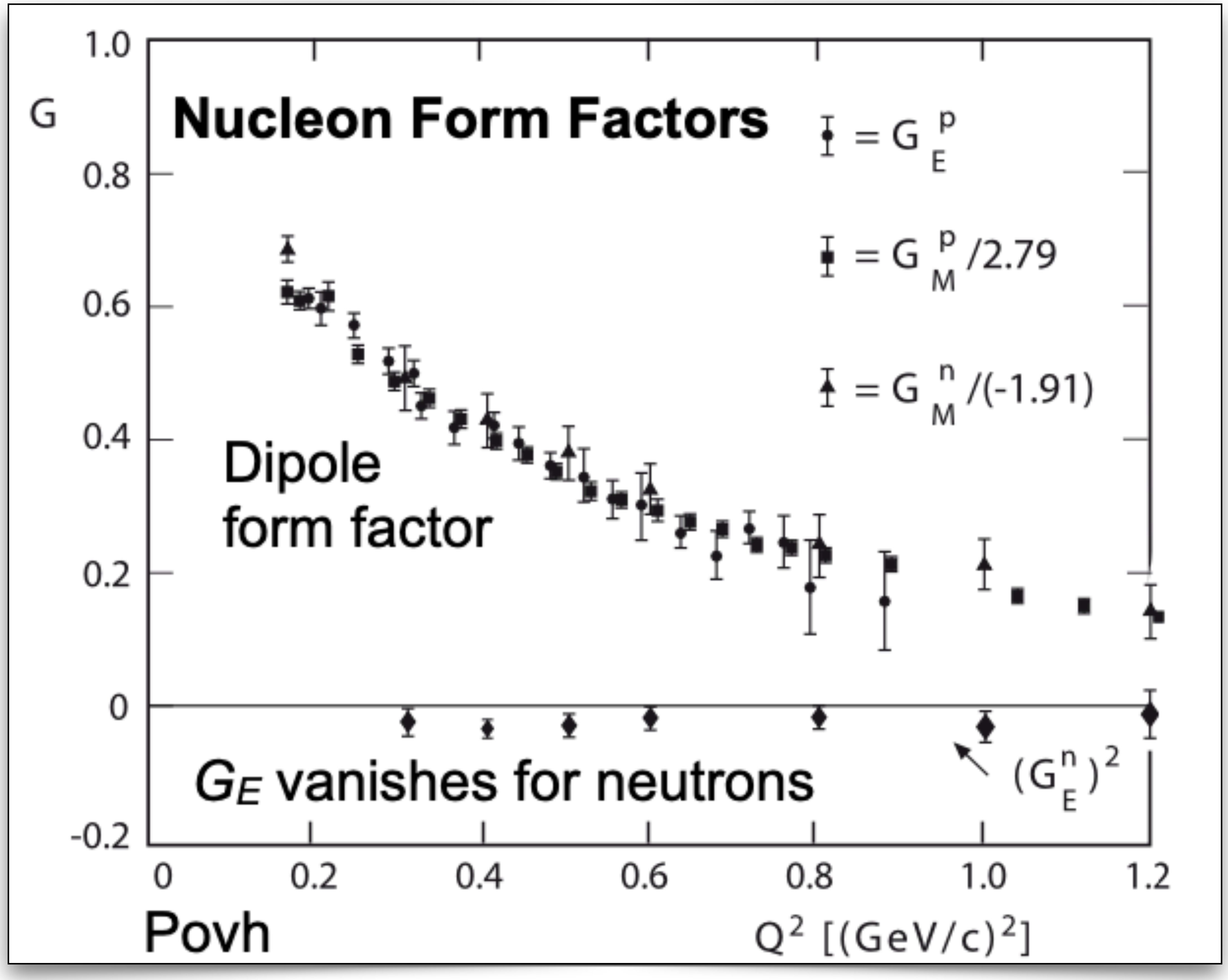
Electron-Proton Scattering: not point-like

$$
\begin{aligned}
G_E(\vec{q}^2) &= \int e^{i\vec{q}\cdot\vec{r}}\rho(\vec{r})d^3r \quad \text{with} \quad G_E(0) = 1 \\
G_M(\vec{q}^2) &= \int e^{i\vec{q}\cdot\vec{r}}\mu(\vec{r})d^3r \quad \text{with} \quad G_M(0) = \mu_p
\end{aligned}
$$

Electron-Proton Scattering: Form factors

- Form factors are functions of momentum transfer

→ **the protons and neutrons are not point like particles!**



Nobel prize 1961

Prize motivation: “for his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the structure of the nucleons”

Source: https://www.nobelprize.org/prizes/physics/1961/hofstadter/facts/

Robert Hofstadter

Born: 5 February 1915, New York, NY, USA

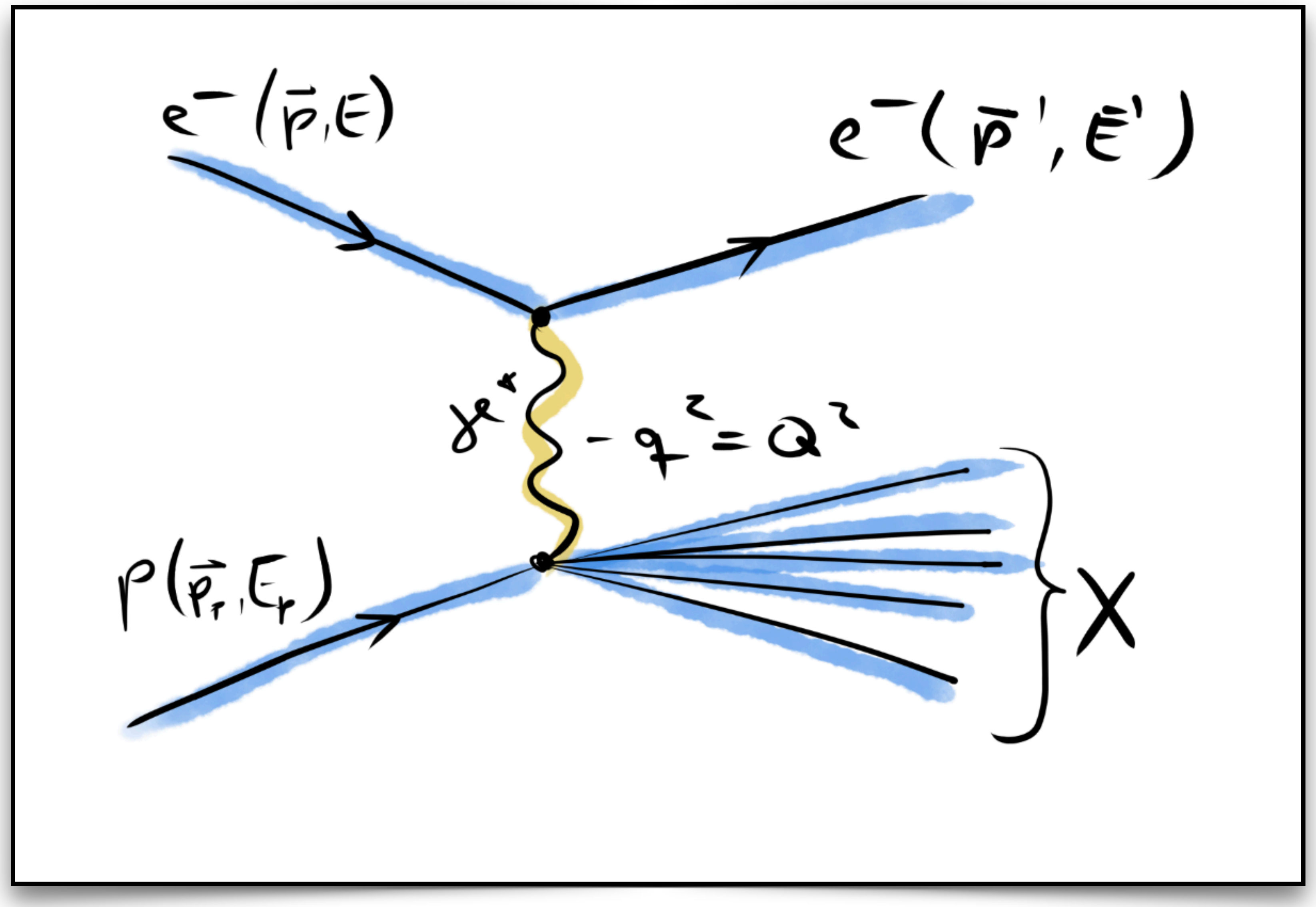
Died: 17 November 1990, Stanford, CA, USA


Grimur


Grimur
Loki

Inelastic Electron-Proton Scattering

- Inelastic scattering:
  - Inelastic energy transfer to target nucleon
  - Target nucleon “shattered”, recoil absorbed by hadronic system with mass W ≥ M
- Kinematics described by Lorentz-invariant observables:
  - Momentum transfer $Q^2 = -q^2 = -(p-p')^2$
  - Energy transfer $\nu = \frac{p_p q}{M} = E_{\text{lab}} - E'_{\text{lab}}$ (in proton restframe)
  - Inelasticity $y = \frac{p_p q}{p_p p} = \frac{E_{\text{lab}} - E'_{\text{lab}}}{E_{\text{lab}}}$ $(0 \leq y \leq 1)$
  - Invariant hadronic mass $W^2 = (p_p + q)^2 = M^2 + 2M\nu - Q^2$



