Karlsruhe Institute of Technology

Particle Physics 1
Lecture 9: QED and QCD

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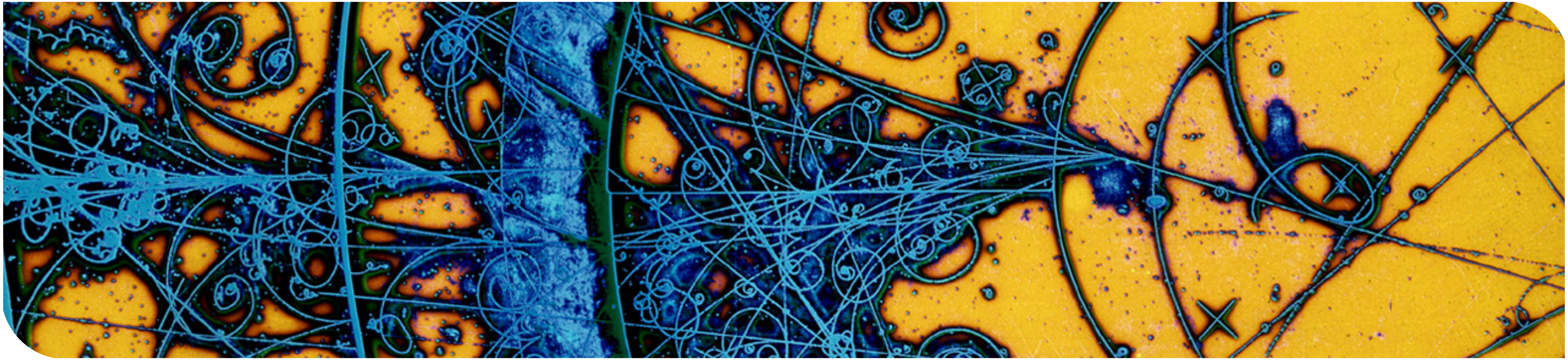


Credit: CERN

Questions from past lectures

Learning goals

- Understand how the QED Lagrangian is derived
- Feynman rules and Feynman graphs
- Understand how the QCD Lagrangian is derived
- Parity invariance of QED (and QCD)
- Explain experimental observations of parity violation

Local Symmetries

- Local ($U = U(x)$, $\alpha = \alpha(x)$) phase transformation of a wave function:

  $$\psi(x) \rightarrow \psi(x)' = U(x)\psi(x) = e^{iq\alpha(x)}\psi(x)$$

  $$\bar{\psi}(x) \rightarrow \bar{\psi}(x)' = U(x)\bar{\psi}(x) = e^{-iq\alpha(x)}\bar{\psi}(x)$$

  - q is a (for now) arbitrary real constant

- Is the transformed Lagrange density of the Dirac equation **invariant** under a **local** phase transformation? (it is under global phase transformations, see lecture 8)

  $$\mathscr{L}' = \bar{\psi}' i\gamma^\mu \partial_\mu \psi' - m\bar{\psi}'\psi' = \mathscr{L} \text{ ?}$$

Local Symmetries

$$m\bar{\psi}'\psi' = m\bar{\psi}\psi$$

$$\partial_\mu e^{iq\alpha(x)}\psi(x) = iq\left(\partial_\mu\alpha(x))\right)e^{iq\alpha(x)}\psi(x) + e^{iq\alpha(x)}\left(\partial_\mu\psi(x)\right) = e^{iq\alpha}\left(iq\psi\partial_\mu\alpha(x) + \partial_\mu\psi\right)$$

$$\begin{aligned}
\mathcal{L}' &= \bar{\psi}' i\gamma^\mu \partial_\mu \psi' - m\bar{\psi}'\psi' \\
&= e^{-iq\alpha(x)}\bar{\psi} i\gamma^\mu e^{iq\alpha(x)}\left(\partial_\mu\psi + iq\psi\partial_\mu\alpha(x)\right) - m\bar{\psi}\psi \\
&= \bar{\psi} i\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi - q\bar{\psi}\gamma^\mu\psi\partial_\mu\alpha(x) \\
&= \mathcal{L} - q\bar{\psi}\gamma^\mu\psi\partial_\mu\alpha(x)
\end{aligned}$$

Not invariant under local gauge transformation… 😔

Covariant derivative

- Invariance can be achieved by replacing the "normal" derivative with the "covariant" derivative: $\partial_\mu \rightarrow D_\mu = \partial_\mu + iqA_\mu(x)$ with arbitrary gauge field $A_\mu(x)$ and the following U(1) transformation behaviour:

$$\begin{aligned}
\psi(x) &\rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x) \\
\overline{\psi}(x) &\rightarrow \overline{\psi}'(x) = \overline{\psi}(x)e^{-i\alpha(x)} \\
A(x)_\mu &\rightarrow A'(x)_\mu = A(x)_\mu - \frac{1}{q}\partial_\mu\alpha(x)
\end{aligned}$$

Covariant derivative

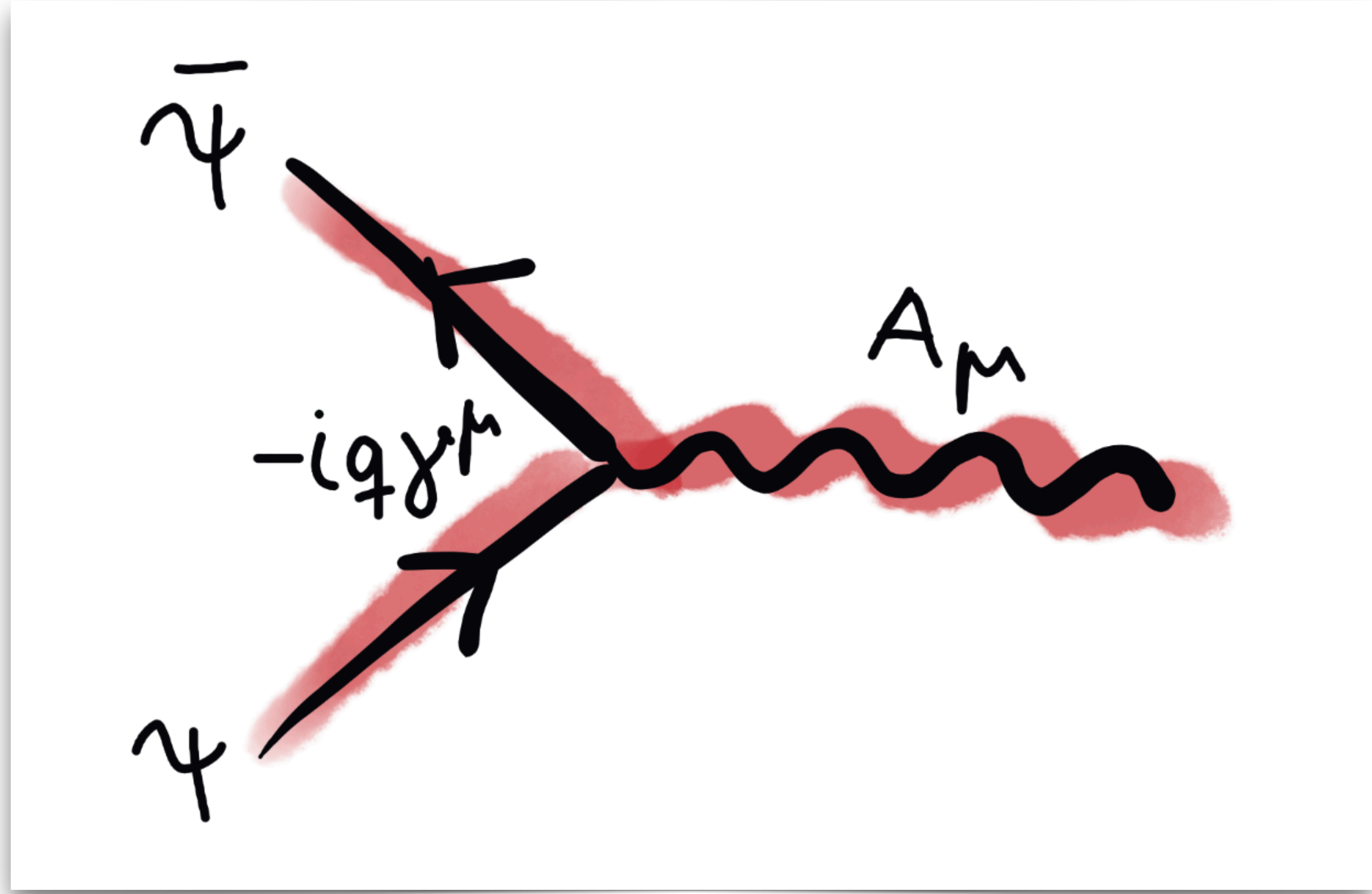
$$\mathcal{L}' = \overline{\psi}'(i\gamma^\mu D'_\mu - m)\psi'$$
$$= \overline{\psi}'(i\gamma^\mu(\partial_\mu + iqA'_\mu) - m)\psi'$$
$$= \overline{\psi}e^{-i\alpha(x)}(i\gamma^\mu(\partial_\mu + iqA_\mu - i\partial_\mu\alpha(x)) - m)e^{i\alpha(x)}\psi$$
$$= \overline{\psi}(i\gamma^\mu(\partial_\mu + i\partial_\mu\alpha(x) + iqA_\mu - i\partial_\mu\alpha(x)) - m)\psi$$
$$= \overline{\psi}(i\gamma^\mu D_\mu - m)\psi = \mathcal{L}$$

