Particle Physics 1
Lecture 18: Neutrino Oscillations

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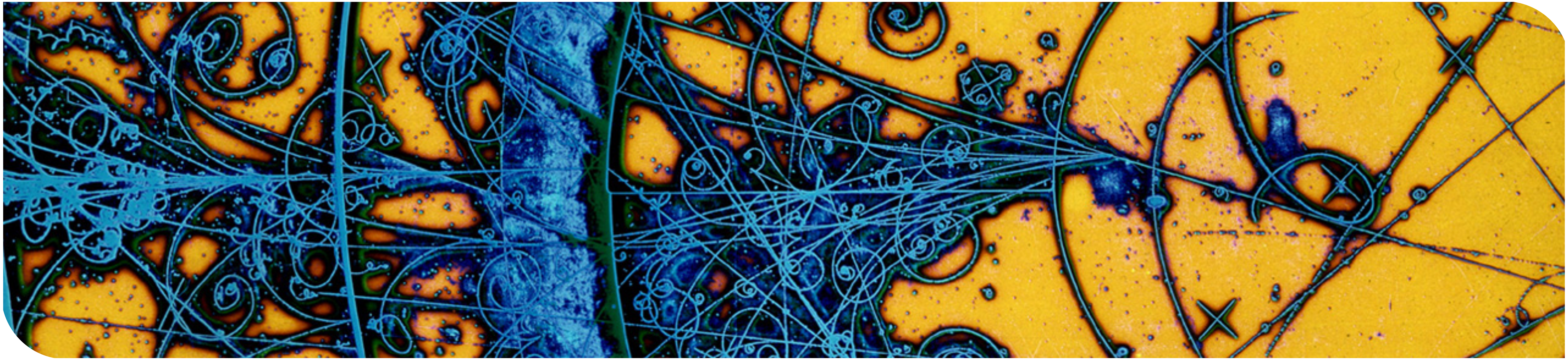
Winterterm 2023/2024



Credit: CERN

Questions from past lectures

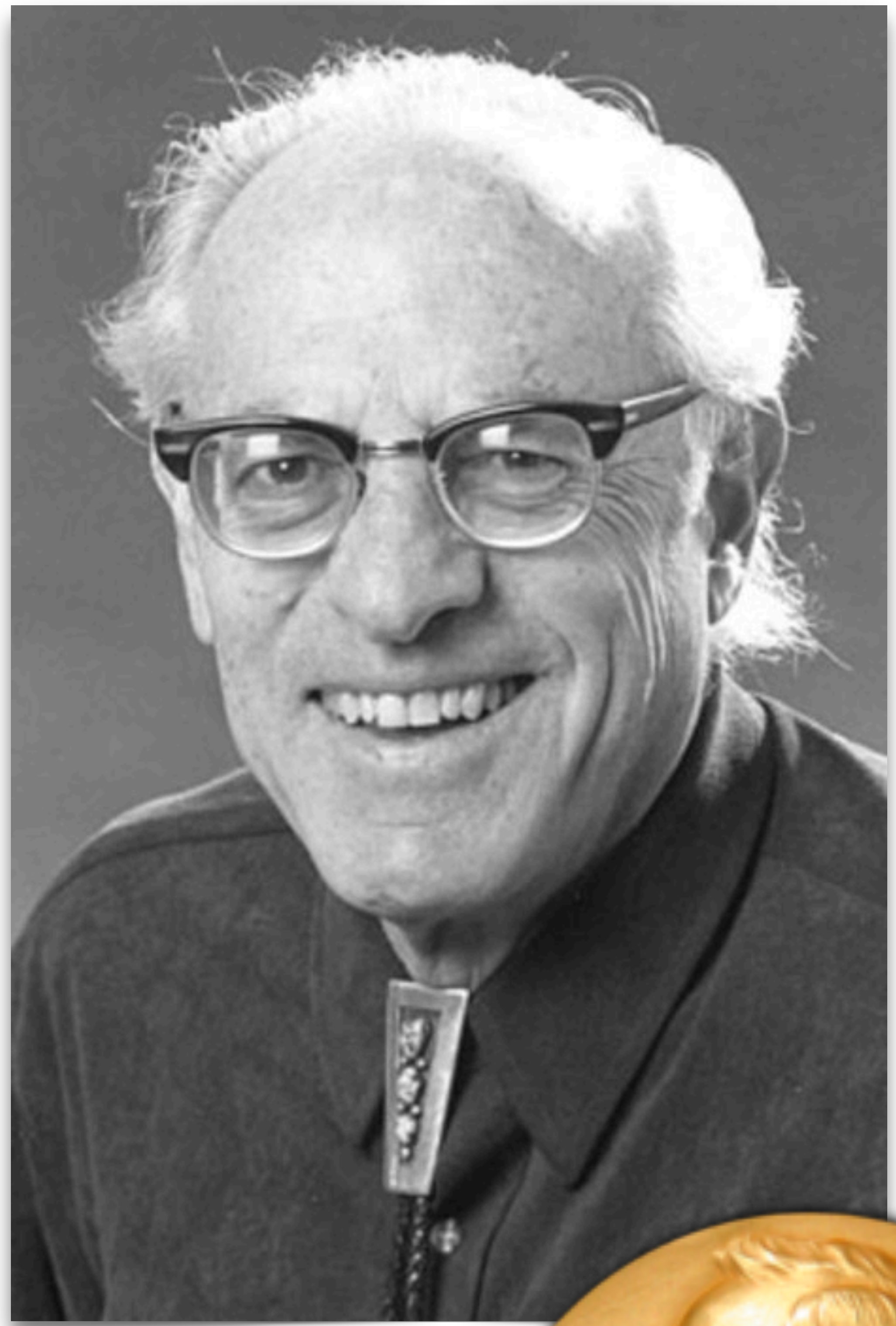
Plan for the rest of the semester

- Today: Neutrino and neutrino oscillations
- 23.01.: Neutrino experiments
- 30.01.: Higgs (T. Chwalek)
- 01.02.-03.0.2.: CERN Trip
- 06.02: Beyond the standard model (BSM) 1: Open questions and anomalies
- 13.02. Dark Matter

Learning goals

- Understand critical steps towards establishing a neutrino a fundamental particle
- Understand neutrino mixing and the PMNS matrix in 2 and 3 flavours
- Understand neutrino mass as a conceptual question of the SM

Nobel Prize 1995



Source: https://www.nobelprize.org/prizes/physics/1995/reines/facts/

Frederick Reines

Born: 16 March 1918, Paterson, NJ, USA

Died: 26 August 1998, Orange, CA, USA

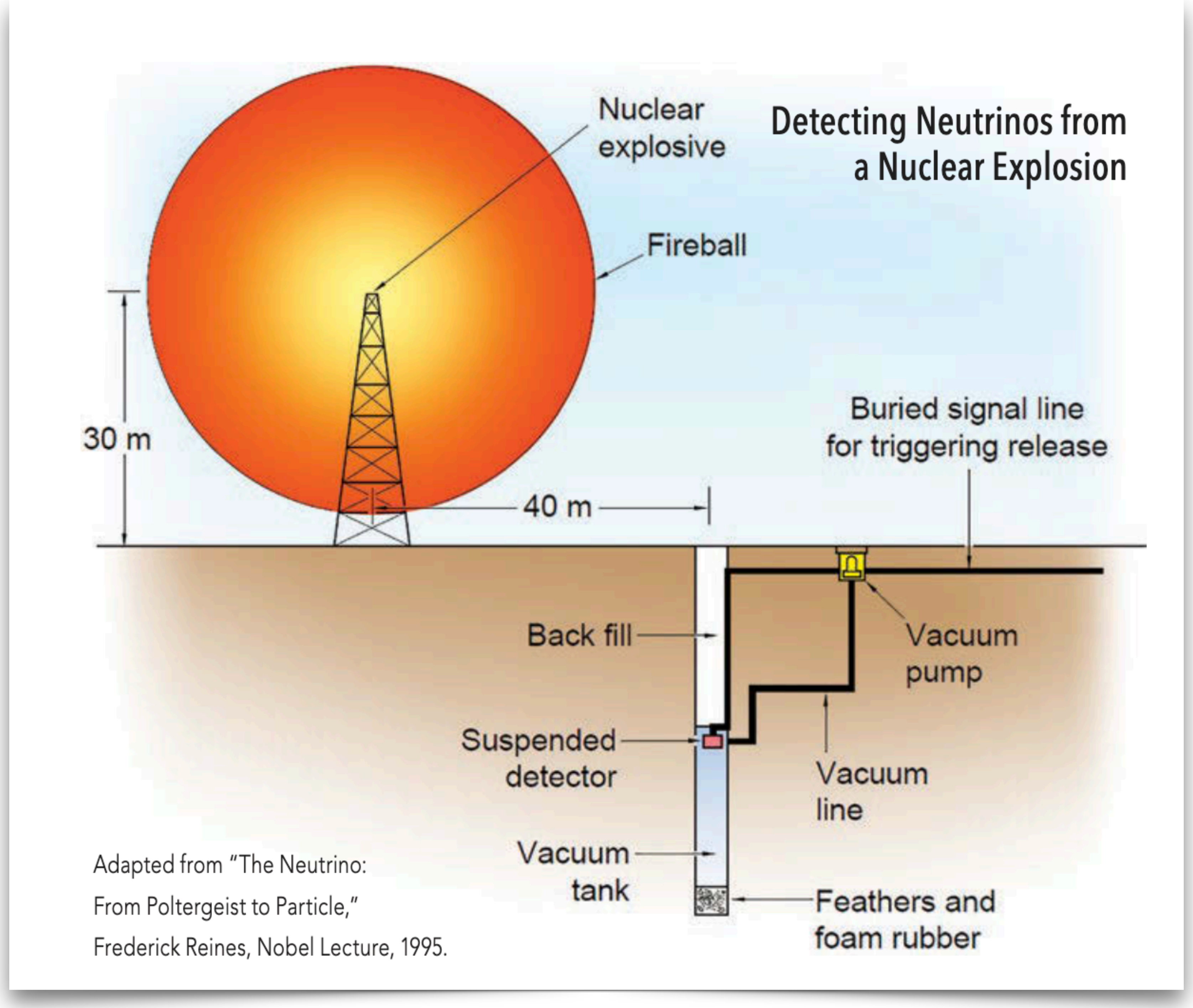
(the co-discoverer **Clyde Cowan** died already 1974)

prize motivation: “for the detection of the (anti-)neutrino”

Project Poltergeist

- Idea 1: 1951, Use a nuclear bomb and measure the neutrinos liquid scintillator detector free falling in vacuum.
  - Detector operated near reactor core but above ground: Large background from cosmic radiation

    Cancelled.



- Idea 2: 1953, Hanford Site, part of the Manhattan project, a gigantic (>1500 km^2) plutonium production facility
  - Detector operated near reactor core but above ground: Large background from cosmic radiation

    No success.