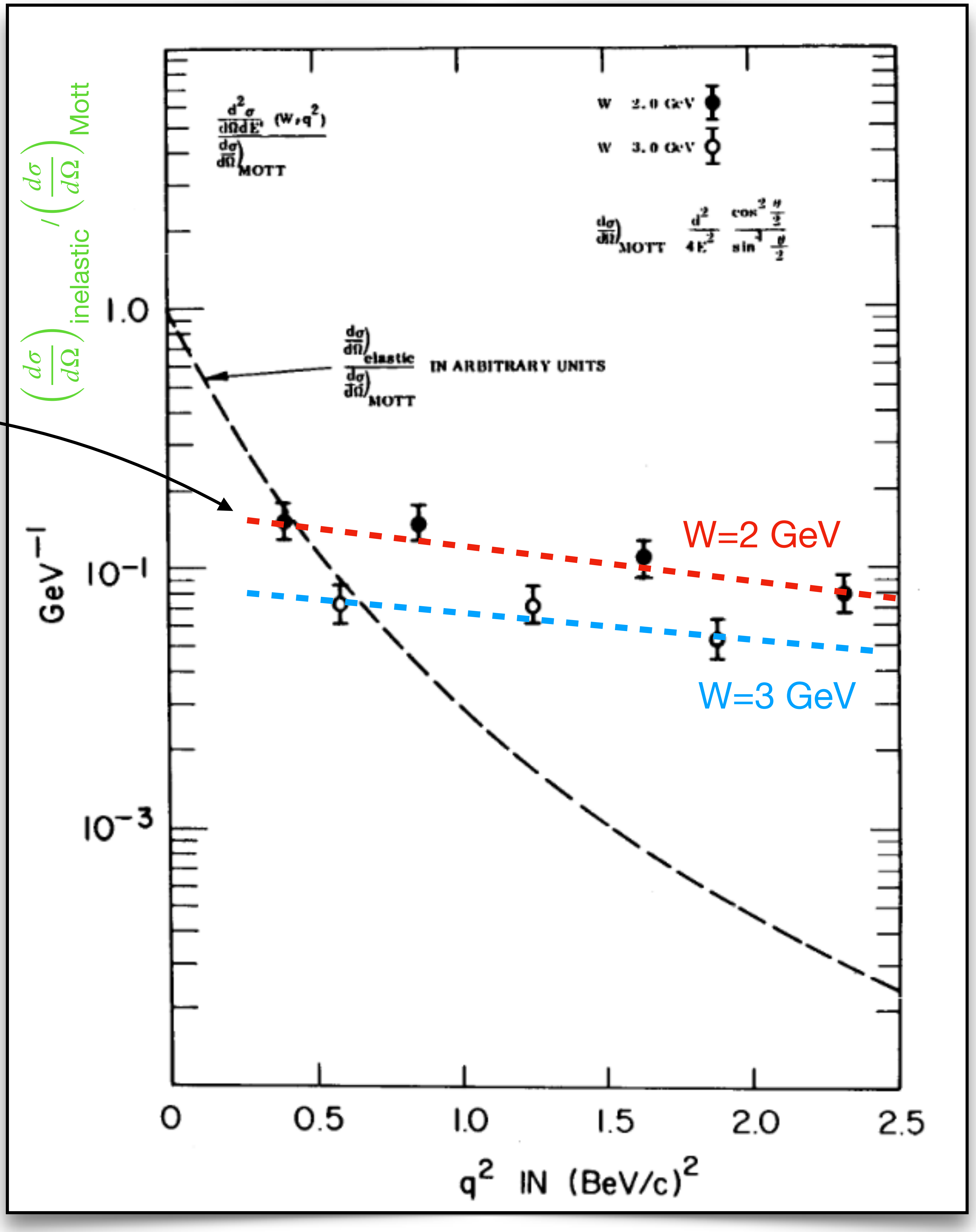
- $W = M$: elastic scattering
- 1 GeV < W < 2 GeV: inelastic resonance excitation
- W> 2 GeV: deep inelastic scattering

Scale invariance

- Early ep scattering experiments at SLAC in the 1960 expected:
  - Very non elastic behaviour of the cross section, almost Mott cross section (in the plot: measurement divided by Mott cross section ≈ constant)
  - Only very weak dependency on Q^2 for $W \gtrsim 2$ GeV
  - Cross section depends (approximately) only on one variable x:

    "Bjorken scaling" or "scaling variable"

$$x = \frac{Q^2}{2M\nu} = \frac{Q^2}{2Pq} \; (0 \leq x \leq 1)$$



https://www.nobelprize.org/uploads/2018/06/kendall-lecture-1.pdf

Structure functions

- Using $Q^2 = 4EE' \sin^2 \theta/2$, rewrite Mott cross section:

$$\left(\frac{d\sigma}{dQ^2}\right) = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\left(\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2(Q^2)\sin^2\frac{\theta}{2}\right)$$

- Using $y = 1 - \frac{E'}{E}\sin^2\frac{\theta}{2}$:

$$\left(\frac{d\sigma}{dQ^2}\right) = \frac{4\pi\alpha^2}{Q^4}\left(\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau}\left(1 - y - \frac{M^2y^2}{Q^2}\right) + \frac{y^2}{2}G_M^2(Q^2)\right)$$

Structure functions

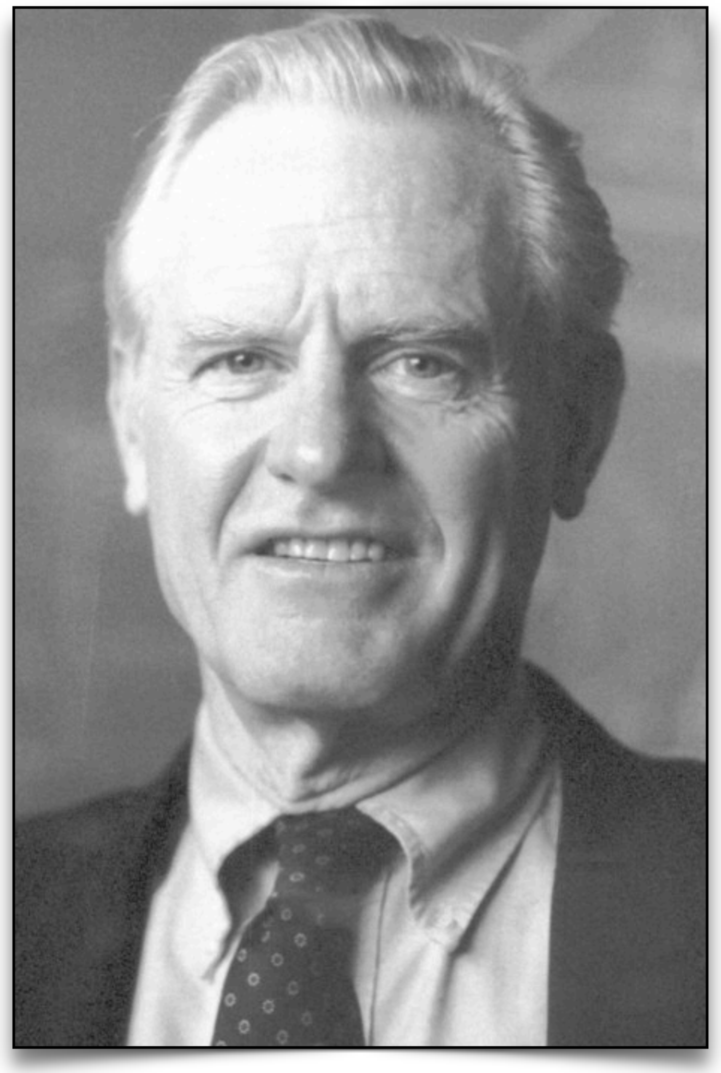
- Double differential cross section:

$$\left(\frac{d\sigma}{dQ^2dx}\right) = \frac{4\pi\alpha^2}{Q^4}\left(\left(\frac{1-y}{x} - \frac{M^2y^2}{Q^2}\right)F_2(x,Q^2) + y^2F_1(x,Q^2)\right)$$

- Dimensionless structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$:
  - $F_1(x,Q^2)$ and $F_2(x,Q^2)$ are not just simple Fourier transformations (since they depend on a second variable, we chose x)
  - Bjorken scaling: $F_1(x,Q^2) \rightarrow F_1(x)$ and $F_2(x,Q^2) \rightarrow F_2(x)$
  - In the high energy limit $Q^2 \gg M^2y^2$ the second term vanishes

Nobel prize 1990

Prize motivation: “for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics”