Invariant under local gauge transformation! 😃

Towards the QED Lagrangian

- Covariant derivative introduces gauge vector field A_μ
- A_μ couples to property q of the spinor field $\psi(x)$
  - q can be identified with the electrical charge
  - A_μ can be identified with the photon field

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi$$
$$= \underbrace{\overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi}_{\text{free fermion}} \underbrace{- q(\overline{\psi}\gamma^\mu\psi)A_\mu}_{\text{interaction}}$$



The QED Lagrangian

- Missing piece to full Lagrangian: Lagrange density of the vector field $A_\mu(x)$ itself:

$$\mathscr{L}_A = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu \text{ (see Proca equation)}$$

- Is the transformed Lagrange density of the Proca equation **invariant** under the local phase transformation?

The QED Lagrangian: Photon kinetic term

transformation behaviour

$$A(x)_\mu \to A'(x)_\mu = A(x)_\mu - \frac{1}{q}\partial_\mu \alpha(x)$$

field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{i}{q}[D_\mu, D_\nu]$$

$$\begin{aligned} F_{\mu\nu} \to F'_{\mu\nu} &= \partial_\mu A'_\nu - \partial_\nu A'_\mu \\ &= \partial_\mu(A_\nu - \partial_\nu \alpha(x)) - \partial_\nu(A_\mu - \partial_\mu \alpha(x)) \\ &= F_{\mu\nu} \end{aligned}$$

Invariant under local gauge transformation! 😃

The QED Lagrangian: Photon mass term

transformation behaviour

$$A(x)_\mu \rightarrow A'(x)_\mu = A(x)_\mu - \frac{1}{q}\partial_\mu \alpha(x)$$

$$m^2 A_\mu A^\mu \rightarrow m^2 A'_\mu A'^\mu = m^2 \left(A^\mu A_\mu - 2A^\mu \frac{1}{q}\partial_\mu \alpha + \frac{1}{q^2}\partial^\mu \alpha \partial_\mu \alpha \right)$$