Project Poltergeist

- Idea 3: Savannah River Site (SRS), a huge (800 km^2) production site to refine nuclear materials for deployment in nuclear weapons, located near Aiken, South Carolina, US
  - Multiple powerful reactors going online around 1954/1955
  - Detector operated 11m underground, 12m from the reactor core
  - Reactor could be switched off to test neutrino rate differences: 3ν per hour, exactly matching the theory prediction

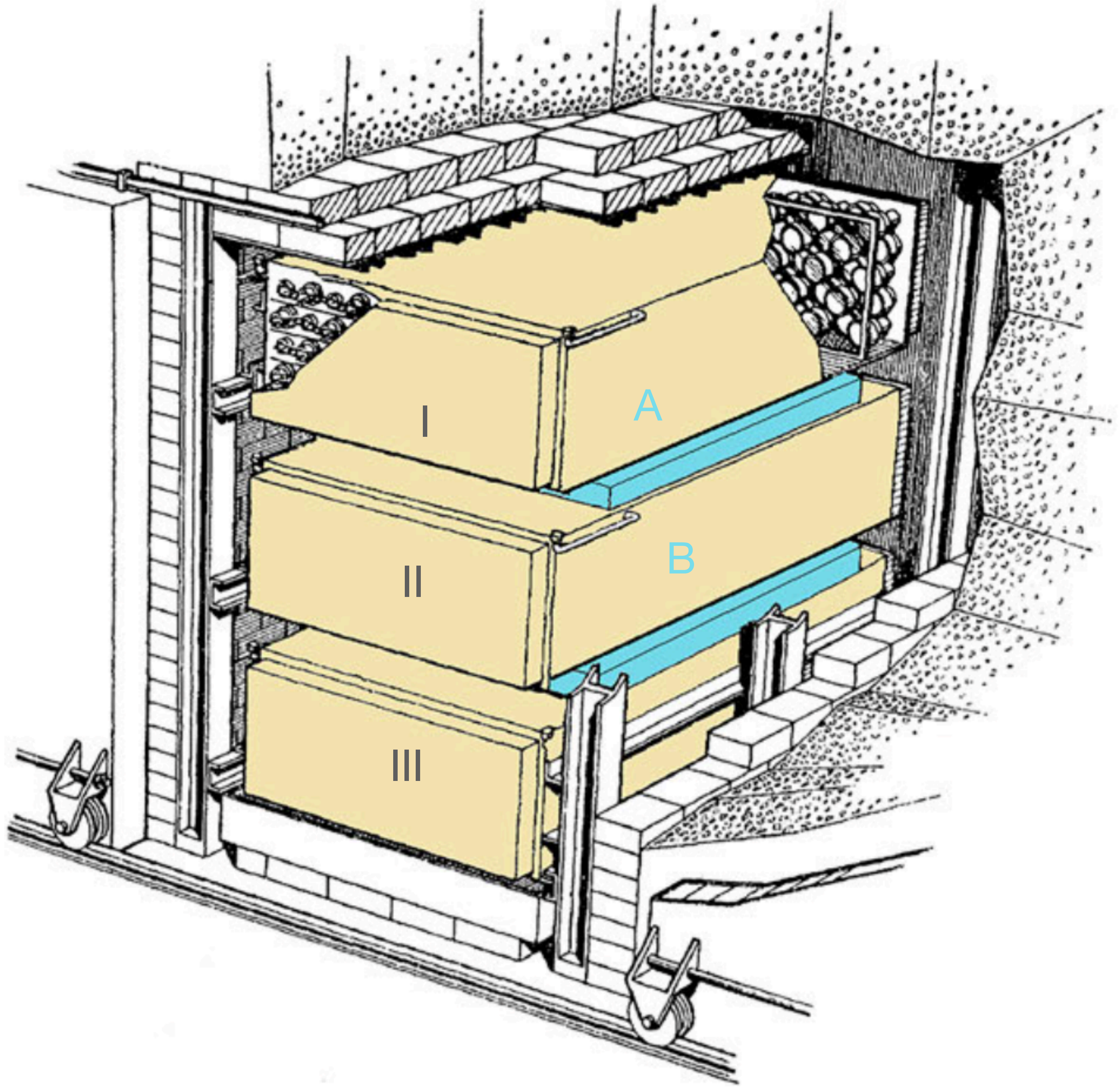
    Success!

Credit: Nuclear Care Partners

Project Poltergeist

- Detector: Three 1400-litre tanks of liquid scintillator (I, II and III), each viewed by 100 phototubes.
- Two smaller tanks (A and B) contained the targets of 200 litres of water doped with cadmium ($CdCl_2$)
- Signal process: **inverse beta decay**

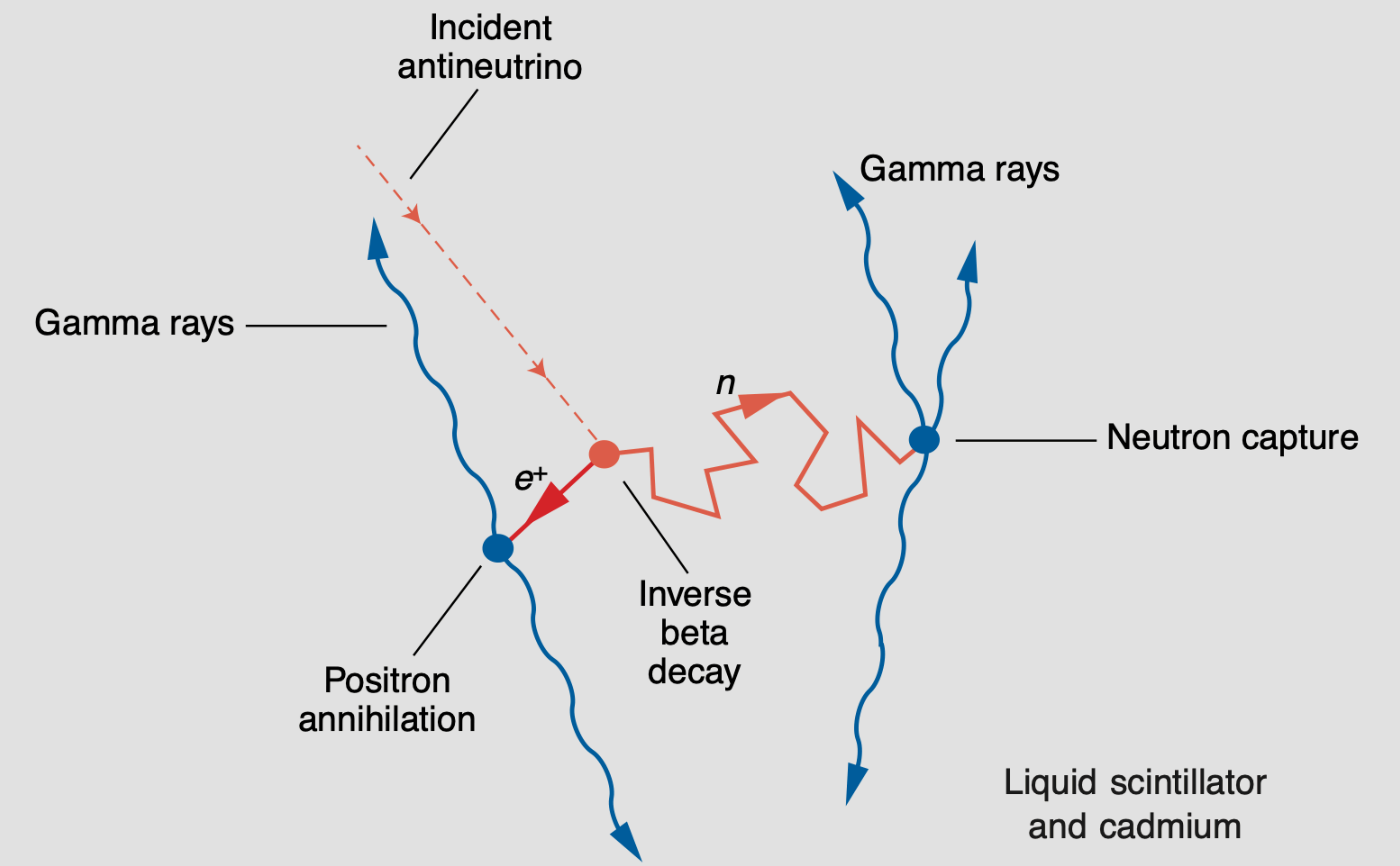
$$\bar{\nu}_e + p \rightarrow n + e^+$$



Credit: Reines et al., Physical Review 117, 159, 1960

Delayed Coincidence

- Detection principle: delayed-coincidence signature
  - Very fast positron annihilation into two 511 keV Photons
  - Neutron captured by Cadmium after O(10 μs): delayed photon emission from at Cadmium dexcitation $n + {}^{108}\text{Cd} \rightarrow n + {}^{109m}\text{Cd} \rightarrow {}^{109}\text{Cd} + \gamma$



Los Alamos Science 25 (1997) 3

Neutrino properties

- Wu-Experiment (1956):
  (Phys. Rev. 105, 1413 (1957))
  - **Maximum parity violation in weak interactions**
- Goldhaber-Experiment (1957):
  (Phys. Rev. 109 1015(1958))
  - **Helicity of neutrinos is -1 → Neutrino are left-handed**
- GARGAMELLE (1973):
  (Phys.Lett. 46B (1973) 138–140)
  - **Discovery of neutral currents** $(\nu + e^- \rightarrow \nu + e^-)$
- LEP (2006):
  (Phys.Rept. 427 (2006) 257–454)
  - **3 light neutrino flavour with** $m_\nu < m_Z/2$

Nobel Prize 1998

Source: https://www.nobelprize.org/prizes/physics/1988/lederman/facts/

Leon. M. Lederman

Born: 15 July 1922, New York, NY, USA

Died: 3 October 2018, Rexburg, ID, USA

prize motivation: "for the neutrino beam method and the demonstration of the doublet structure of the leptons through the discovery of the muon neutrino"

Source: https://www.nobelprize.org/prizes/physics/1988/schwartz/facts/

Melvin Schwartz

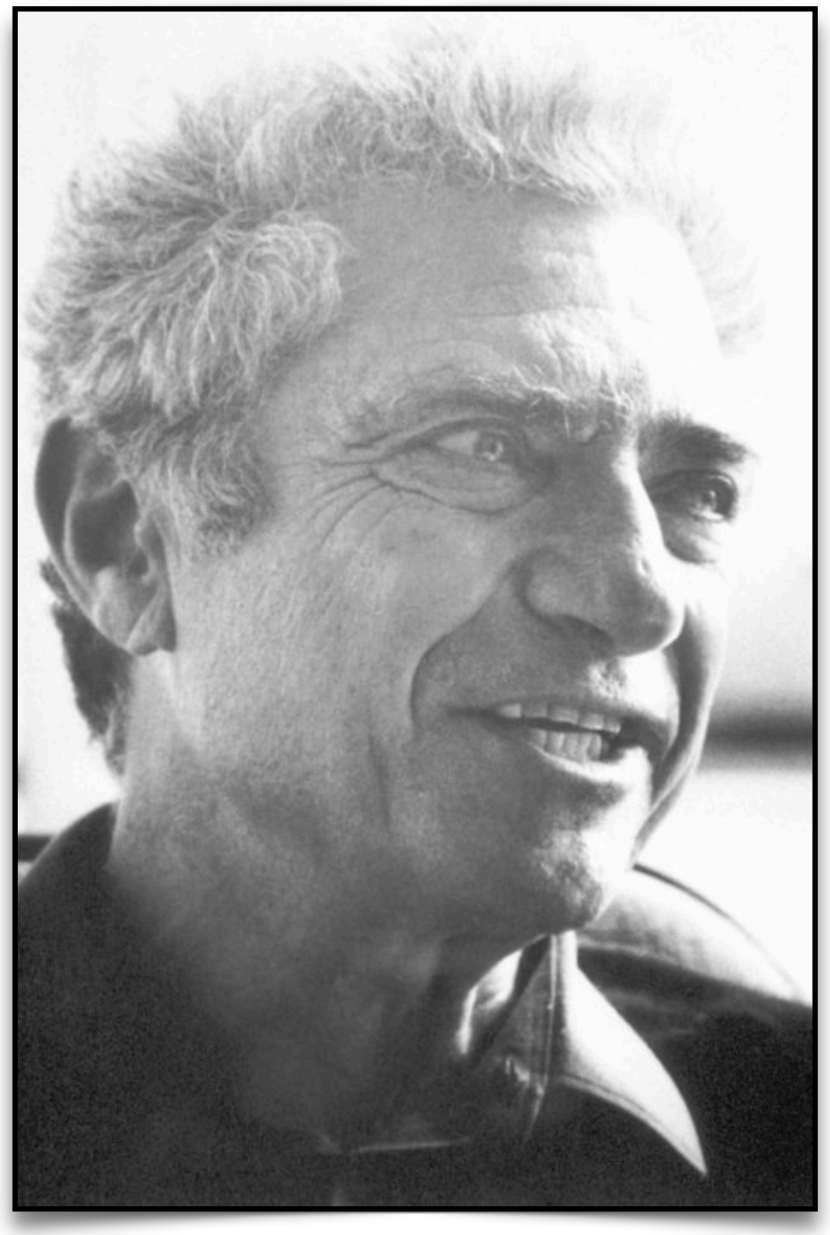
Born: 2 November 1932, New York, NY, USA