Source: https://www.nobelprize.org/prizes/physics/1990/kendall/facts/

Henry W. Kendall

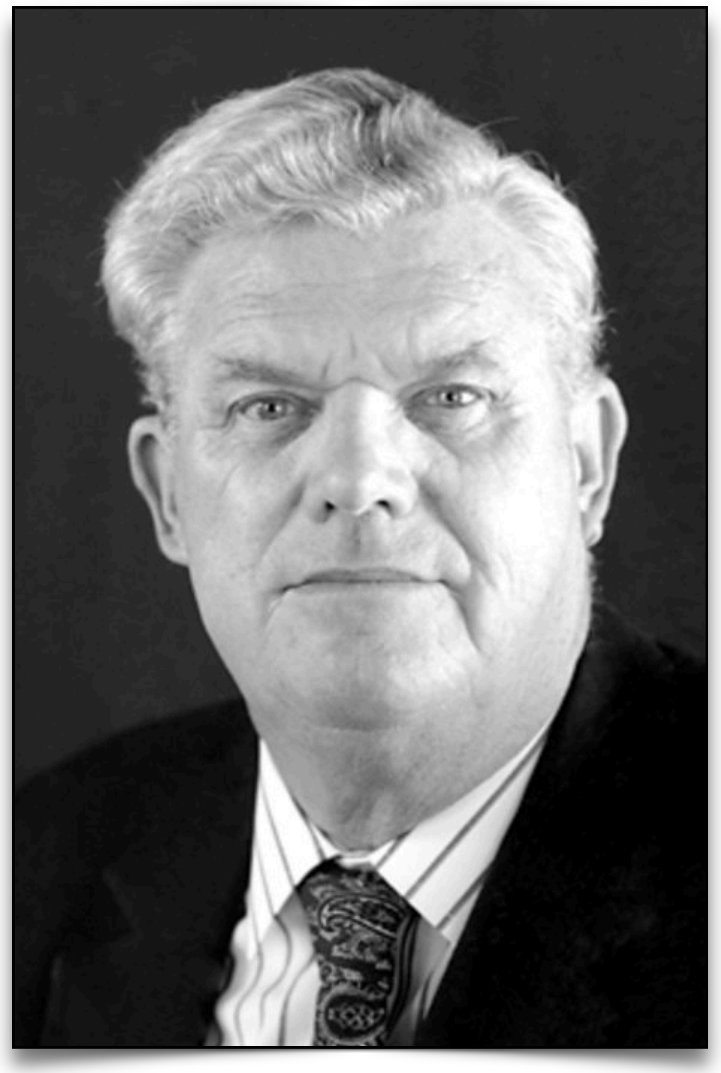
Born: 9 December 1926, Boston, MA, USA

Died: 15 February 1999, Wakulla Springs State Park, FL, USA

Richard E. Taylor

Born: 2 November 1929, Medicine Hat, Alberta, Canada

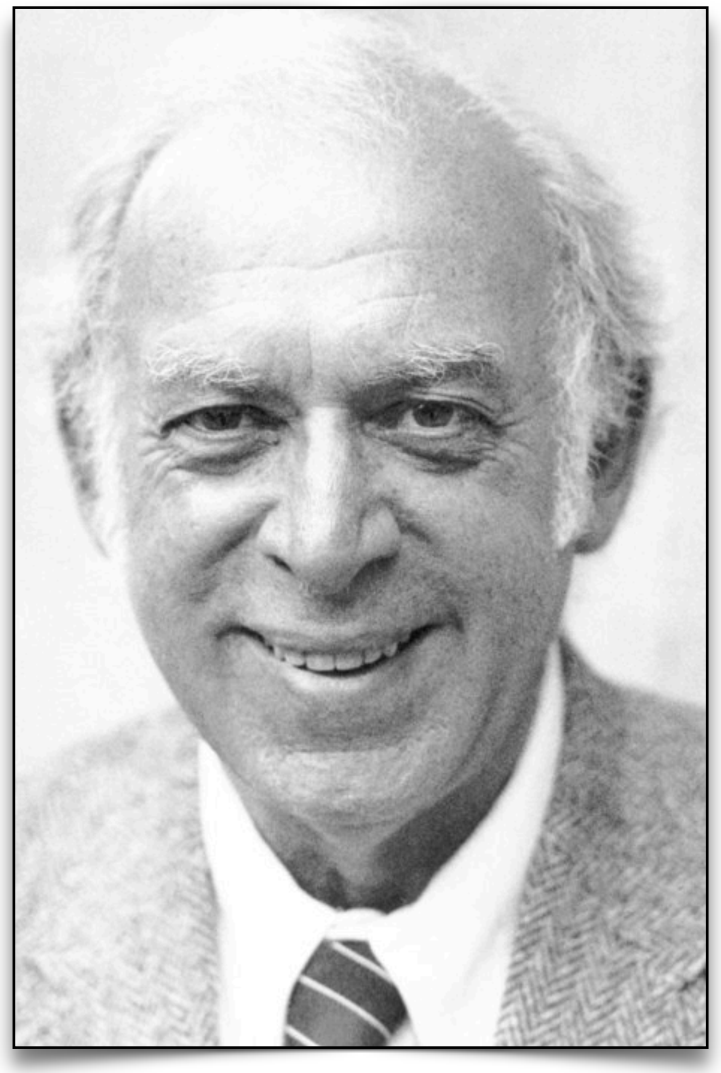
Died: 22 February 2018, Stanford, CA, USA