Not invariant under local gauge transformation… 😔

… unless we make the vectorfield massless (m=0)! 😃

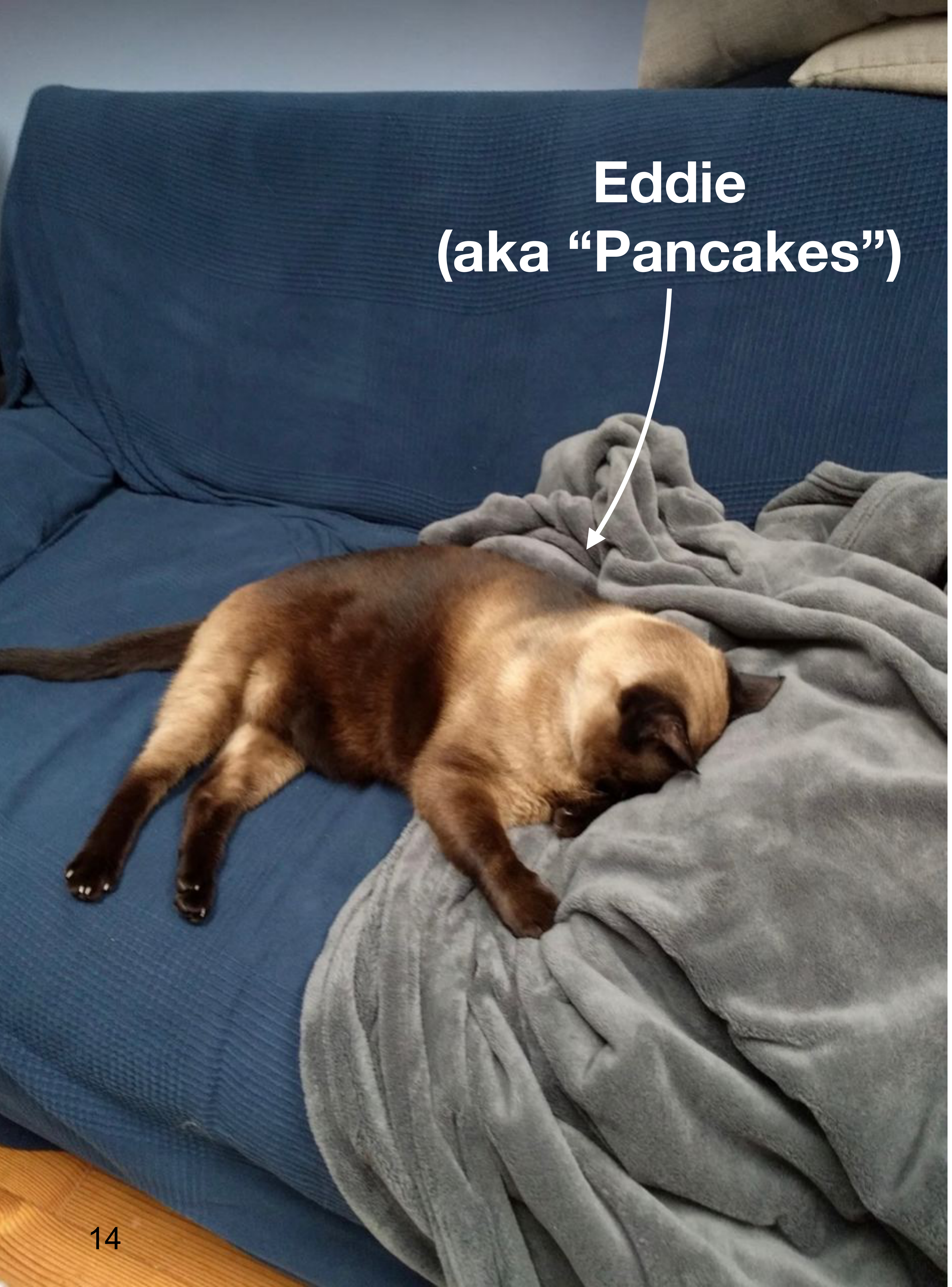
The QED Lagrangian

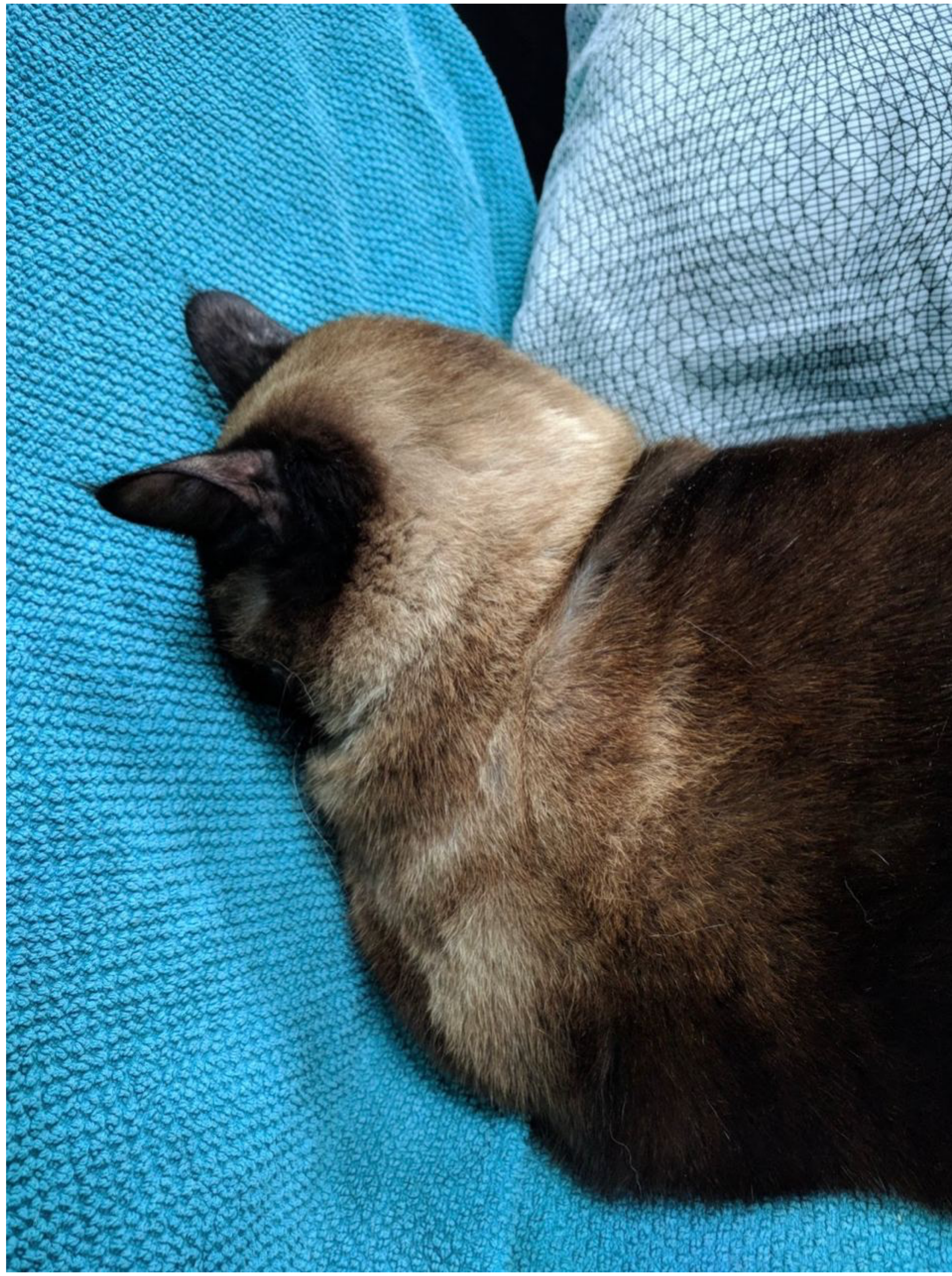
$$
\begin{aligned}
\mathcal{L}_{\text{QED}} &= \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} \\
&= \underbrace{\overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi}_{\text{free fermion}} - \underbrace{q(\overline{\psi}\gamma^\mu\psi)A_\mu}_{\text{interaction}} - \underbrace{\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{\text{gauge field}}
\end{aligned}
$$

Summary QED

- Postulation of U(1) local gauge symmetry lead to Lagrangian of QED with a massless photon
  - Start with wave function that represents fermions
  - Postulate gauge invariance (i.e. gauge transformation $\psi(x) \rightarrow \psi(x)' = e^{iq\alpha(x)}\psi(x)$ should leave theory invariant)
  - “Unwanted” extra term is created by the derivative
  - Gauge invariance can be restored by adding a vector-field A(x) (the photon)

- All currently known models of particle physics incorporate gauge symmetry (i.e. local symmetry)


Eddie
(aka “Pancakes”)