Died: 28 August 2006, Twin Falls, ID, USA

Jack Steinberger

Born: 25 May 1921, Bad Kissingen, Germany

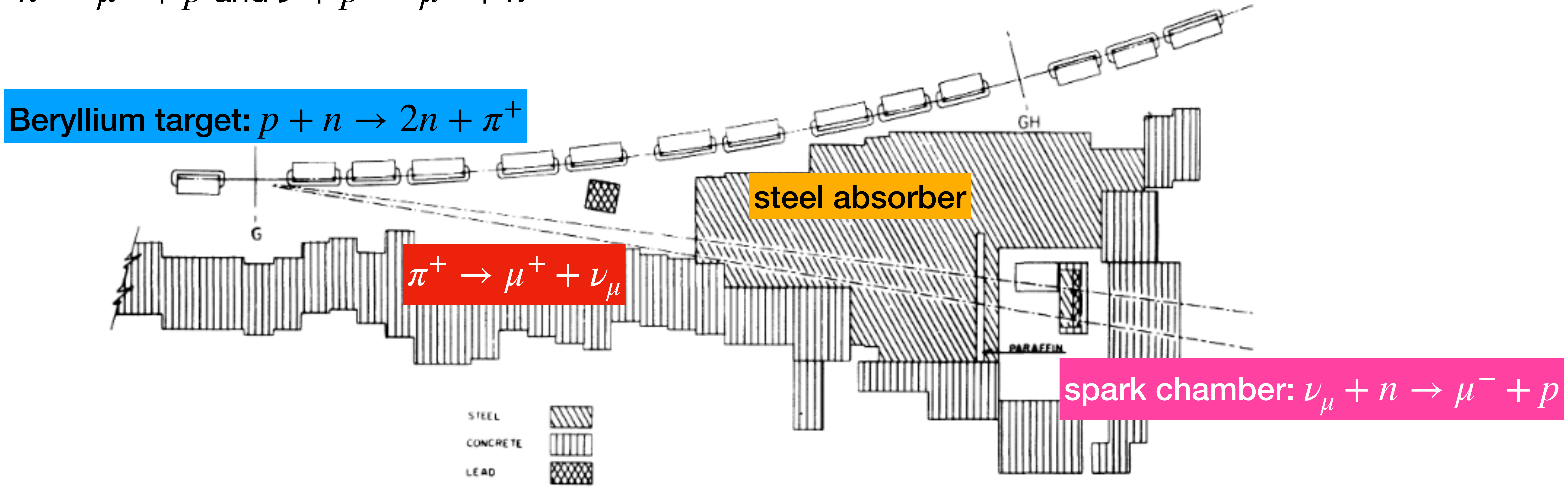
Died: 12 December 2020, Geneva, Switzerland



Source: https://www.nobelprize.org/prizes/physics/1988/steinberger/facts/

Discovery of the muon neutrino

- Production at the Alternating-Gradient-Synchrotron (AGS) at the Brookhaven National Lab (BNL): first neutrino beam from accelerator
  - Protons on target → pions → decay involving neutrinos
- Signal signature: neutrinos produce muons but no electrons in reactions with target nuclei (neutrons and protons)
  - $\nu + n \rightarrow \mu^- + p$ and $\bar{\nu} + p \rightarrow \mu^+ + n$

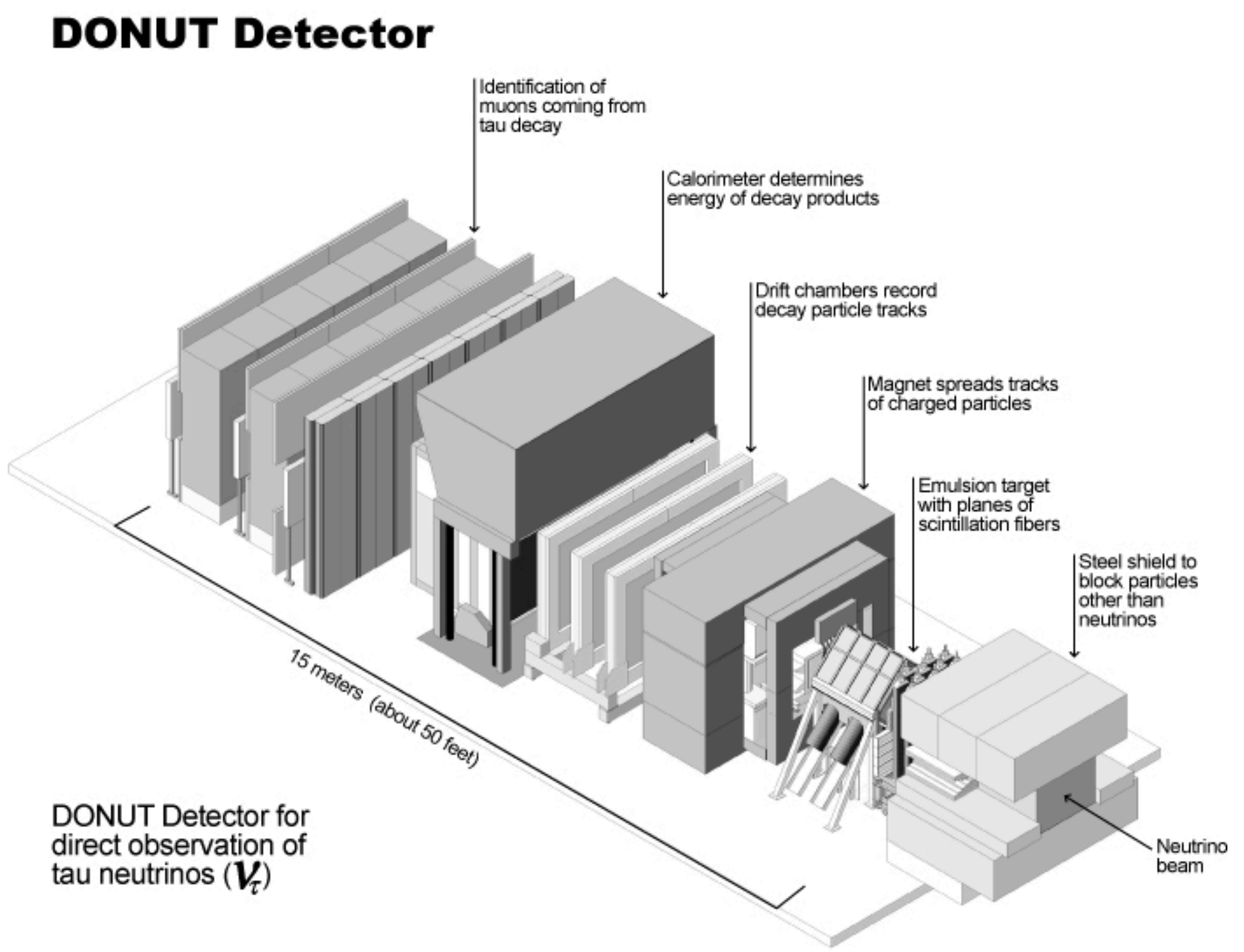


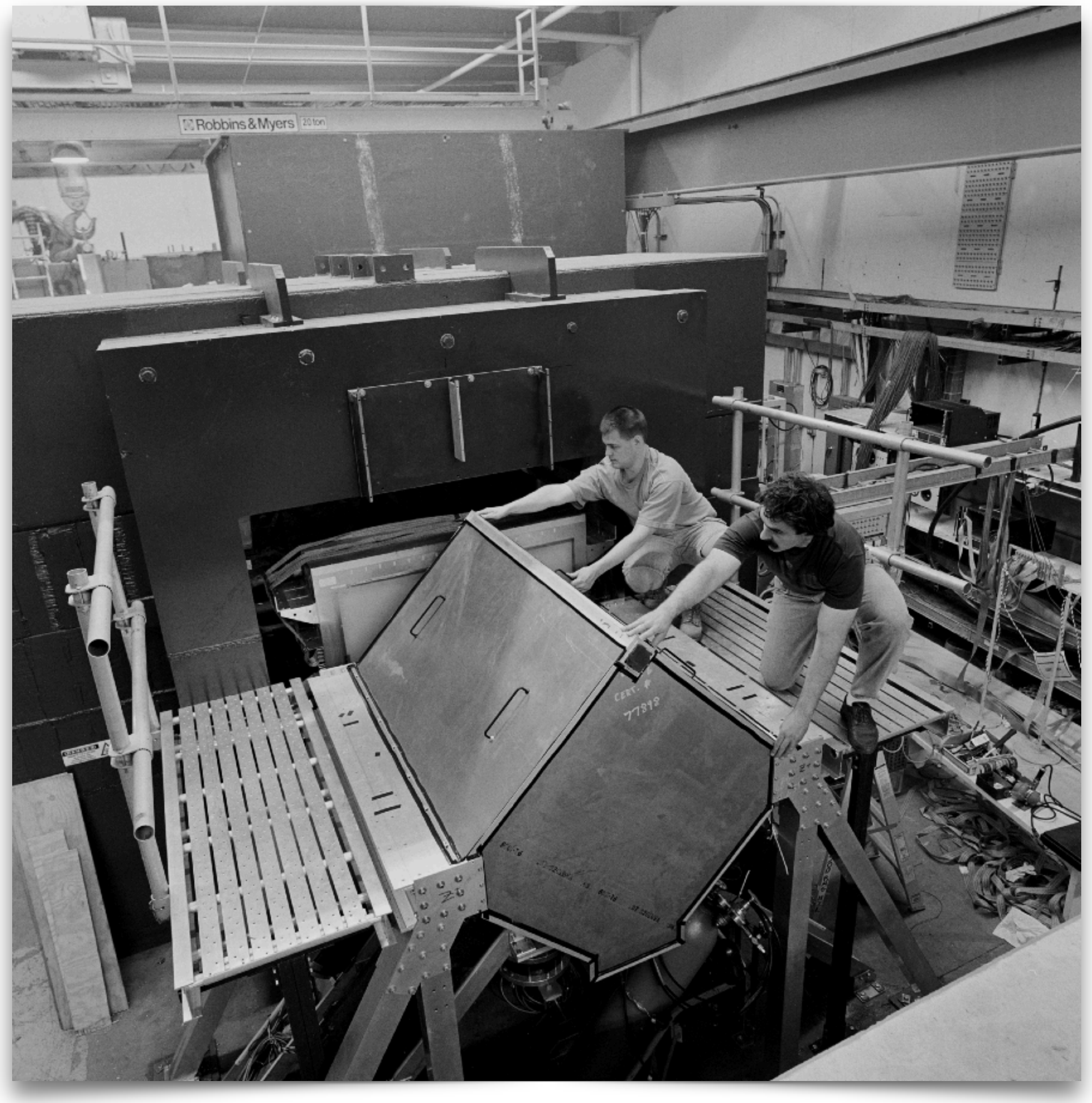
Source: Phys.Rev.Lett. 9 (1962) 36–44

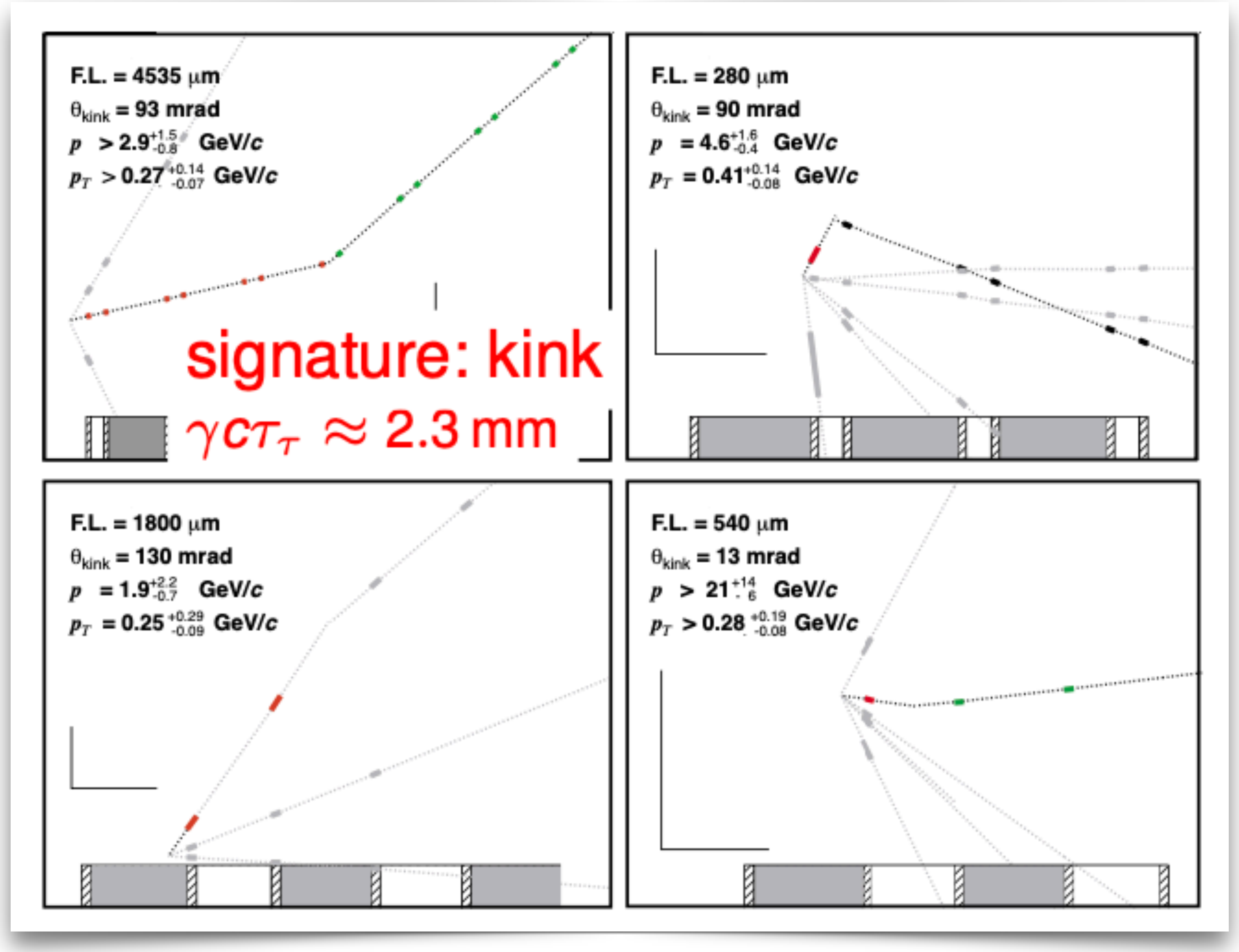
Discovery of the tau neutrino

Background?
Cross-section?