Source: https://www.nobelprize.org/prizes/physics/1990/taylor/facts/

Jerome I. Friedman

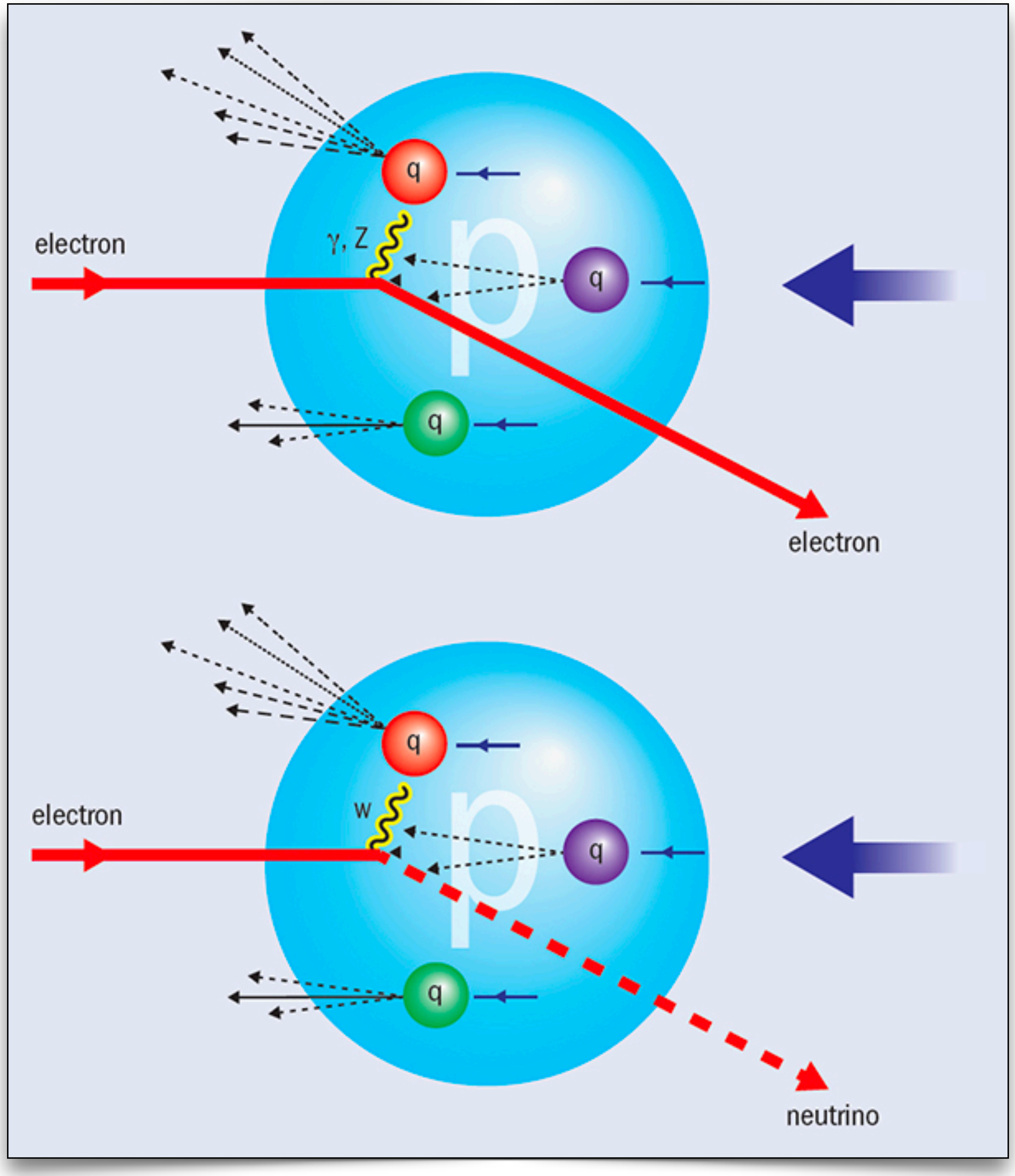
Born: 28 March 1930, Chicago, IL, USA

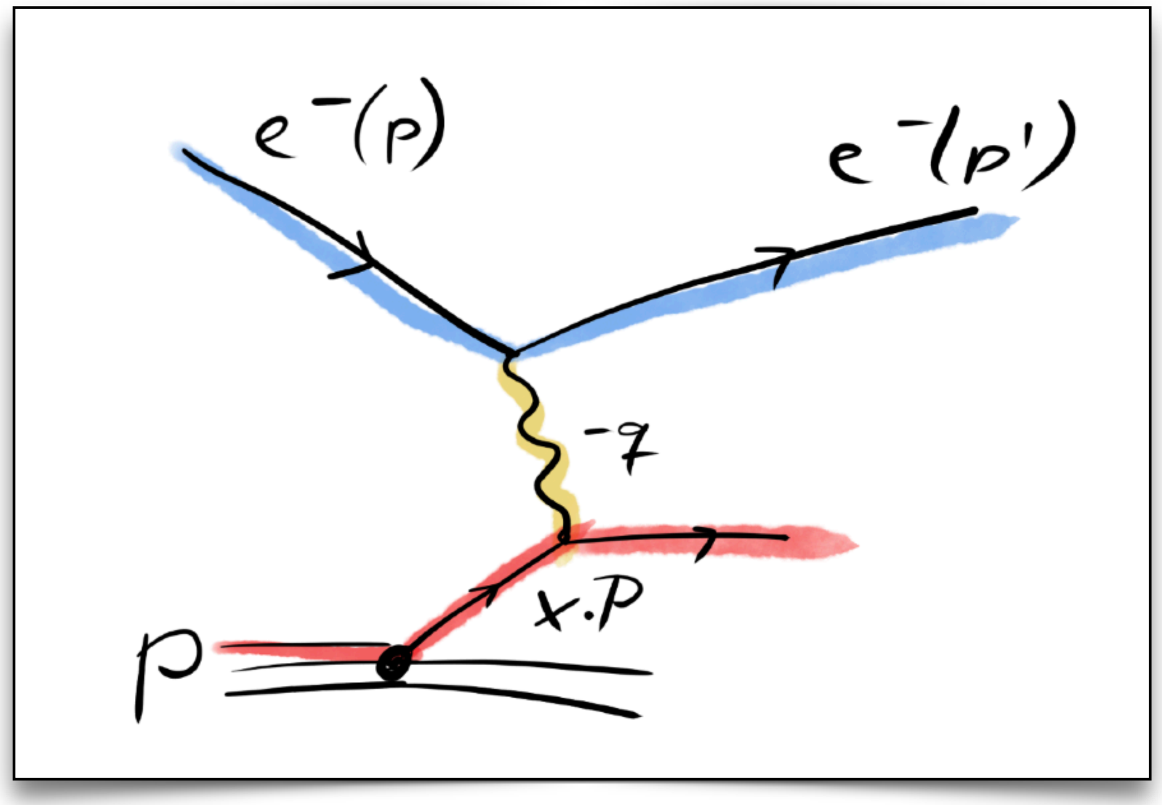


Source: https://www.nobelprize.org/prizes/physics/1990/friedman/facts/

Naive parton model

- Deep inelastic *ep* scattering: incoherent superposition of elastic scattering processes with point-like spin-1/2 particles (“partons”)
  → relation between structure functions and partons
- Bjorken scaling variable x (obtained from observed kinematics) is the fraction of the hadron momentum carried by the parton that scatters



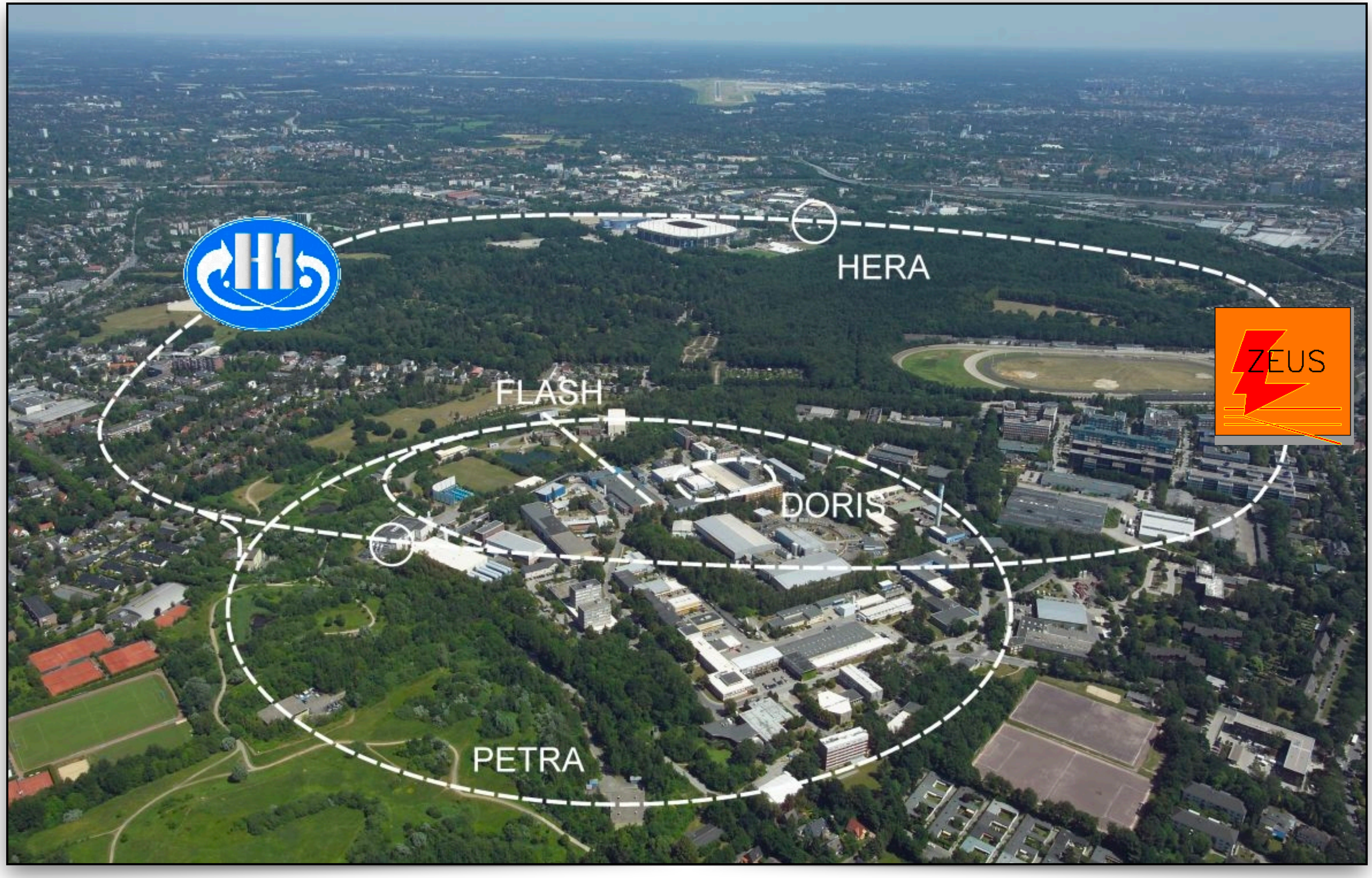


Naive parton model

- Protons (or neutrons) may carry multiple partons, and each parton carries a momentum fraction x of the overall proton momentum
- Identify structure function with sum over all parton scatters:
  - $F_2(x) = x \sum_i e_i^2 \left(q_i(x) + \bar{q}_i(x)\right)$
  - $F_1(x) = \frac{1}{2x} F_2(x)$ ("Callan-Gross-relation", only holds for spin-1/2 partons)
- $q(x)$ are called parton distribution functions (PDFs) that must be determined experimentally