From Lagrange density to observables

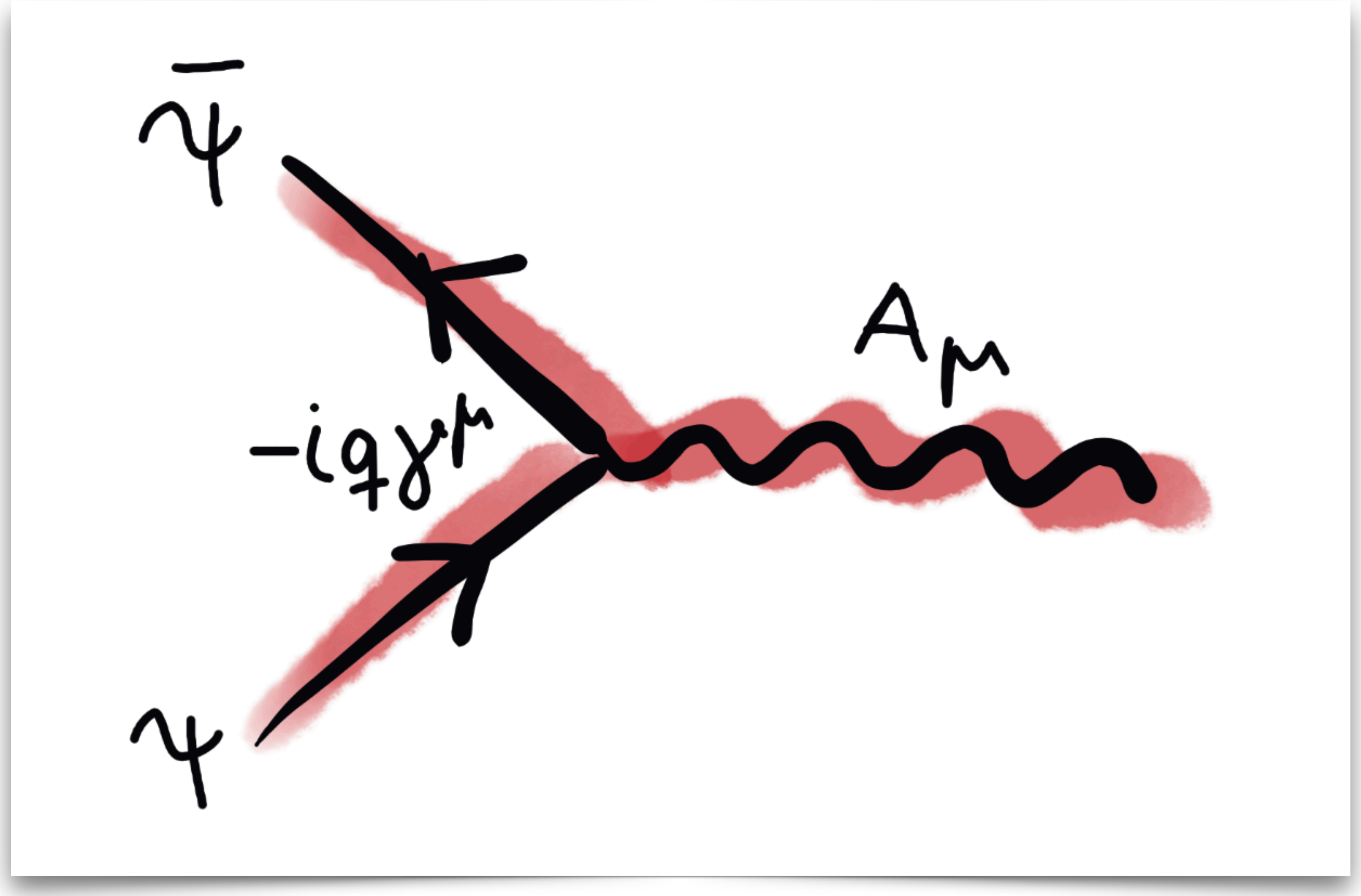
- Rate of a process given by (Fermi's golden rule)

$$\frac{dN}{dt} = \frac{|\text{matrix element}|^2}{\text{flux of incoming particles}} \cdot \text{phase space}$$

- All dynamics of the process encoded in matrix element $\mathscr{M}$
  - Element of scattering matrix S that transforms initial state into outgoing final state
- Rules how to compute matrix element in perturbation theory: **Feynman rules**
- Graphical representation (this is **not** reality): **Feynman graphs**

Perturbative series

- Matrix element $\mathscr{M} = \mathscr{M}_{if} = \psi_f^\dagger \psi_{\text{scat}} = \psi_f^\dagger \mathcal{S} \psi_i$
  - ψ_f: final state after scattering of initial state ψ_i
- Example fermion scattering in QED:
  - $(i\gamma^\mu \partial_\mu - m)\psi_{\text{scat}} = -e\gamma^\mu A_\mu \psi_{\text{scat}}$
  - Can not be solved analytically 😔



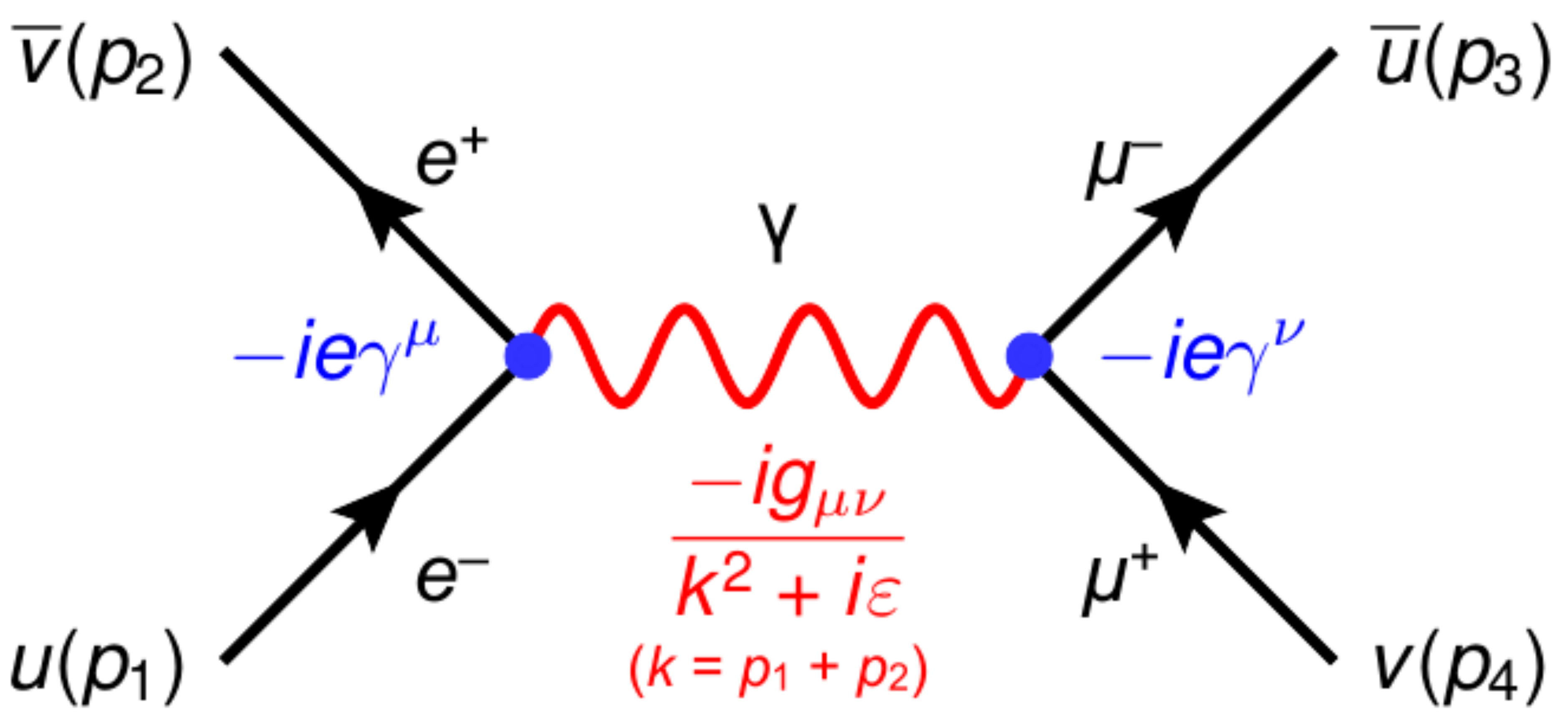
- But since $\alpha = e^2 \ll 1$ solution can be expanded in orders of coupling constant 😃

$$\psi_{\text{scat}} = \mathcal{S}\psi_i = \left[\sum_{n=0}^{\infty} \alpha^n S_n\right] \psi_i$$