- DONUT (Detector for direct observation of tau neutrinos) experiment (Fermilab, 2000)
  - Tau neutrino from accelerator neutrino beam (800 GeV protons) via decays of charmed mesons
  - Detection: τ decay ($\tau^- \rightarrow \nu_\tau + \ell + \nu_\ell$ and $\tau^- \rightarrow \nu_\tau + h + X$) in emulsion targets and spectrometer, in total observed 9 events



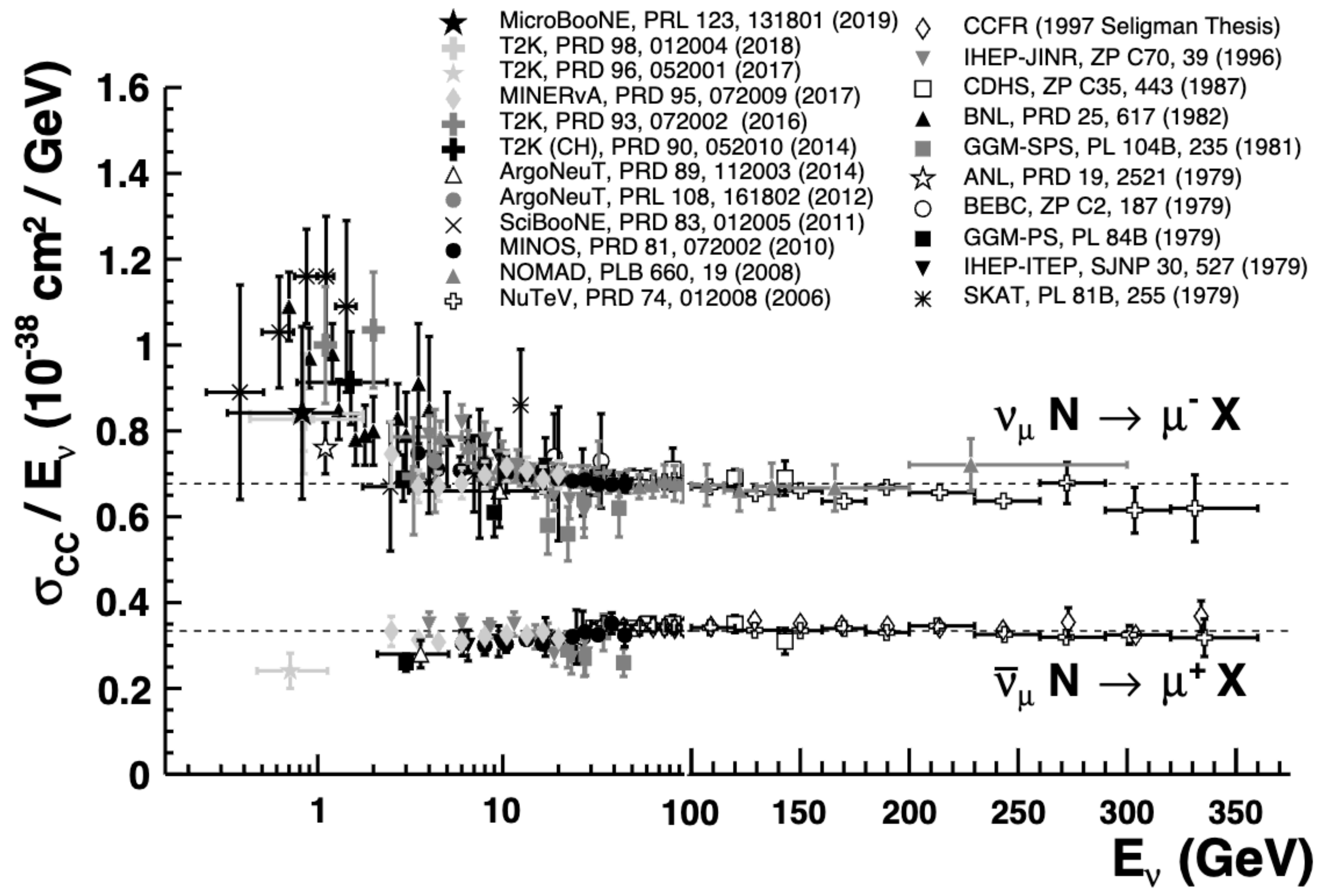




- Fun fact: Neither DONUT nor OPERA have seen anti-tau-neutrinos (consistent with expectation)

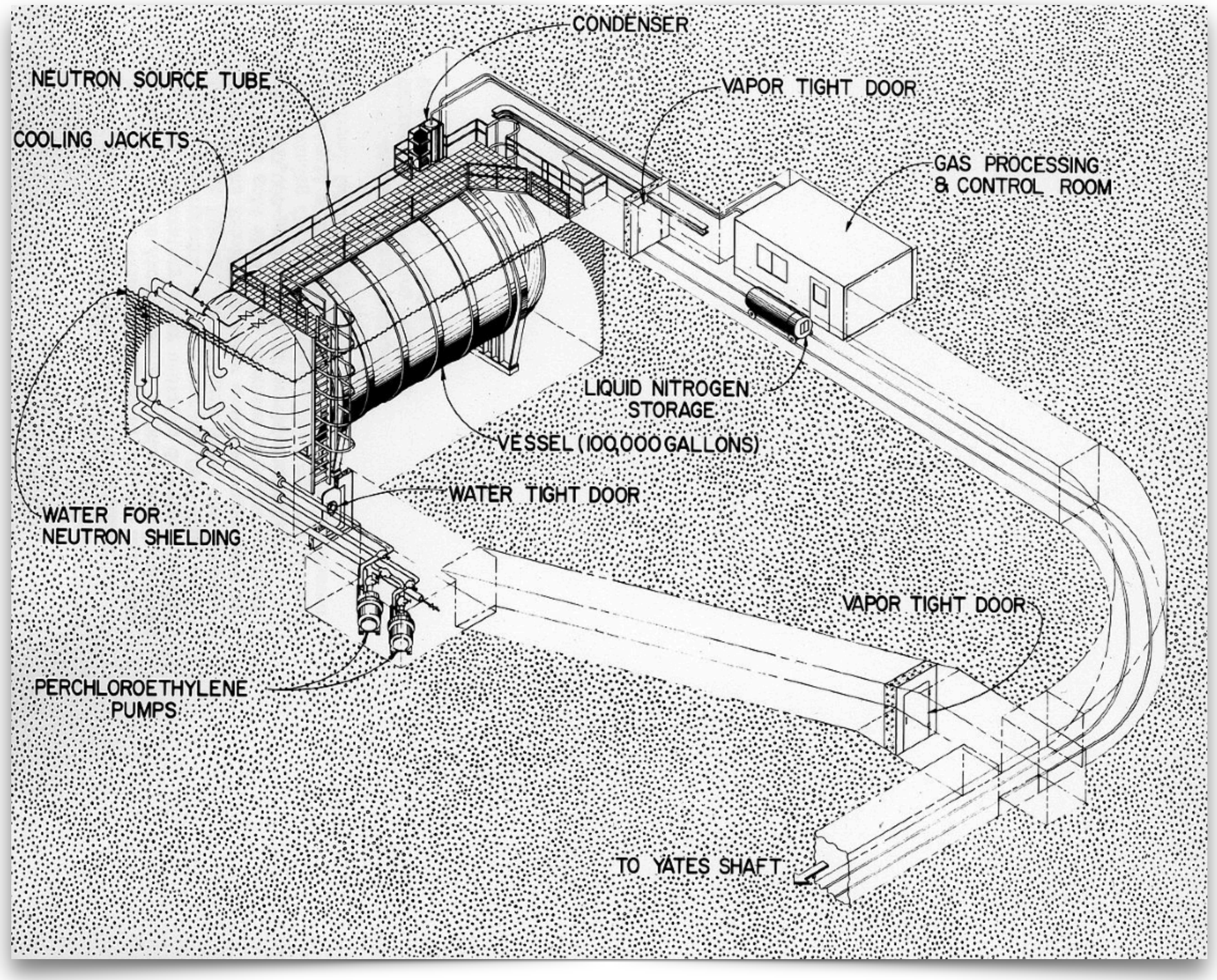
Source: Phys.Lett. B504 (2001) 218-224

Neutrino-nucleus cross section



Solar neutrinos

- 1968: R. Davis Jr. and J. Bahcall, Homestake experiment
  - Radiochemical detection of solar neutrinos in 615 t of tetrachloroethylene (C_2Cl_6), 1500m underground
    - solar neutrinos (~10 MeV) produced in the sun via nuclear fusion
  - Detect $^{37}\text{Cl} + \nu_e \rightarrow {}^{37}\text{Ar} + e^-$ via detection of ^{37}Ar (radioactive with a lifetime of 35 days)
- Expected ~0.5 neutrino interaction per day, observed only 1/3 of that…
  - Solar model wrong?
  - Neutrino cross section model wrong?
  - BSM neutrino interactions (e.g. decays)?
  - Experimental techniques wrong?
- “Solar neutrino problem” unsolved for more than 30 years