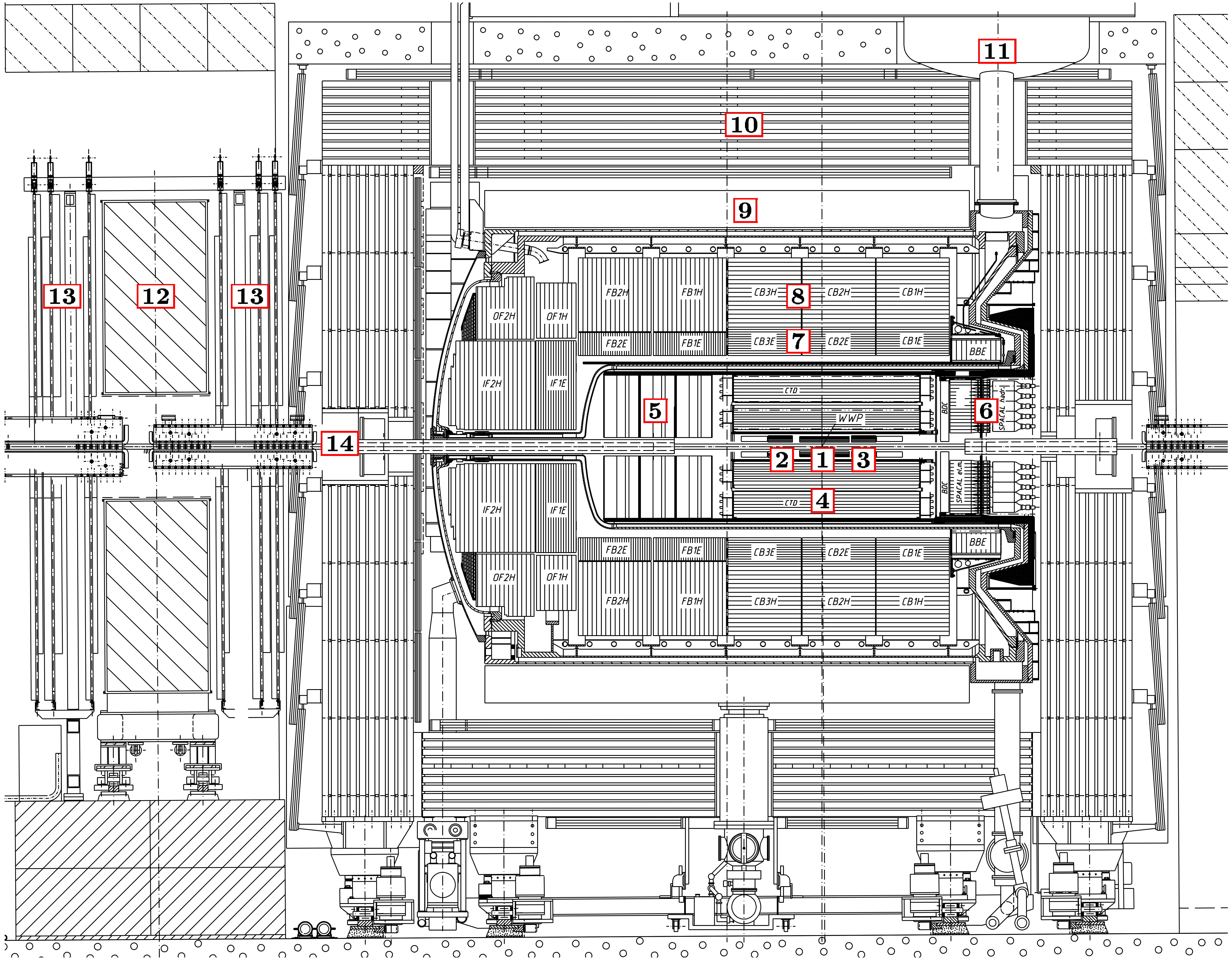
HERA

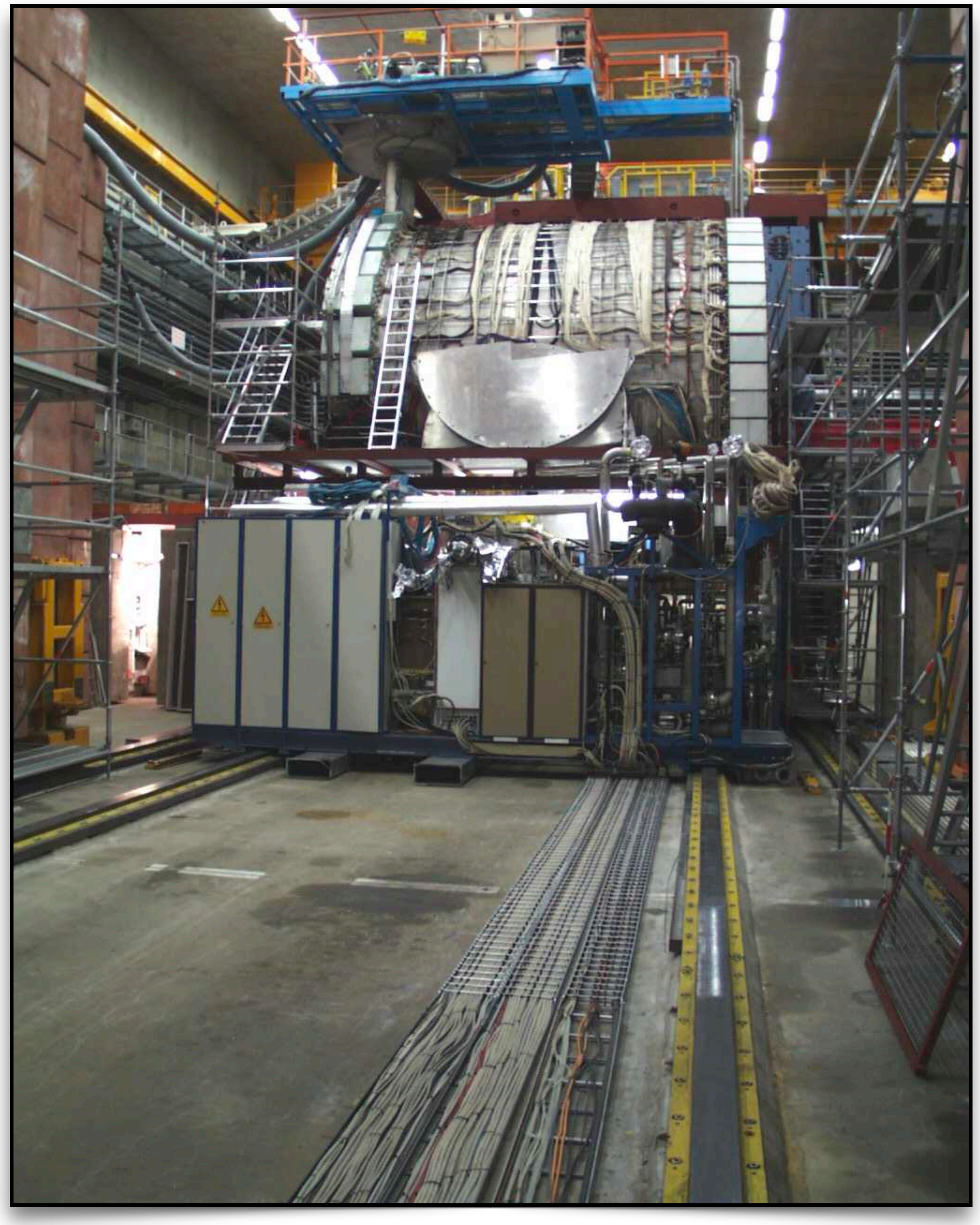
- Data taking: 1992-2007
- 920 GeV p
  27.5 GeV e^- or e^+
- 4 experiments:
  - H1 and ZEUS (ep)
  - Hermes (e fixed target)
  - HERAb (p fixed target)