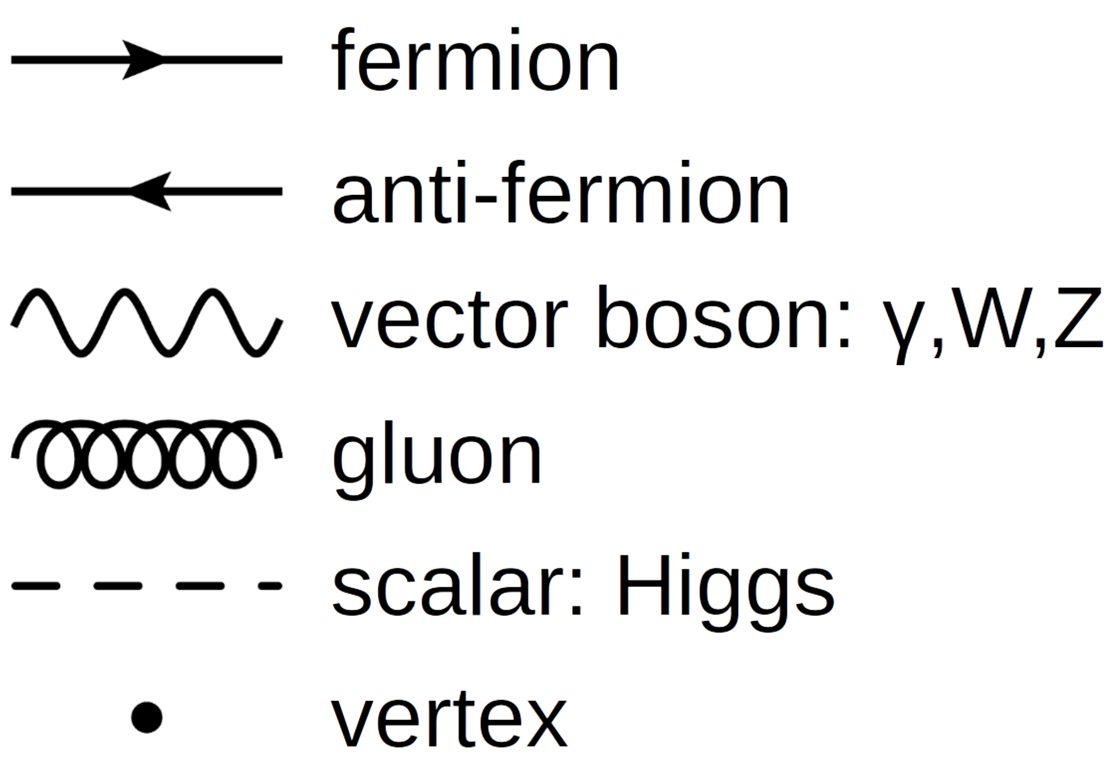
  - S_n can be computed with Feynman rules
  - Each term of this perturbation series is associated with a distinct process

Feynman rules

- Elements of Feynman rules:
  - External lines: Incoming/outgoing particles
  - Vertices: coupling between particles, energy and momentum is conserved at each vertex
  - Propagators (=internal lines): Exchange of virtual particles during scattering process (Green's function of free field equation in momentum space)

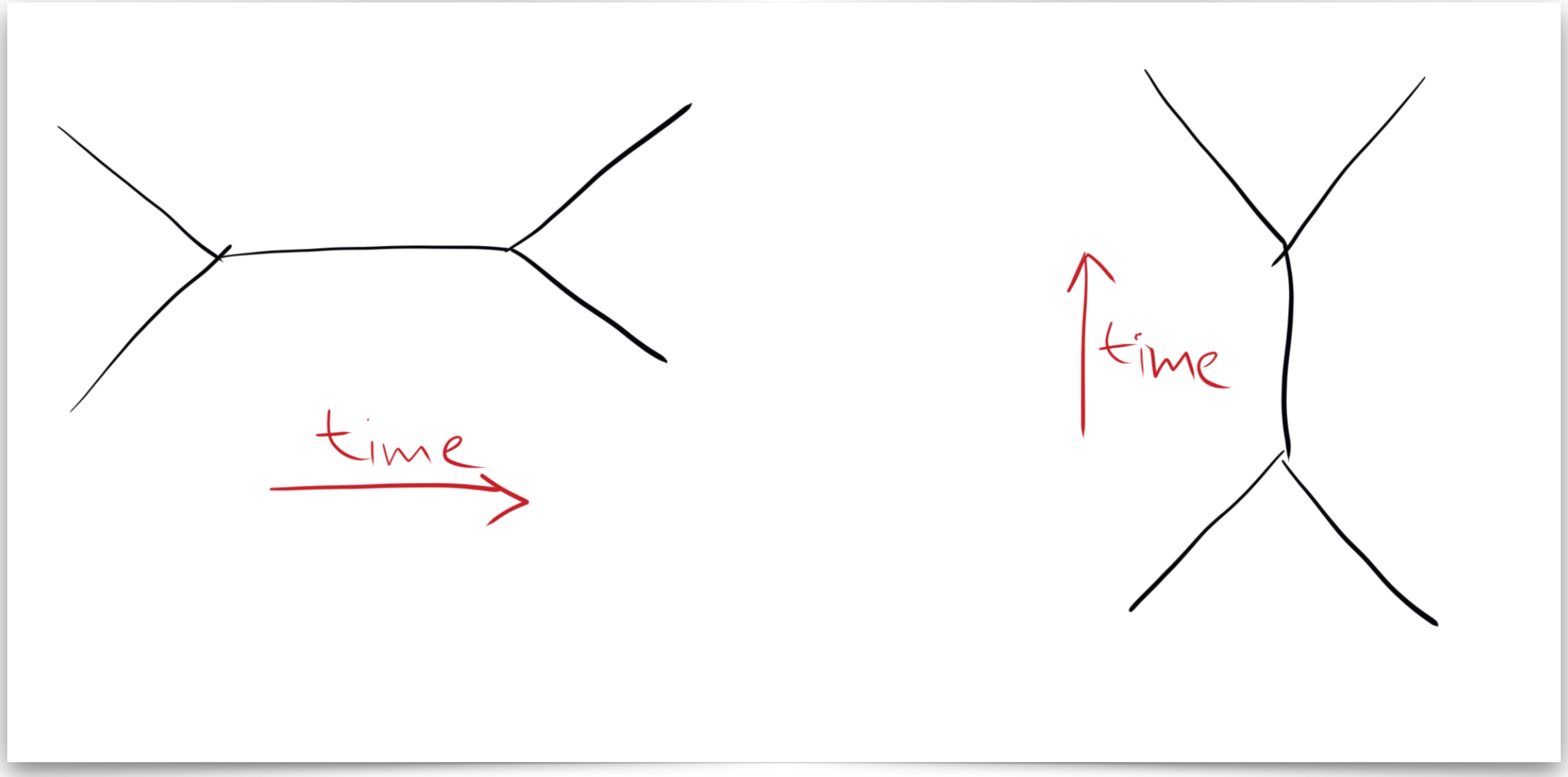


$$\mathcal{M} = \bar{v}(p_2)(-ie\gamma^\mu)u(p_1)\frac{-ig_{\mu\nu}}{(p_1+p_2)^2 + i\varepsilon}\bar{u}(p_3)(-ie\gamma^\nu)v(p_4)$$