Nobel Prize 2002

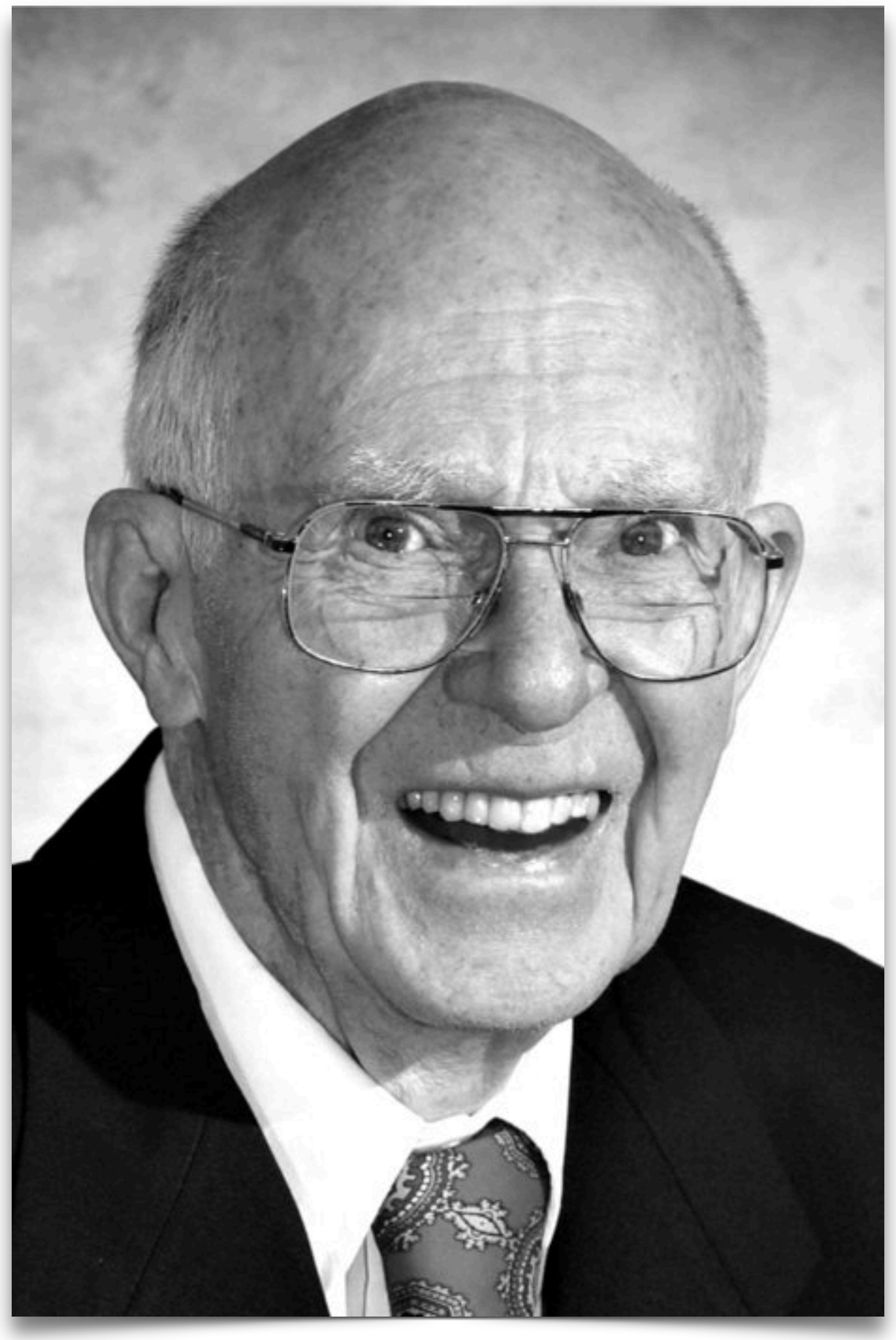
Source: https://www.nobelprize.org/prizes/physics/1995/reines/facts/

Masatoshi Koshiba

Born: 19 September 1926, Toyohashi, Japan

Died: 12 November 2020, Tokyo, Japan

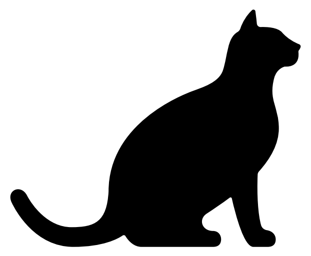
prize motivation: "for pioneering contributions to astrophysics, in particular for the detection of cosmic neutrinos"



Raymond Davis Jr.

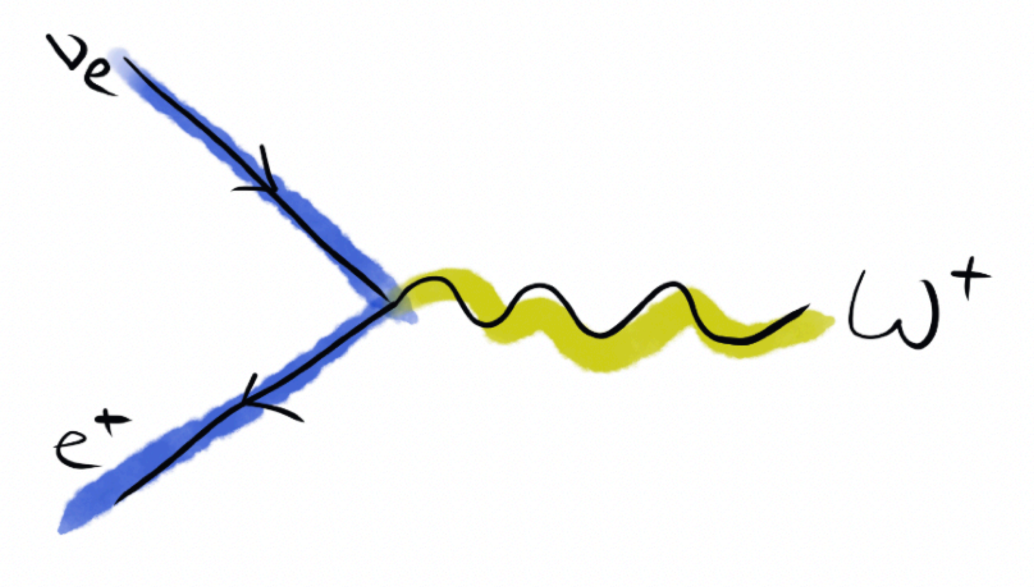
Born: 14 October 1914, Washington, D.C., USA

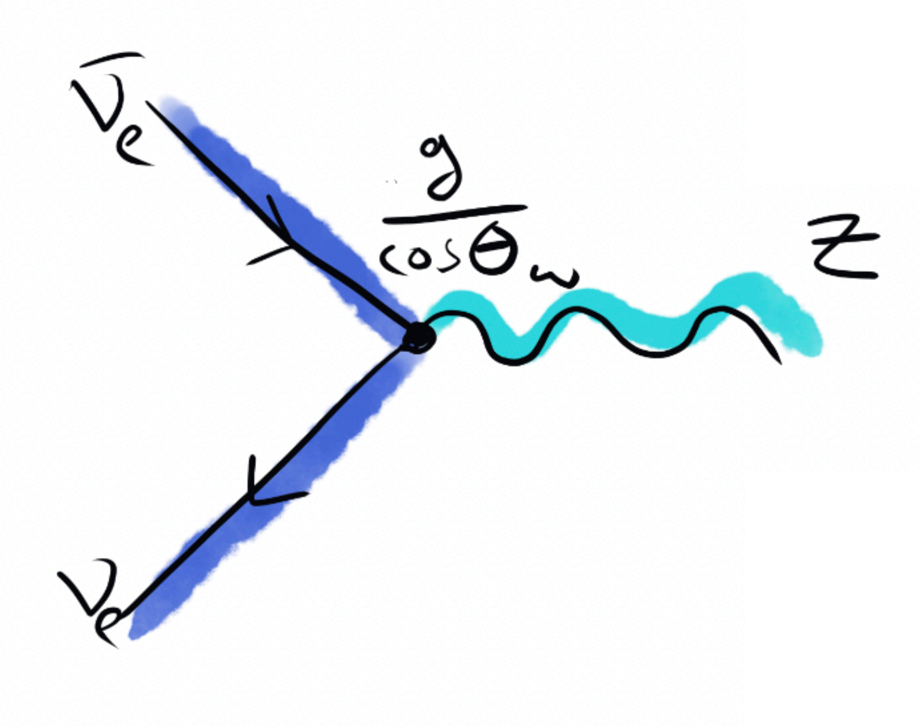
Died: 31 May 2006, Blue Point, NY, USA



Neutrinos in the Standard model

- 3 left-handed neutrinos as isospin partners of the charged e$^-$, μ$^-$, and τ$^-$
- No right-handed neutrinos ↔ neutrinos are massless
  - Unlike the photon or the gluon, this does not come from a fundamental symmetry
- Only weak interactions:
  - via W-bosons: "charged current (CC) interaction"
  - via Z-boson: "neutral current (NC) interaction"
- Cross section very small:
  - $\sigma(\nu_e + e^-) \approx \frac{G_F^2 s}{3\pi} \approx 10^{-38}\,\mathrm{cm}^{-2} = 1\,\mathrm{fb}^{-1}$ for E_ν=1GeV





Neutrino masses

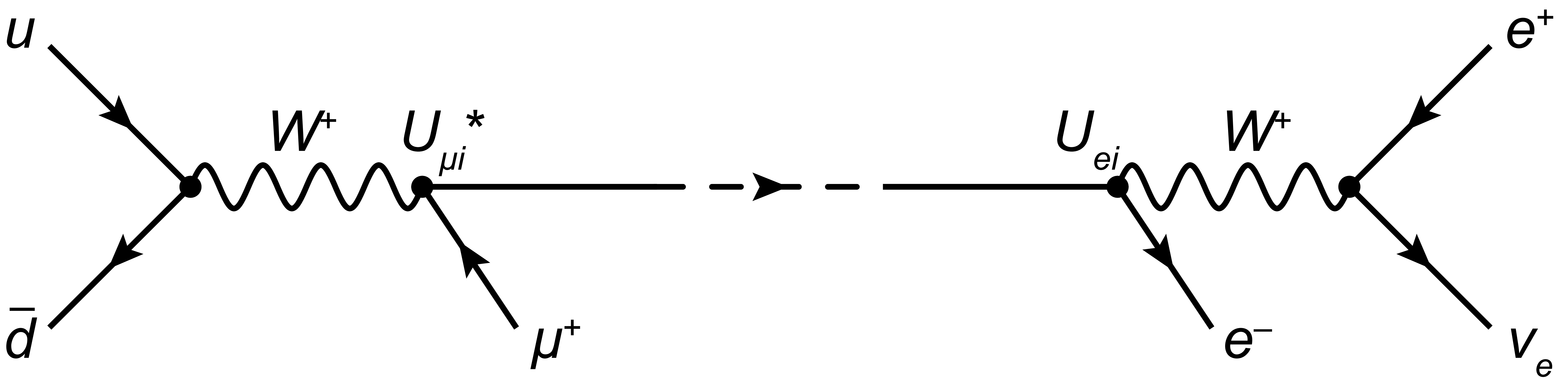
- No direct experimental evidence for absolute neutrino masses:
  - Upper limit from astrophysical observations: $\sum m_\nu < 0.12$ eV
  - Upper limit from beta spectrum (KATRIN): $m_\nu < 0.8$ eV https://www.nature.com/articles/s41567-021-01463-1
- Indirect evidence for neutrino masses and mass differences:
  - Neutrino oscillations suggested by B. Pontecorvo (1957) as explanation of the solar neutrino problem
  - First evidence by Super-Kamiokande (1998) and SNO (2001)
  - Very active experimental program since 20 years:
    - All flavour transitions confirmed (assuming CPT holds), some anomalies found, …

Neutrino oscillations

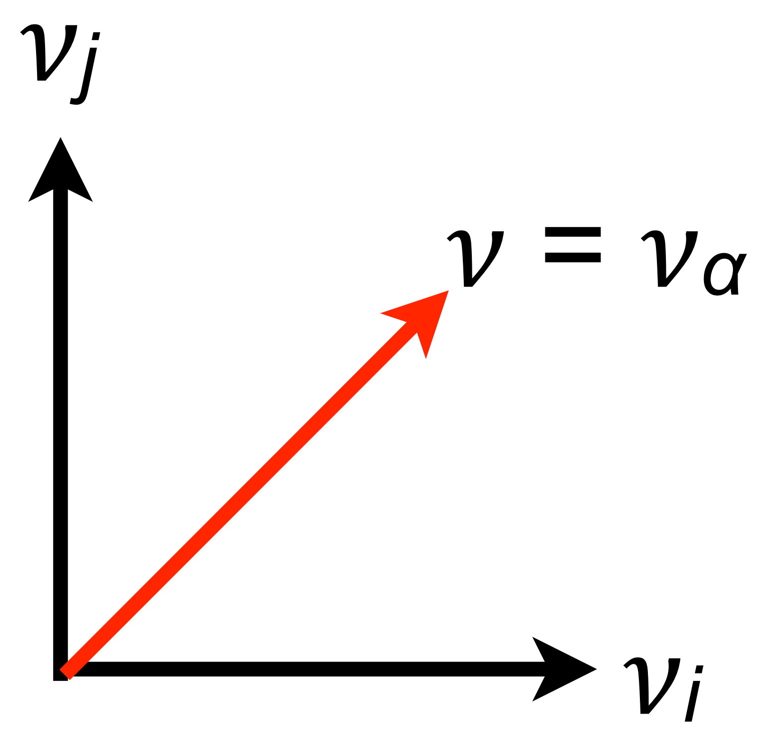
- General idea very similar to quark mixing:
  - Flavour Eigenstates $|\nu_\alpha\rangle$ $(\alpha = e, \mu, \tau)$ are not identical to the mass Eigenstates $|\nu_i\rangle$ $(i = 1,2,3)$
  - Neutrino flavour Eigenstates are defined by the production in association with a charged lepton in CC interactions
- Neutrino mixing matrix is a unitary 3×3 matrix: Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS matrix) with entries $U_{\alpha i}$

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i} |\nu_\alpha\rangle \qquad |\nu_\alpha\rangle = \sum_\alpha U^*_{\alpha i} |\nu_i\rangle$$