Credit: https://h1.desy.de/

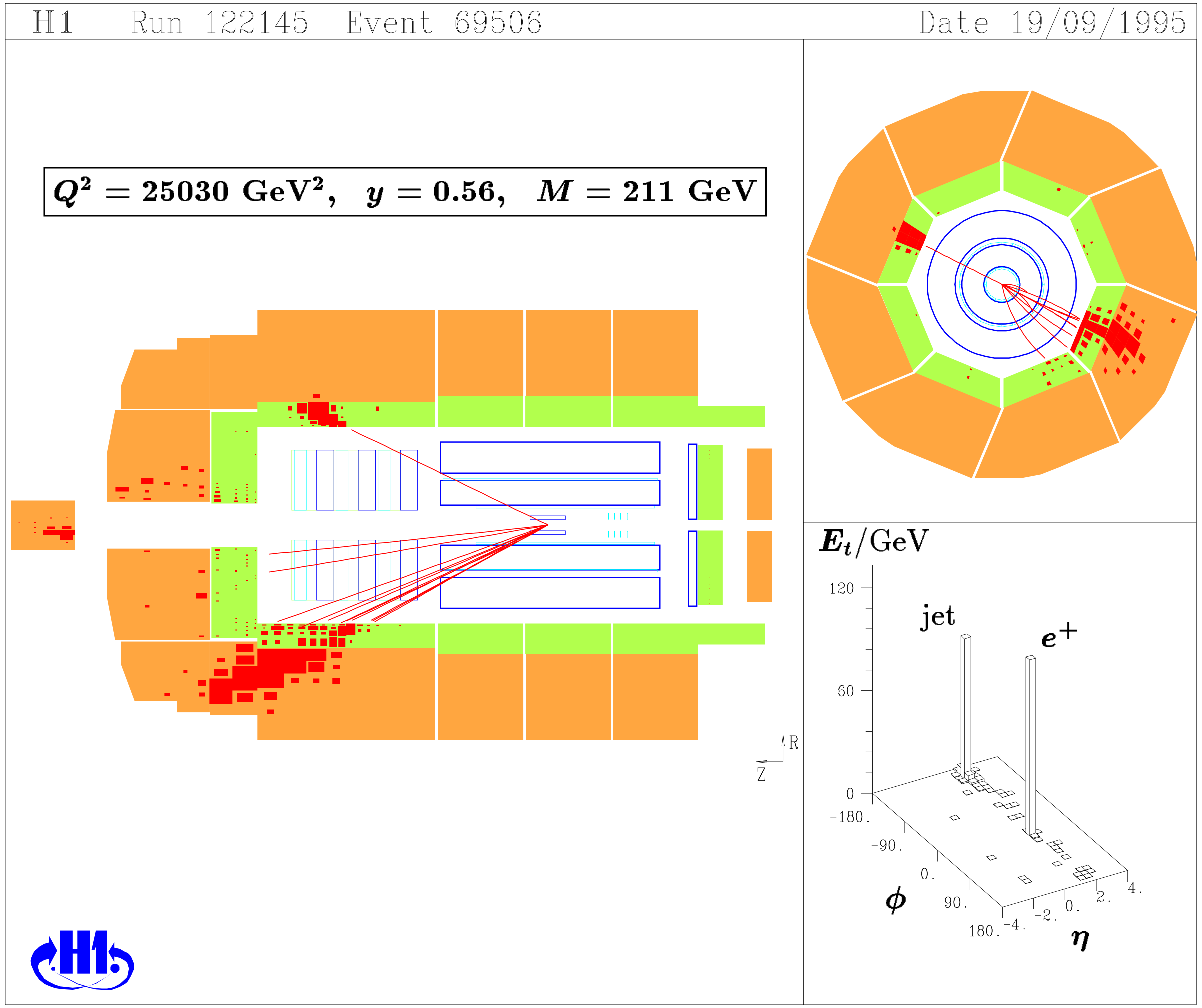
H1





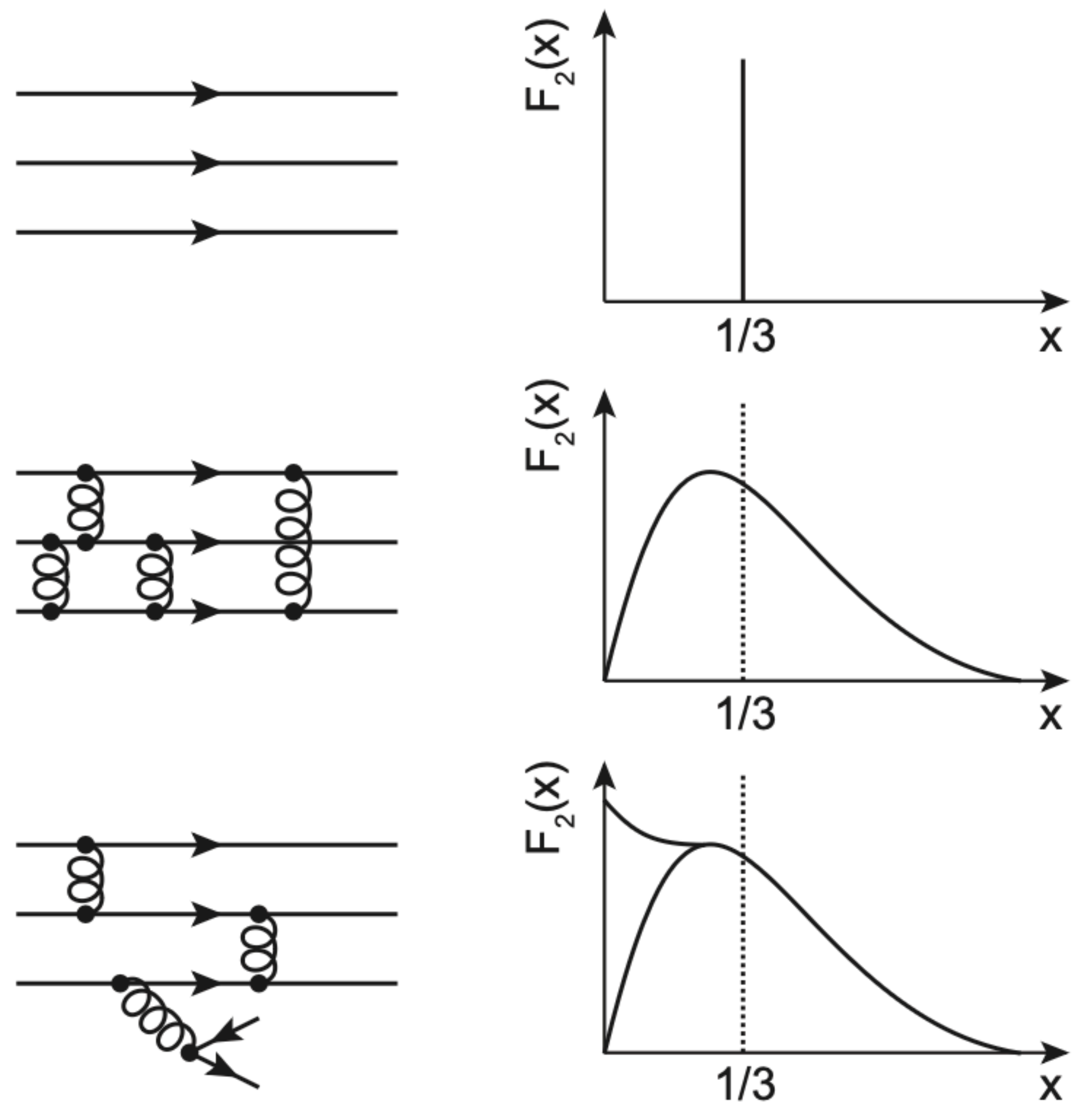
1	Central Silicon Tracker (CST)	8	Hadronic Calorimeter (LAr)
2	Forward Silicon Tracker (FST)	9	Superconducting Coil
3	Backward Silicon Tracker (BST)	10	Instrumented Iron (CMD)
4	Central Tracking Detector (CTD)	11	Liquid Argon Cryostat
5	Forward Tracking Detector (FTD)	12	Muon Toroid Magnet
6	Spaghetti Calorimeter (SpaCal)	13	Forward Muon Detector (FMD)
7	Electromagnetic Calorimeter (LAr)	14	Beam Pipe

H1 typical event



Structure functions after HERA

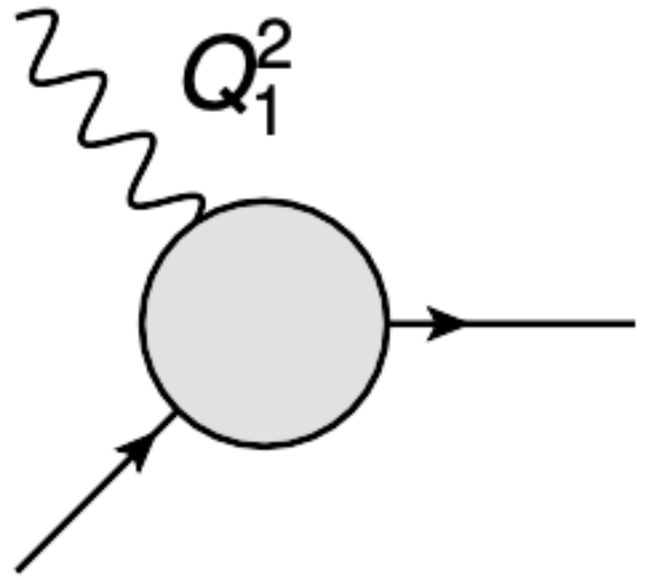
- Simplest model: Three independent valence quarks $F_2 = \delta(1/3)$
- Gluon exchange between vales quarks: F_2 centered at 1/3, but smeared
- Gluon exchange and gluon radiation: sea quarks and gluons

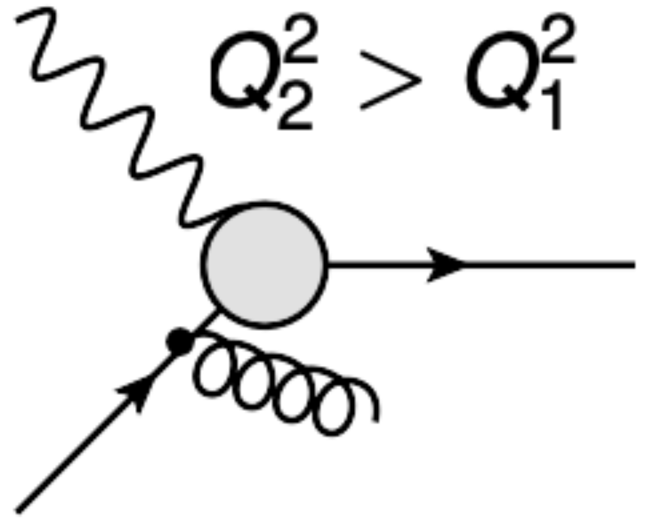


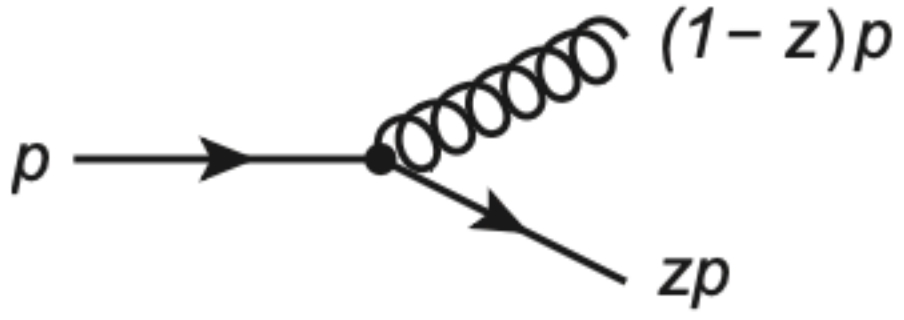
after: Halzen, Martin, Quarks & Leptons

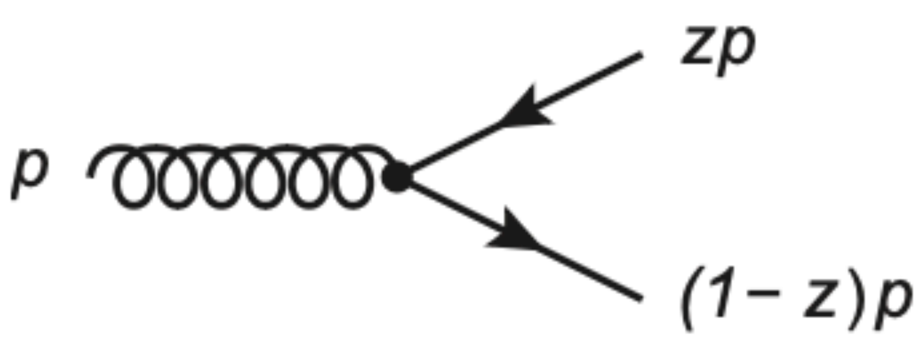
QCD evolution of PDFs

- Simplified picture: electron beam = microscope
  - Small four-momentum transfer Q^2 → photon-parton vertex only coarsely resolved
  - Large four-momentum transfer Q^2 → high-resolution "image" of photon-parton vertex, gluon-induced effects become visible
- Consequence: scaling violations = deviation from Bjorken scaling → PDFs become Q^2-dependent
- Relevant processes:
  - Gluon radiation (also: gluon bremsstrahlung): quark momenta shifted to smaller x, additional gluons at small x
  - Gluon splitting: additional quark-antiquark pairs at small x