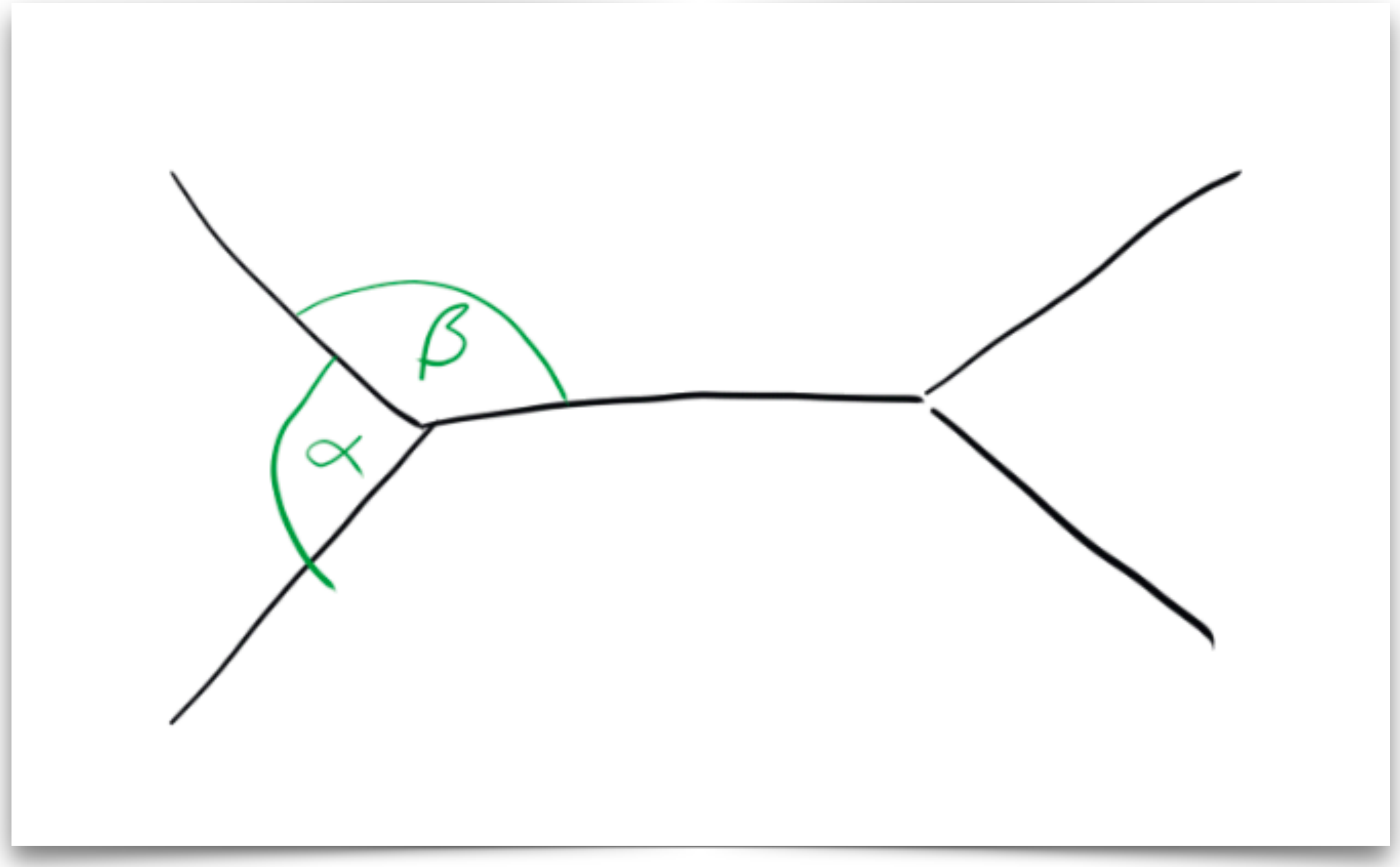
Feynman diagrams

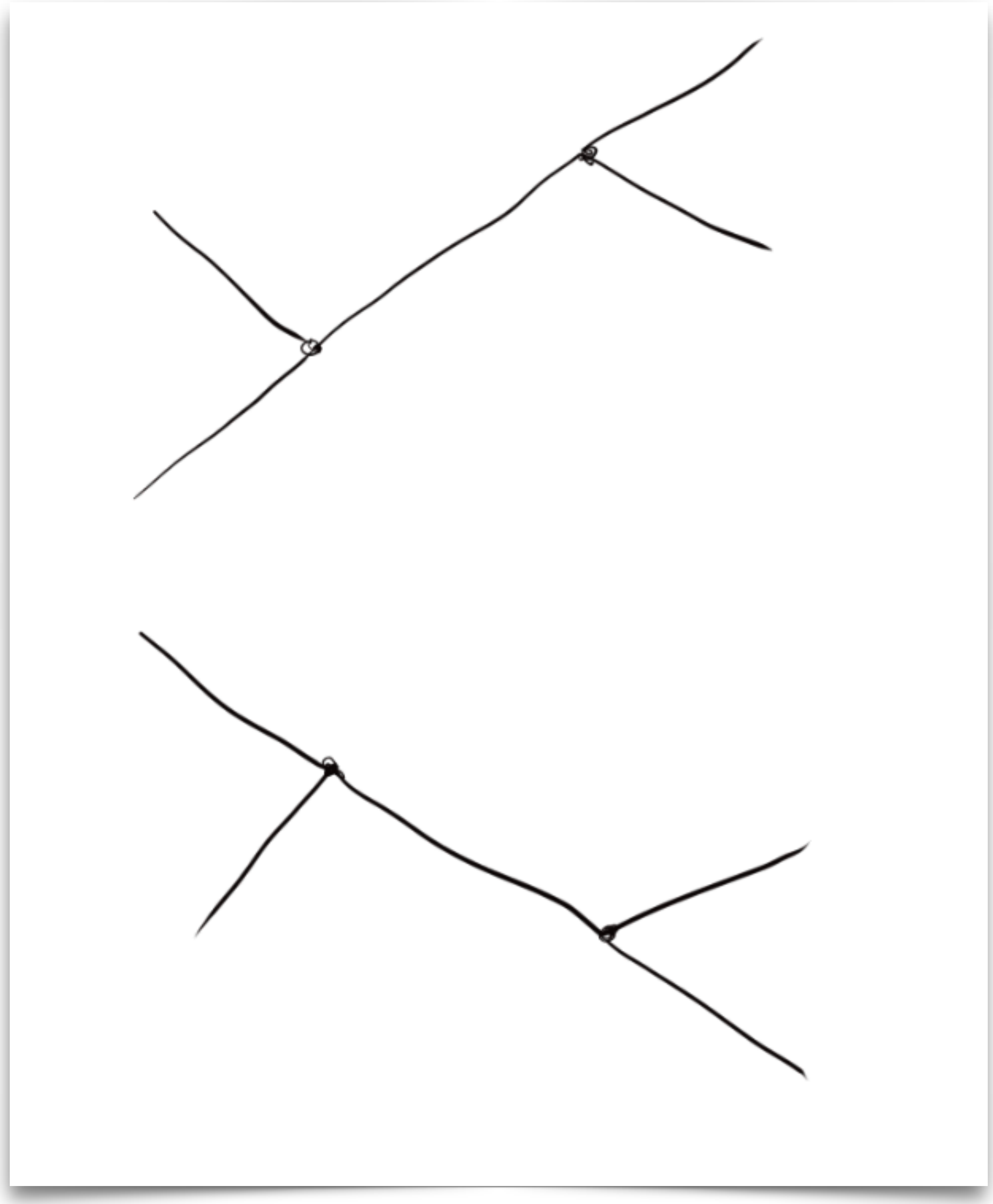
- Direction of time is convention (I like left to right)