Production and decay

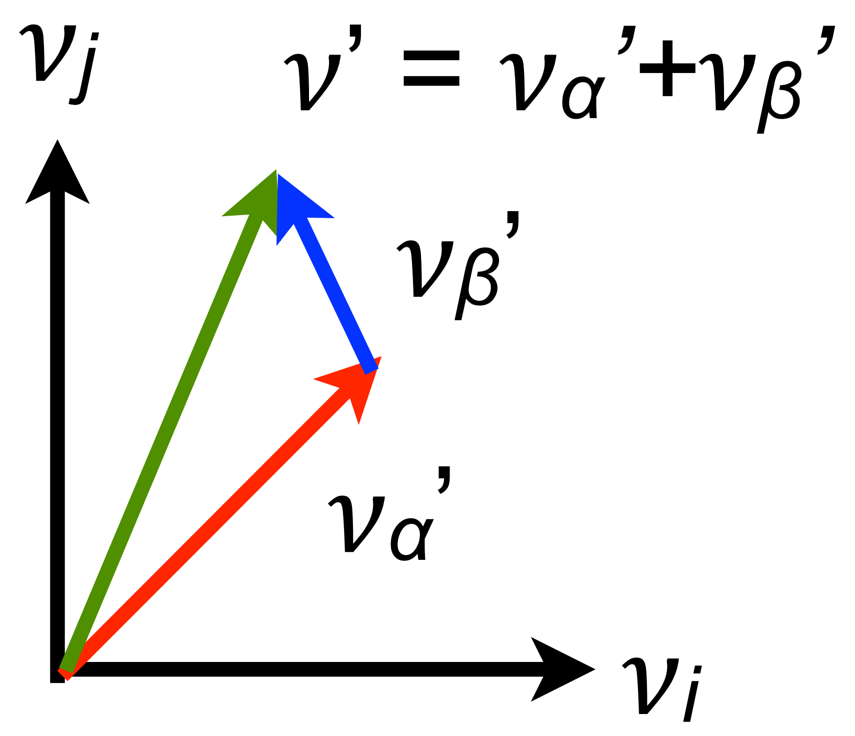


Production process
e. g. pion decay
$\rightarrow$ muon neutrinos



Propagation $\nu \rightarrow \nu'$
physical particle (mass eigenstate): ***different masses*** $\rightarrow$ ***different propagation*** with time/distance

Detection
e. g. electron scattering
$\rightarrow$ projection onto ν_e state

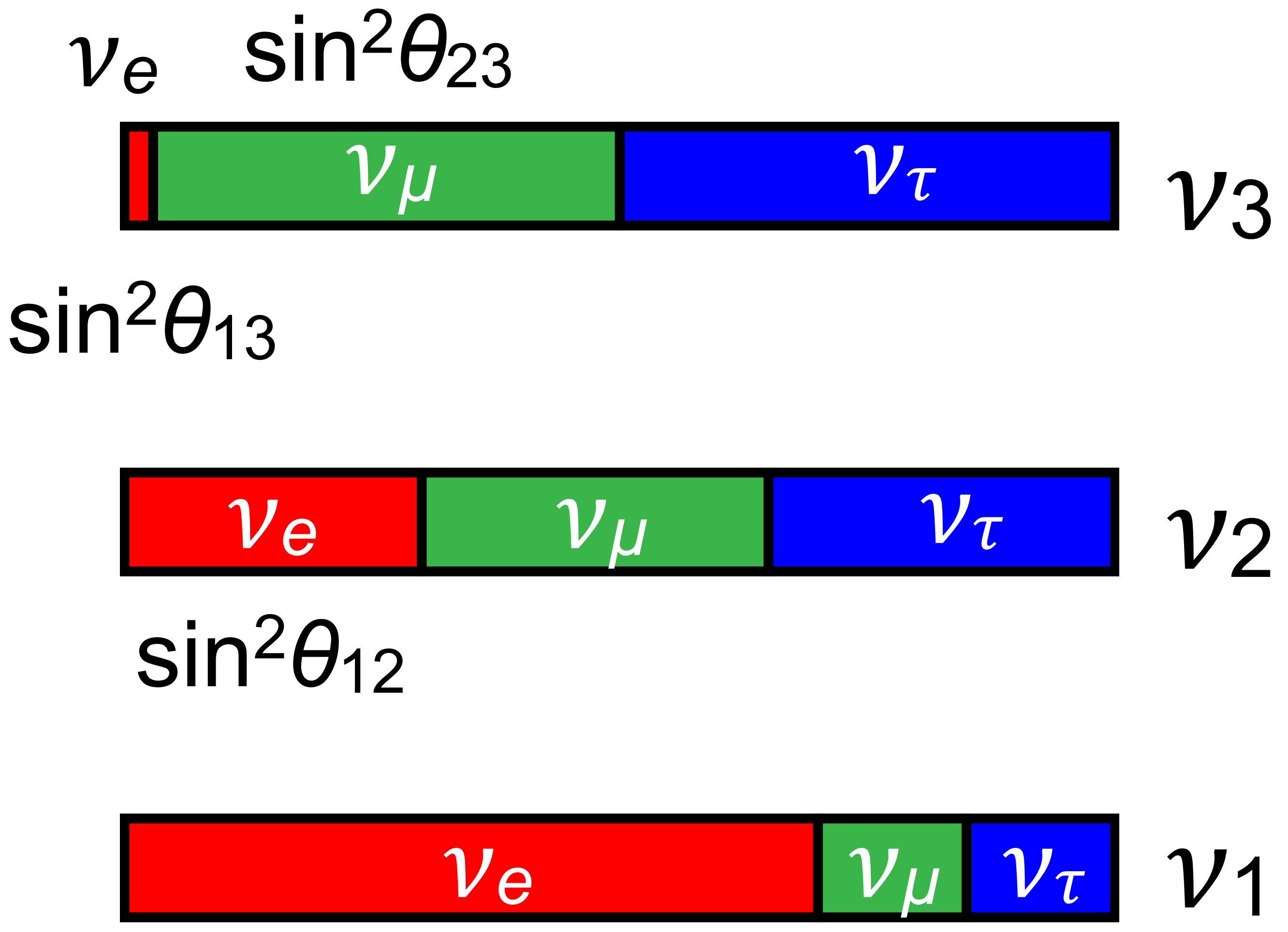


PMNS matrix

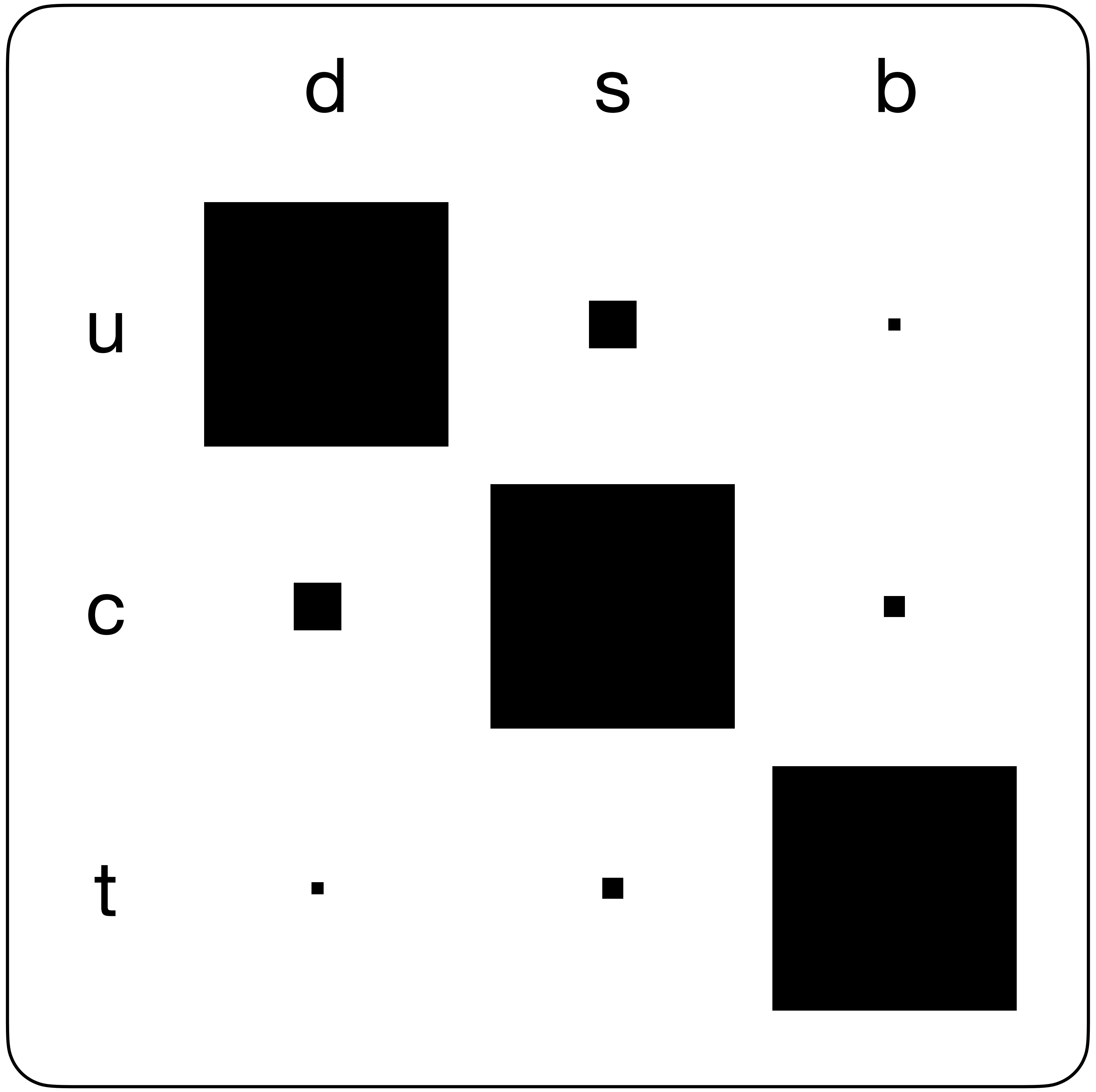
- Standard parametrization with three Euler angles and one CP violating phase ($\cos\theta_{ij} = c_{ij}$ and $\sin\theta_{ij} = s_{ij}$)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

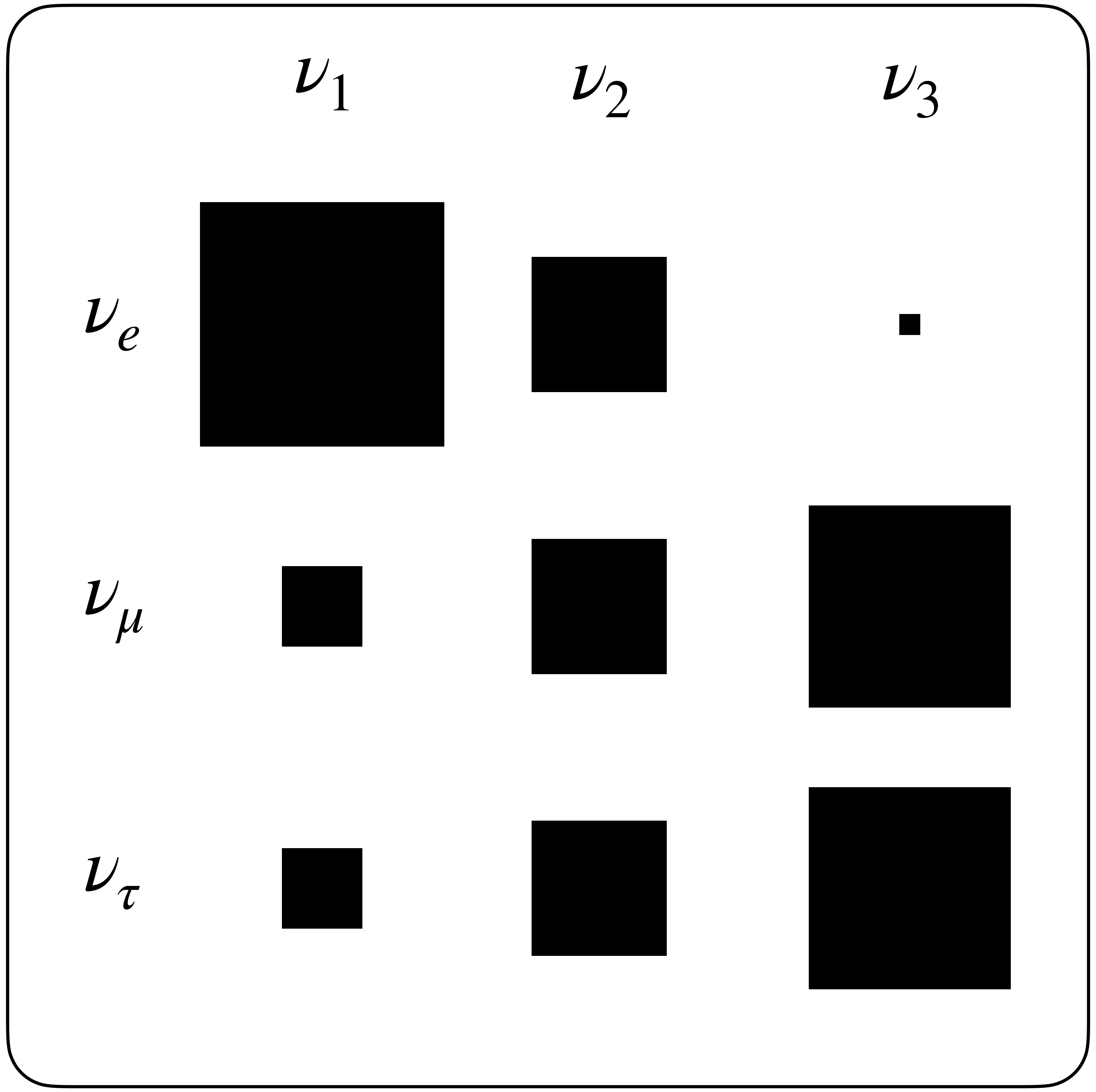
- $\theta_{23} \approx 48°,\ |\Delta m^2_{23}| \approx 2.45 \times 10^{-3}\mathrm{eV}^2$
- $\theta_{12} \approx 34°,\ |\Delta m^2_{21}| \approx 7.4 \times 10^{-5}\mathrm{eV}^2$
- $\theta_{13} \approx 9°,\ |\Delta m^2_{31}| \approx 2.45 \times 10^{-3}\mathrm{eV}^2$
- δ still rather uncertain