Grimur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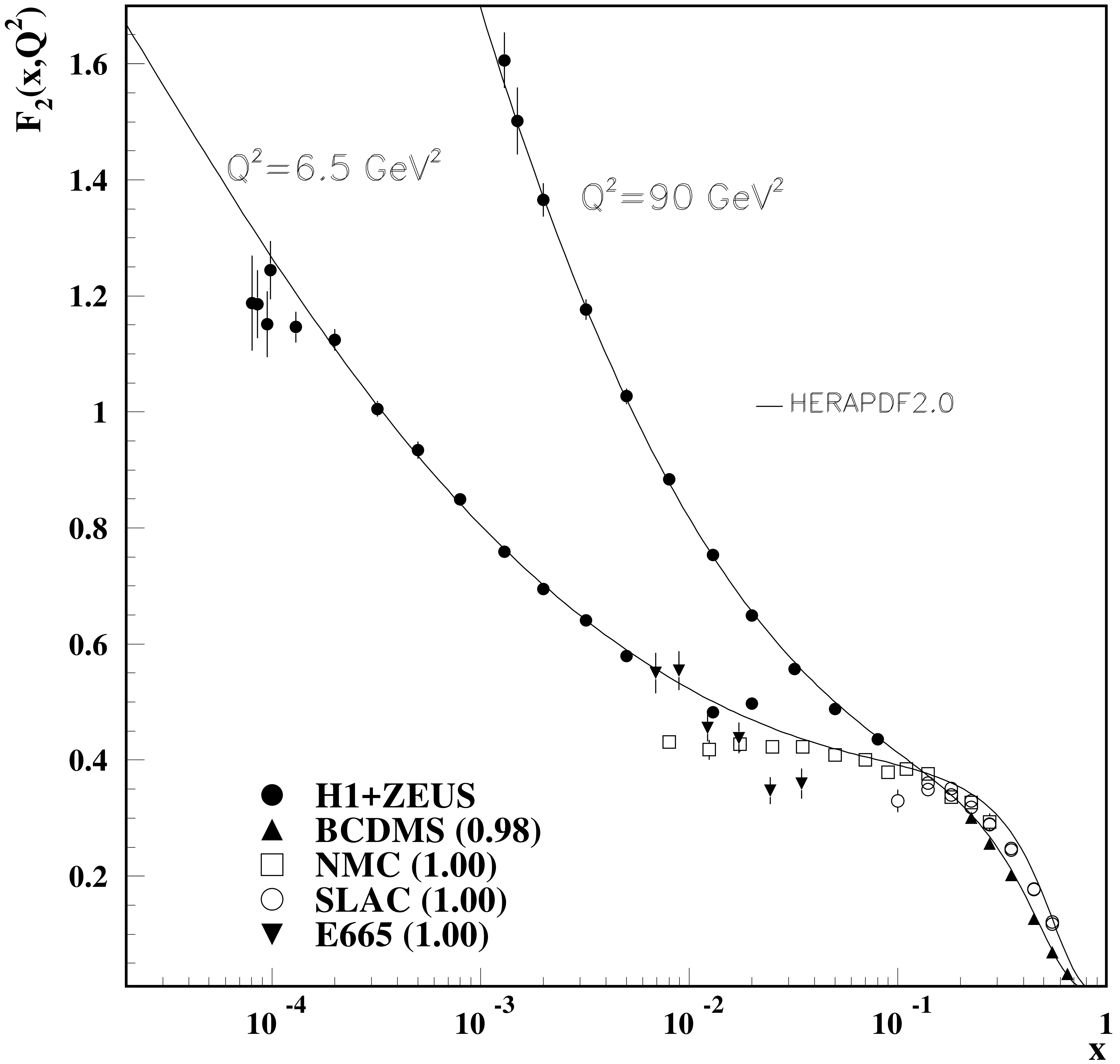
Measurements of $F_2(x, Q^2)$



$$\frac{d^2\sigma}{dQ^2\,dx} = \frac{4\pi\alpha^2}{x\,Q^4}\left(1 - y + \frac{y^2}{2}\right)\cdot F_2(x, Q^2)$$

Measure differential cross section

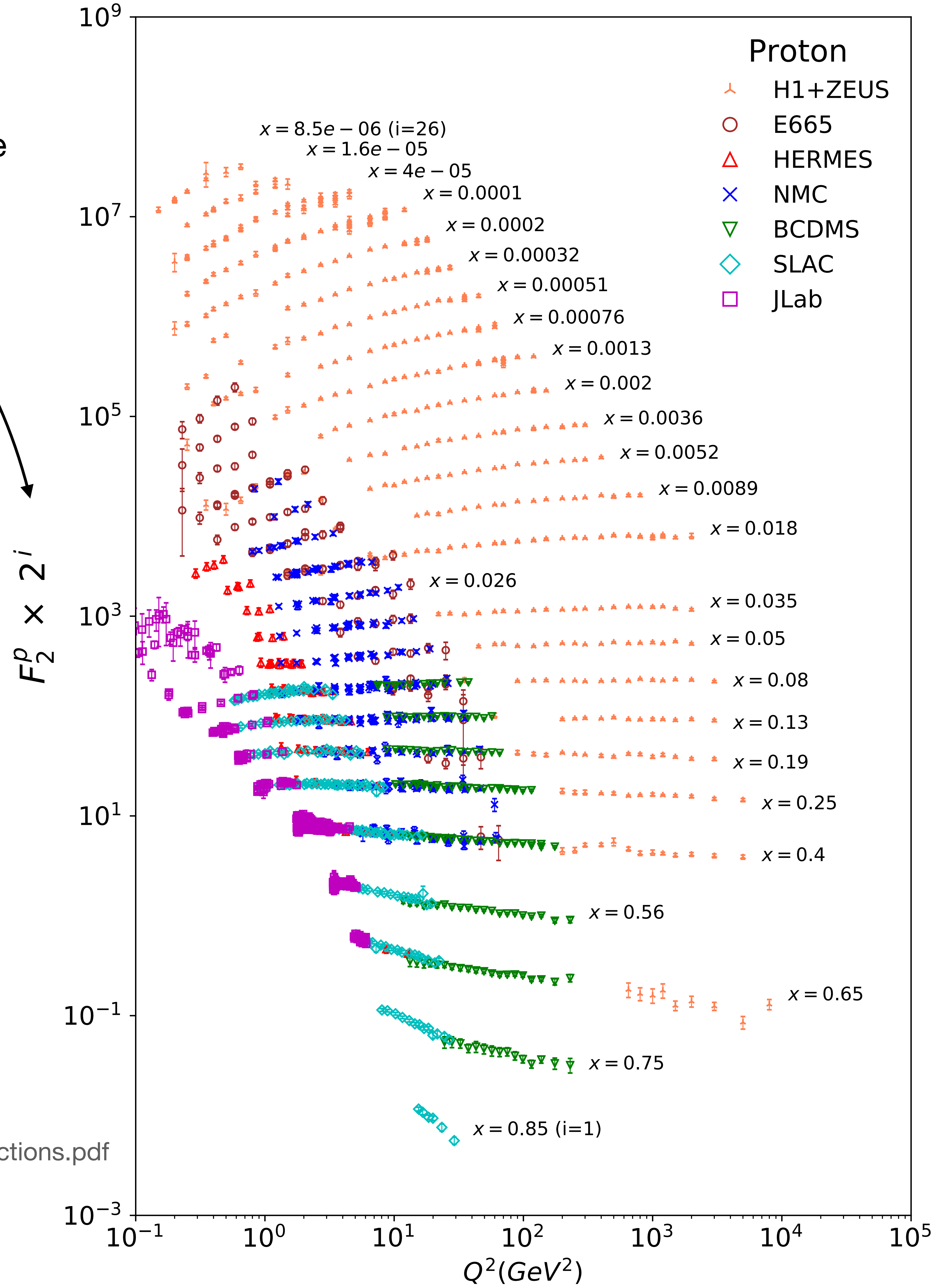
Determine kinematics of scattering

Result: F_2

https://pdg.lbl.gov/2020/reviews/rpp2020-rev-structure-functions.pdf

Measurements of $F_2(x, Q^2)$

Lines displaced by 2^i (they would be all mostly be on top of each other)



https://pdg.lbl.gov/2020/reviews/rpp2020-rev-structure-functions.pdf

$$\frac{d^2\sigma}{dQ^2\,dx} = \frac{4\pi\alpha^2}{x\,Q^4}\left(1 - y + \frac{y^2}{2}\right)\cdot F_2(x, Q^2)$$

Measure differential cross section

Determine kinematics of scattering

Result: F_2

“Scaling violation” visible:

Large x: F_2 decreases with Q^2

Small x: F_2 increases with Q^2

QCD evolution of PDFs

- Momentum conservation → **normalization** condition:

$$\sum_i \int_0^1 x f_i(x) dx = 1$$

- Experimentally for protons:

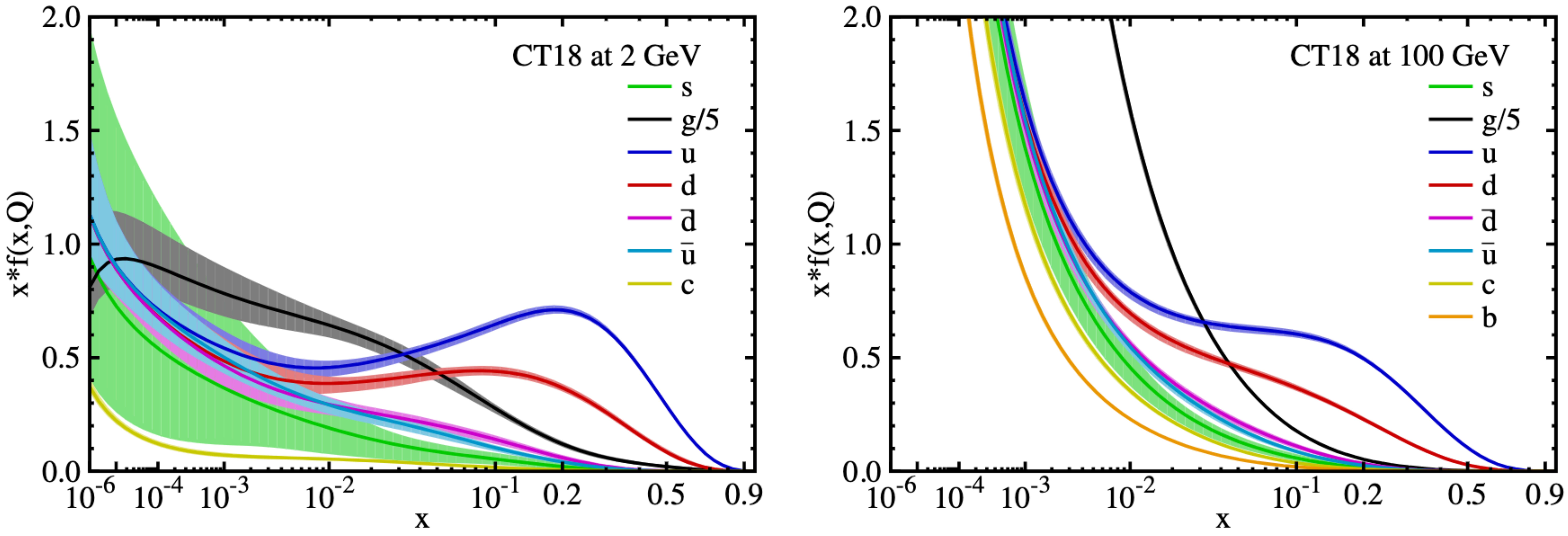
$$\int x \left(u(x) + \bar{u}(x) + d(x) + \bar{d}(x) + \ldots \right) dx \approx 0.5$$

→ 50% of the proton momentum is carried by gluons!