Feynman diagrams

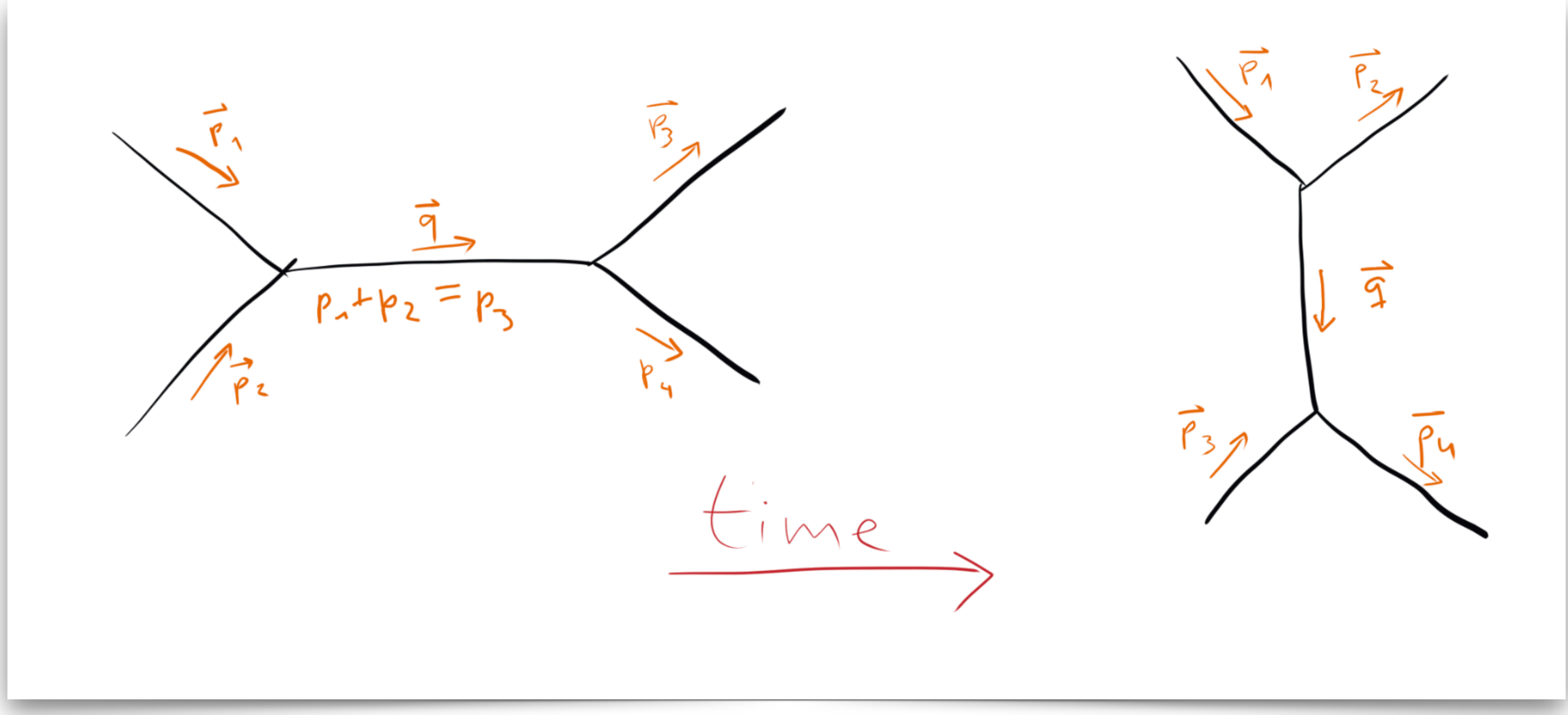
- The angles between the different lines have no spatial meaning
- The two diagrams on the right are considered identical





Feynman diagrams

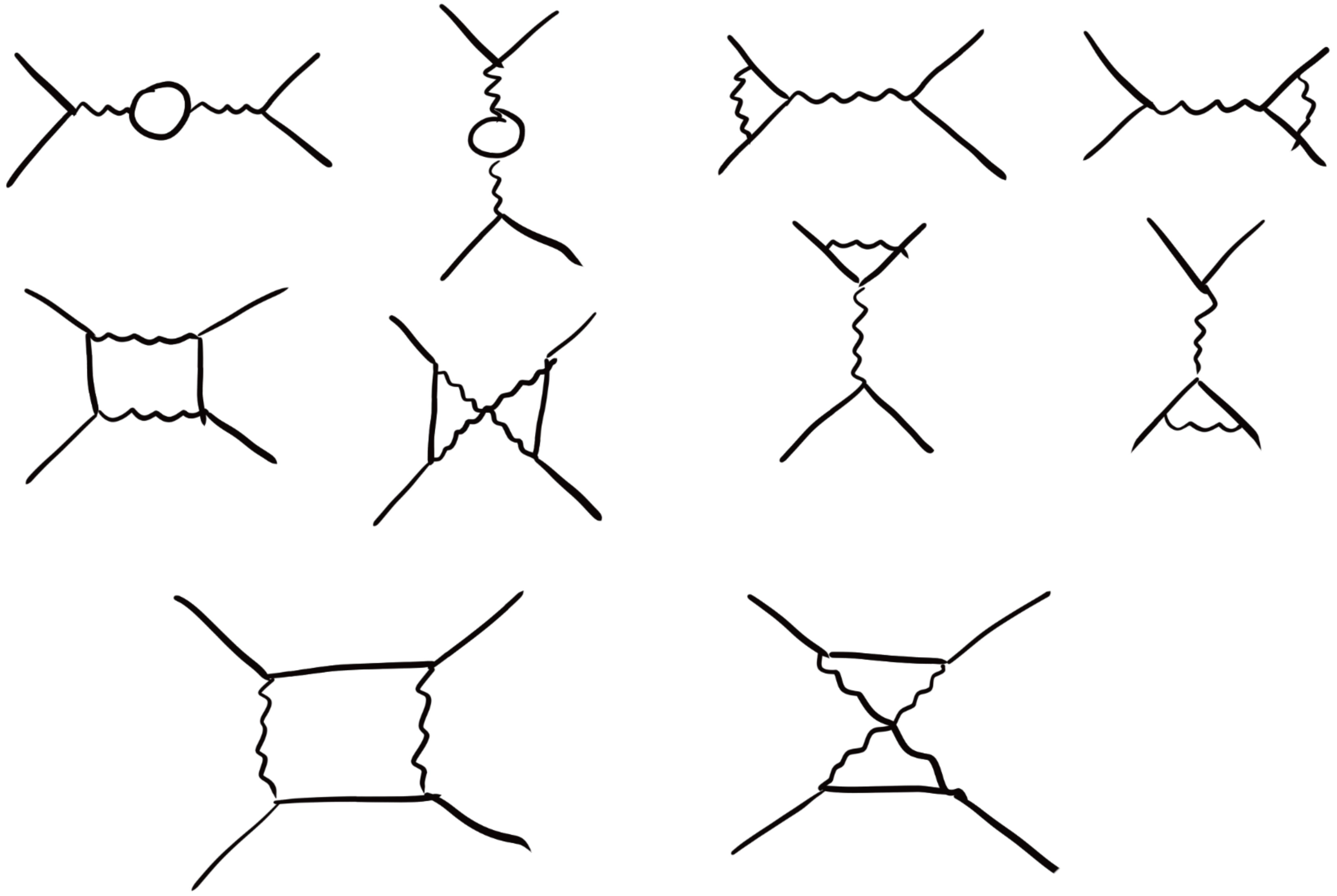
- Once you choose a convention, those diagrams describe different physics processes!