PMNS matrix



CKM



PMNS

Majorana phases

- If neutrinos are their own anti-particles, there can be two additional complex, so called Majorana-phases:

$$
\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}
$$

Oscillation amplitudes and probabilities*

- Propagation of mass eigenestates can be describe by plane wave solutions:

$$|\nu_j(t)\rangle = e^{-i(E_j t - \vec{p}_j \vec{x})} |\nu_j(0)\rangle$$ with t and $\vec{x}$ relative to starting point

- Ultrarelativistic limit, here without neglecting the mass:

$$E = (p + m)^{1/2} = p(1 + \frac{m^2}{p^2})^{1/2} \approx p(1 + \frac{m^2}{2p^2}) = p + \frac{m^2}{2p} \approx E + \frac{m_j^2}{2E}$$

- Using $v \approx c$ (and hence in natural units $t \approx L$):

$$|\nu_j(L)\rangle = e^{-i\left(\frac{m_j^2 L}{2E}\right)} |\nu_j(0)\rangle$$

* A quantum mechanically more correct treatment (with identical result) can be found in: https://pdg.lbl.gov/2020/reviews/rpp2020-rev-neutrino-mixing.pdf

$$(1+x)^{1/2} = 1 + \frac{x^2}{2} - \frac{x^4}{8} + \dots$$

Oscillation amplitudes and probabilities

- Transition **probability**:

$$P(\nu_\alpha \to \nu_\beta, t) \equiv |\mathcal{A}(\nu_\alpha \to \nu_\beta, t)|^2$$

$$= \left| \sum_i U^*_{\alpha i} U_{\beta i} e^{-i\frac{m_i^2}{2E}t} \right|^2$$

$$= \sum_i \sum_j U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} e^{-i\frac{(m_i^2 - m_j^2)}{2E}t}$$

$$= \sum_i \sum_j U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} e^{-i\frac{\Delta m_{ij}^2}{2E}L}$$

squared mass difference (unknown)

distance from production to detection (known)

neutrino energy (known-ish)

PMNS matrix elements (unknown)

Oscillation amplitudes and probabilities

- Transition **probability**:

“survival probability” for β=α

$$P(\nu_\alpha \to \nu_\beta, L) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right)$$

different sign for neutrino and antineutrino

$$\pm 2\sum_{i>j} \Im\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin\left(\frac{\Delta m^2_{ij}L}{2E}\right)$$

Imaginary part only non-zero if CP phase δ is non-zero

Careful: Cross-section and production modes very different for neutrinos and anti-neutrinos.

Example: 2 Flavour Oscillations

- Probability of flavour transition: Appearance of a new flavour

$$\boxed{\begin{aligned} |\langle \nu_\beta | \nu_\alpha \rangle|^2 = |\langle \nu_\alpha | \nu_\beta \rangle|^2 &= \sin^2(2\theta)\sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right) \\ &= \sin^2(2\theta)\sin^2\left(1.27\Delta m_{ij}^2[\mathrm{eV}^2]\frac{L[\mathrm{km}]}{4E[\mathrm{GeV}]}\right) \end{aligned}}$$

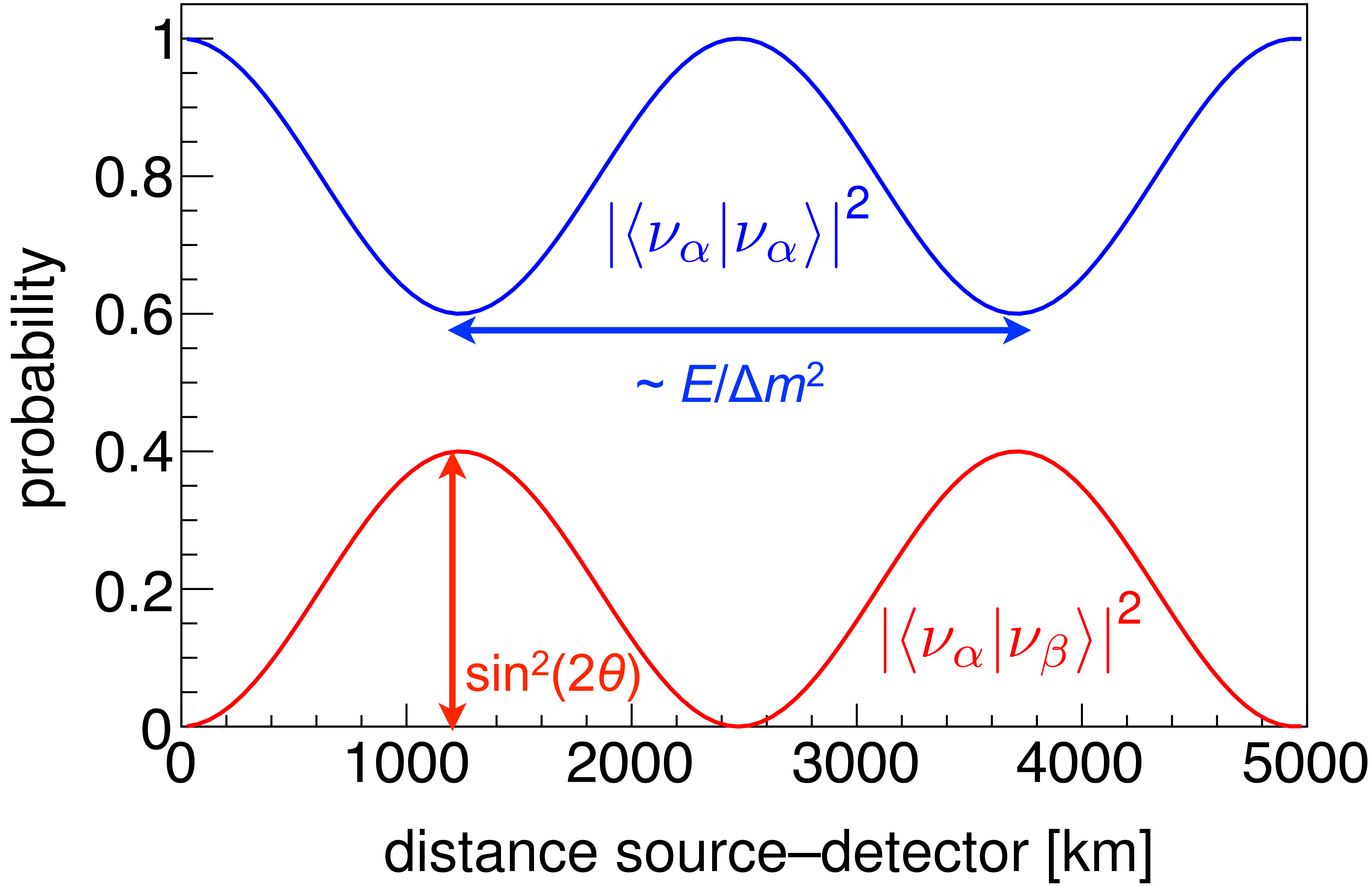
- Probability of flavour survival: Disappearance of a flavour

$$\boxed{|\langle \nu_\beta | \nu_\beta \rangle|^2 = |\langle \nu_\alpha | \nu_\alpha \rangle|^2 = 1 - \sin^2(2\theta)\sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right)}$$

- Mixing angle $\sin^2(2\theta)$: Amplitude of the oscillation
- Squared mass difference Δm^2: Frequency of the oscillation

$$\frac{\Delta_{jk}(mc^2)^2\,L}{4\hbar c\,E} = \frac{\mathrm{GeV\,fm}}{4\hbar c} \times \frac{\Delta_{jk}m^2}{\mathrm{eV}^2}\frac{L}{\mathrm{km}}\frac{\mathrm{GeV}}{E} \approx 1.27 \times \frac{\Delta_{jk}m^2}{\mathrm{eV}^2}\frac{L}{\mathrm{km}}\frac{\mathrm{GeV}}{E}$$

Example: 2 Flavour Oscillations



$\sin^2(2\theta) = 0.4$
$\Delta m^2 = 0.001\,\mathrm{eV}^2$
$E = 1\,\mathrm{GeV}$

Credit: U. Husemann

PINGO

- **PINGO:**
  - Umfrage: Teilchenphysik 1 (WS 23/24)
  - Zugangsnummer: 434521
  - Link: https://pingo.coactum.de/events/434521