QCD evolution of PDFs

- Renormalization group equations for PDFs:
  - PDFs cannot be determined from first principles
  - Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP-) equations: RGE for evolution of quark and gluon PDFs
- Several competing PDF datasets: CT/CTEQ, MSTW/MMHT, NNPDF, HERAPDF, ABM, …
  - Strategy: joint fit of parameterized PDF shape to many measurements (“PDF fit”)
- PDF evolution to desired Q^2 using DGLAP equations and fit to experimental data

Example: CT18 PDFs



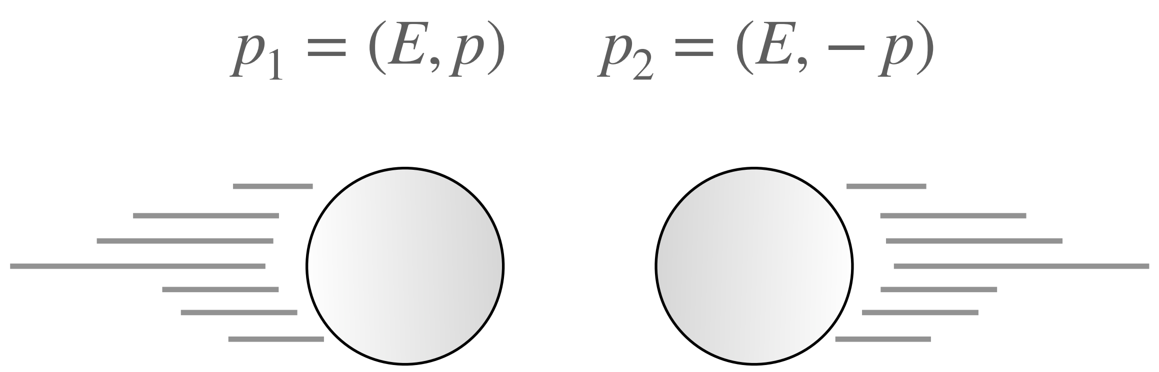
https://arxiv.org/pdf/1912.10053.pdf

Top pair production

- Protons at the LHC collide at $\sqrt{s} = 13.6$ TeV.
  What is the minimum x required to produce a $t\bar{t}$ pair?
  - Hint 1: There are two incoming partons involved.
  - Hint 2: There are two outgoing partons involved.
  - Hint 3: What fraction of the total center of mass energy is available in the parton-parton collision?

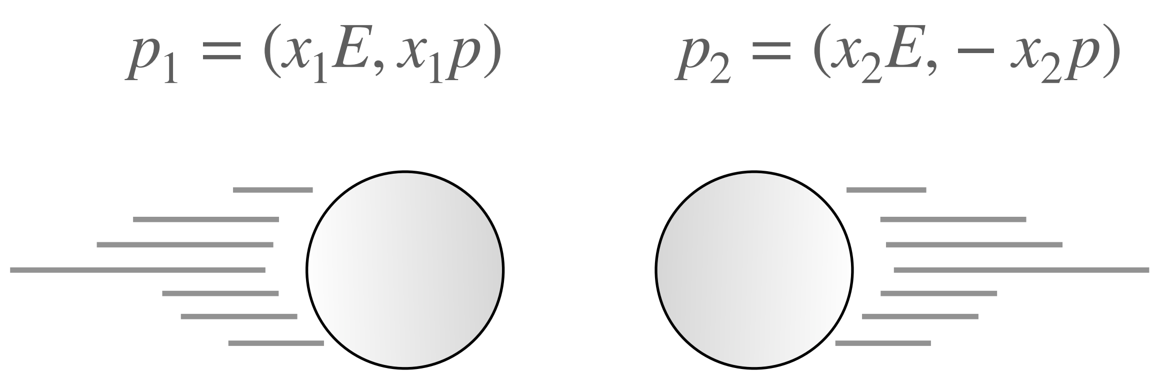
Center of mass energy $\sqrt{s}$

- Symmetric collider (e.g. LHC, LEP)



$$s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1p_2$$
$$= (E^2 - p^2) + (E^2 - p^2) + 2(E^2 + p^2) = 4E^2$$

with partons:



$$\hat{s} = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1p_2$$
$$= m_1^2 + m_2^2 + 2(x_1Ex_2E + x_1px_2p)$$
$$\approx 4x_1x_2E^2 = x_1x_2s$$

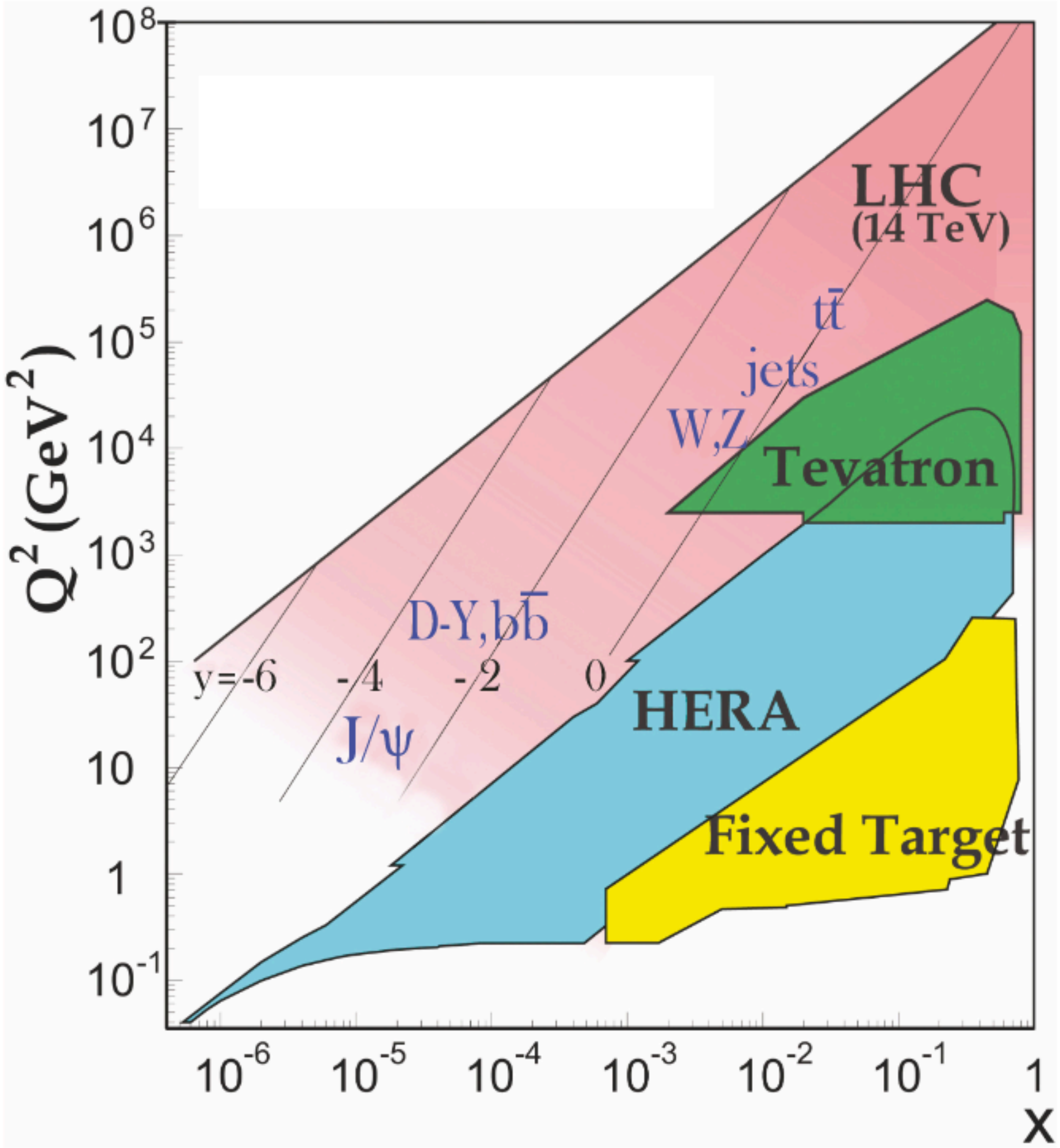
Top pair production

- Condition: $x_1 x_2 s \geq (2m_t)^2 \rightarrow x_2 \geq \dfrac{(2m_t)^2}{x_1 s}$
- For $x_1 \rightarrow 1$ (maximal possible value):

$$x_2 \geq \left(\frac{364\ \text{GeV}}{13.6 \times 10^3\ \text{GeV}} \right)^2 \approx 6 \times 10^{-4}$$

PDFs for the LHC

- LHC collisions probe parton pdfs at very high Q^2 and very low x values
- Requires very precise theory calculations for the PDFs
- Requires very precise experimental inputs (reminder: RGE only predicts the evolution of the PDF, it needs starting values)



What questions do you have?