PINGO: Neutrino Oscillations

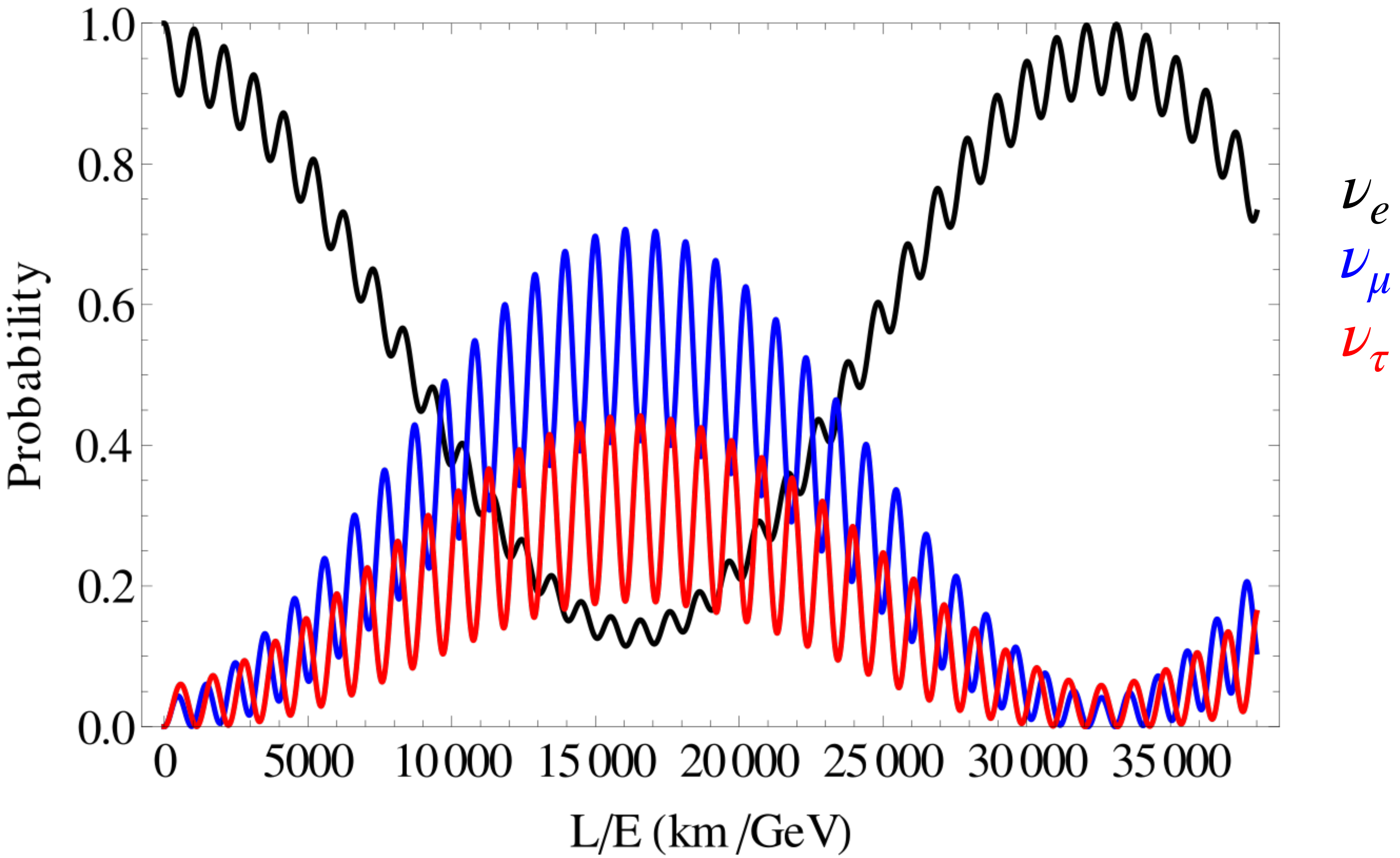
- In an experiment on neutrino oscillations, which parameters can be tuned experimentally to increase the sensitivity?
  - Mixing angle θ_{ij}
  - Majorana phases $\alpha_{1,2}$
  - Difference of squared masses Δm_{ij}^2
  - Distance source–detector L
  - Beam energy E

PINGO: Neutrino Oscillations

- In an experiment on neutrino oscillations, which parameters can be tuned experimentally to increase the sensitivity?
  - Mixing angle θ_{ij}
  - Majorana phases $\alpha_{1,2}$
  - Difference of squared masses Δm_{ij}^2
  - **Distance source–detector L**
  - **Beam energy E**
- Design goal for experiments: place detectors at distance L/E such that different structures can be resolved optimally: Several experiments complementing each other

- End of lecture 16.01.2024

Example: 3 Flavour Oscillations

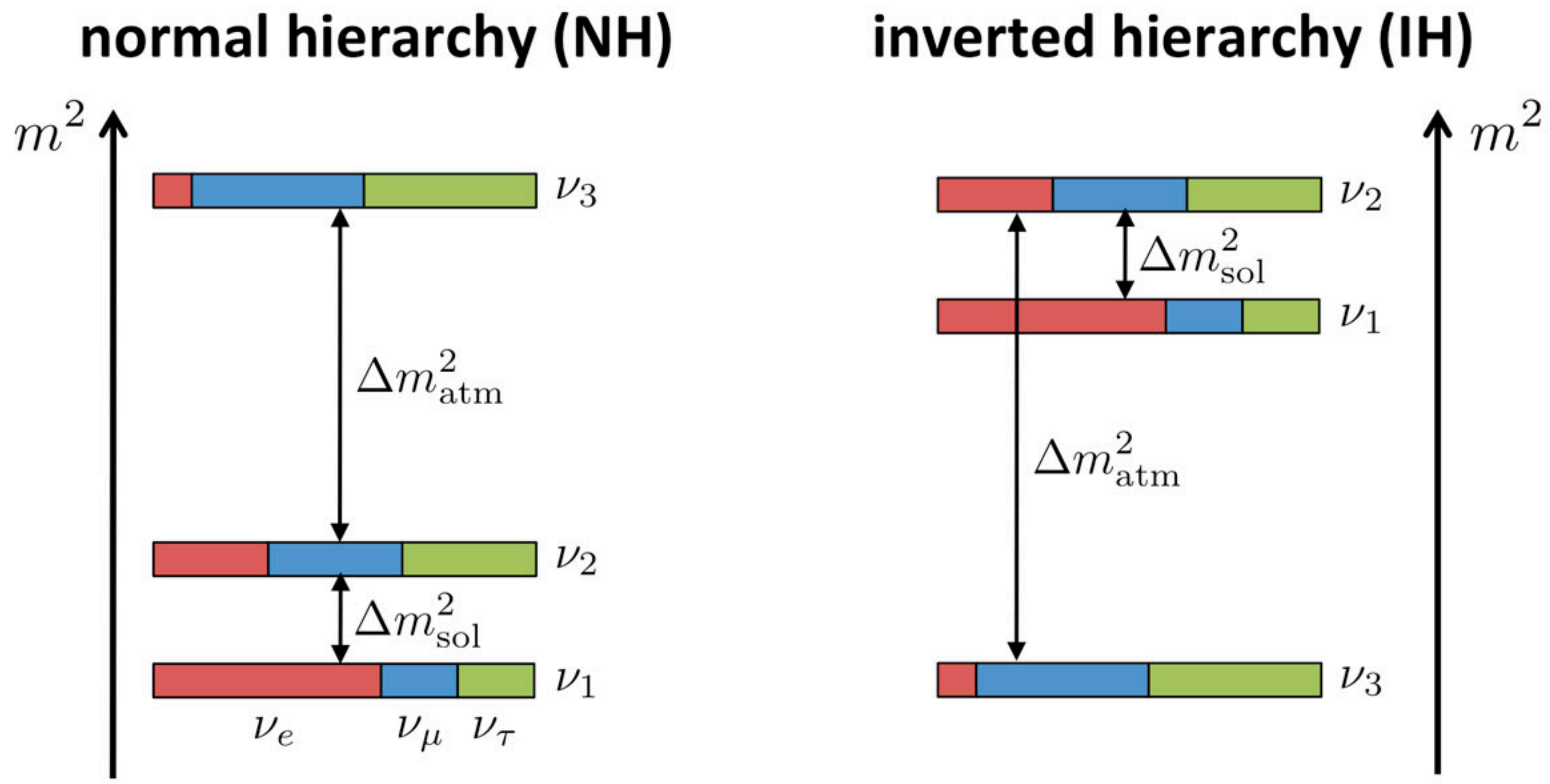


- Qualitative example with initial ν_e (with realistic choice of parameters)
  - $\approx$ 2 different amplitudes because $\theta_{13} \ll \theta_{12} < \theta_{23}$
  - 2 different frequencies because $\Delta m^2_{21} \ll |\Delta m^2_{31}| \approx |\Delta m^2_{32}|$

Source: https://en.wikipedia.org/wiki/Neutrino_oscillation

Neutrino mass hierarchy

- Neutrino oscillations with 3 flavours
  - Large mass difference between m_3 and $m_{1,2}$
  - Small mass difference between m_2 and m_1
- No conclusion about the mass hierarchy, i.e. the sign of Δm_{31}^2
  - Known: $\Delta m_{21}^2 > 0$ (MSW effect, later)
  - Normal mass hierarchy (NH): $m_1 < m_2 \ll m_3$
  - Inverted mass hierarchy (IH): $m_3 \ll m_1 < m_2$
  - Quasi-degenerate mass hierarchy (QD): $m_1 \approx m_2 \approx m_3$



Source: Fermilab

Neutrino masses in the Standard model

- Neutrino masses are a requirement for neutrino oscillations: BSM!
- Neutrino mass term in the Standard model do not exist, because there are no right handed neutrinos (→ never observed experimentally):
- Possible solution 1: “Dirac neutrinos” (not in the SM!)

$$\mathcal{L}_{\mathrm{D}} = -m^D \bar{\nu}_L \nu_R + \mathrm{h.c.}$$

  - Introduction of right-handed neutrinos (and left-handed antineutrinos) as $SU(2)_L$ singlet
  - no interaction with W and Z bosons (“sterile neutrinos”)

  No experimental evidence for the existence of sterile neutrinos!

Neutrino masses in the Standard model

- Possible solution: “Majorana neutrinos” (not in the SM!)
  - Neutral particles identical to their anti-particles, e. g. photons
  - note that in the SM left-handed neutrinos are not neutral under electroweak charge ($I_3 = +1/2, Y = -1$, i.e. non-zero weak isospin)

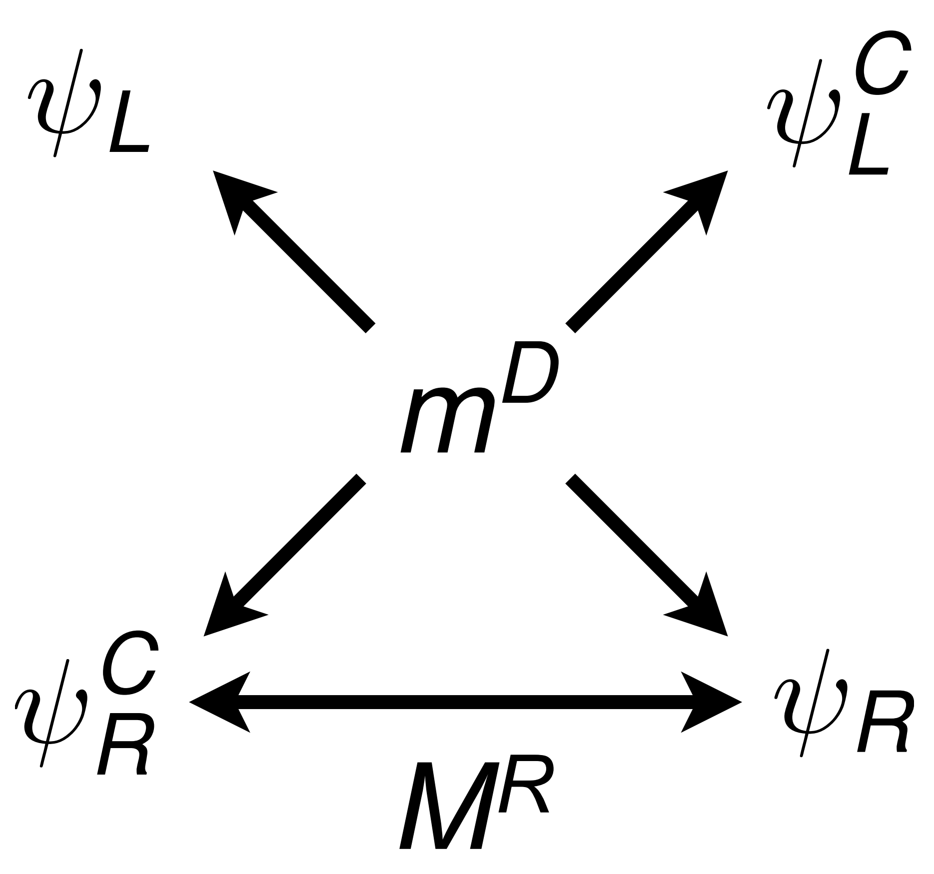
$$\mathcal{L}_{\mathrm{M}} = -\frac{1}{2} M^R \bar{\nu}_R^C \nu_R + \text{h.c.}$$

- Rather severe consequences:
  - Transition between neutrino and charge conjugated neutrino: lepton number violation (not just lepton flavour violation as in neutrino oscillations)
  - Experimental test: Neutrinoless double-beta decay (later)

No experimental evidence for the existence of Majorana neutrinos (yet)!

Seesaw mechanism

- Appealing possibility to explain why neutrino masses are so tiny: Neutrinos with both Dirac and Majorana masses:



$$\mathcal{L}_{\mathrm{DM}} = -\tfrac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{\nu}_R^C \end{pmatrix} \begin{pmatrix} 0 & m^D \\ m^D & M^R \end{pmatrix} \begin{pmatrix} \nu_L^C \\ \nu_R \end{pmatrix} + \text{h.c.}$$

- Diagonalize with assumption $M^R \gg m^D$
  - Eigenvectors $\nu_{\text{light}} = \nu_L + \nu_L^C$ and $\nu_{\text{right}} = \nu_R + \nu_R^C$
  - Eigenvalues (=masses of physical neutrinos):

$$m_{\text{light}} \approx \frac{\left(m^D\right)^2}{M^R} \text{ and } m_{\text{heavy}} \approx M^R$$

What questions do you have?