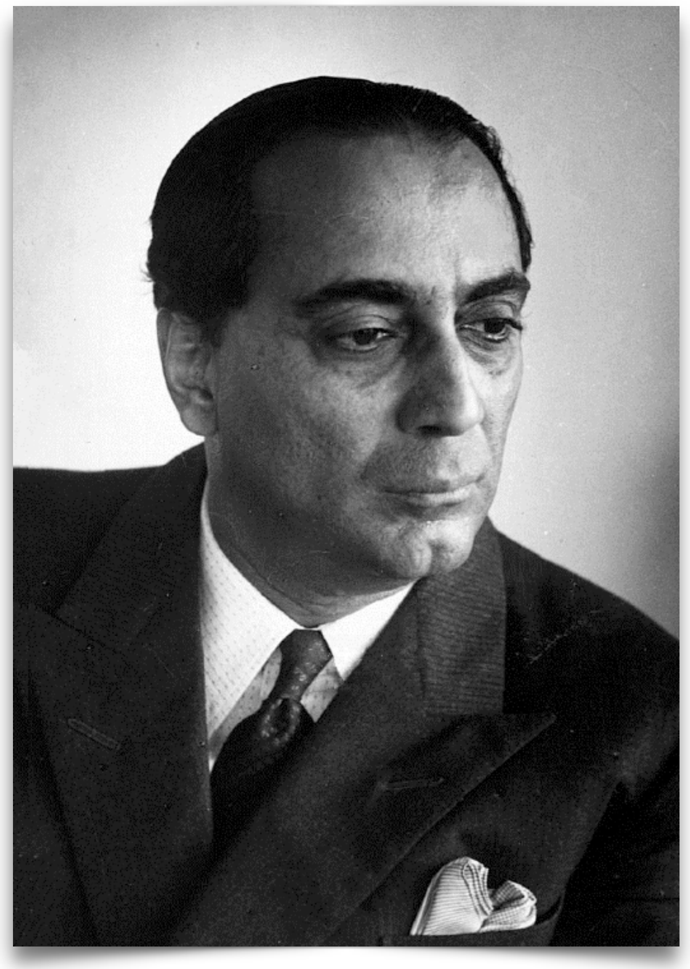
Feynman rules: Examples Bhabha scattering $e^+e^- \rightarrow e^+e^-$

First order:

Second order:

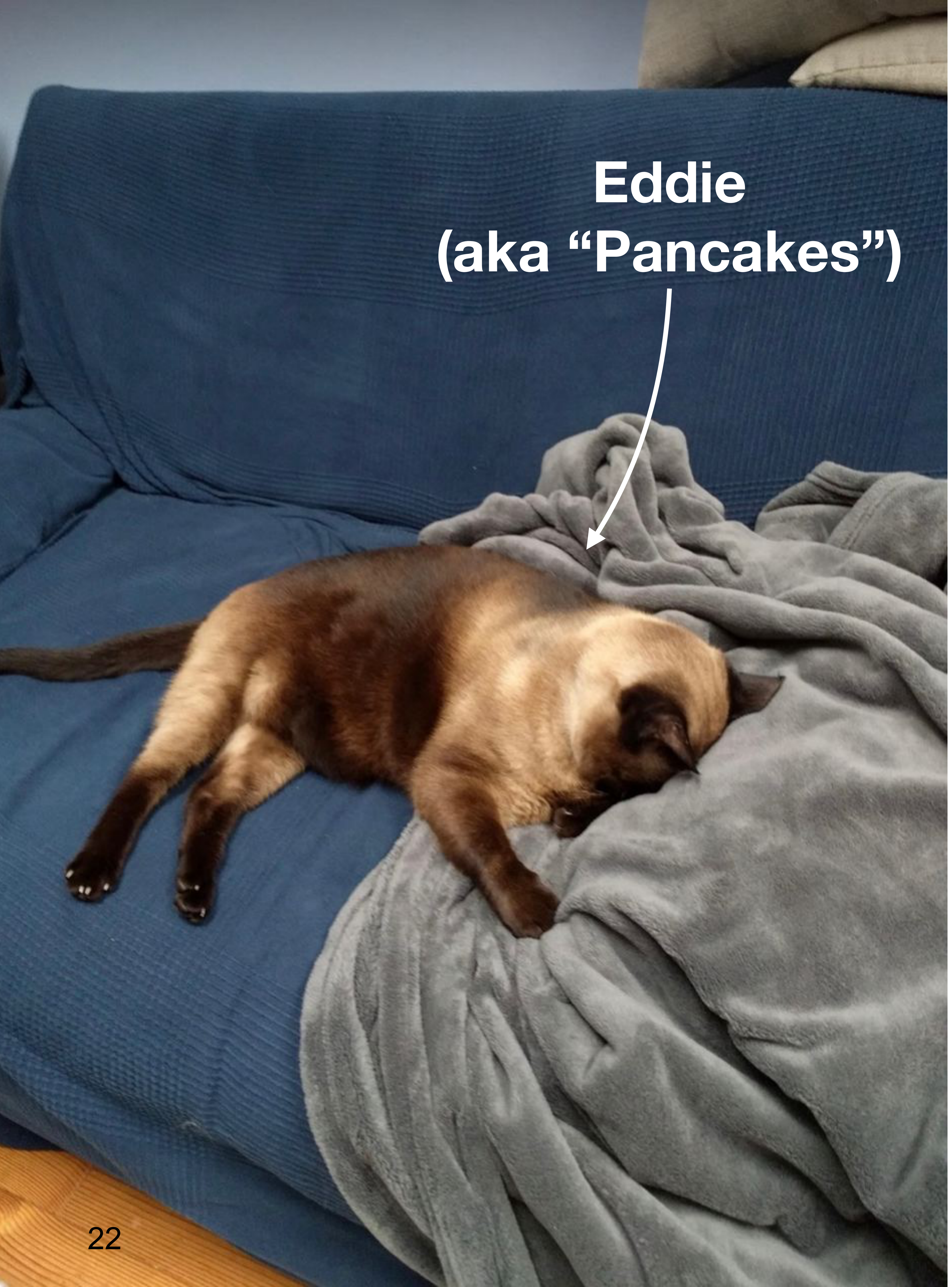


Third order: 122 diagrams...



Credit: Public Domain

Process named after the Indian physicist Homi Jehangir Bhabha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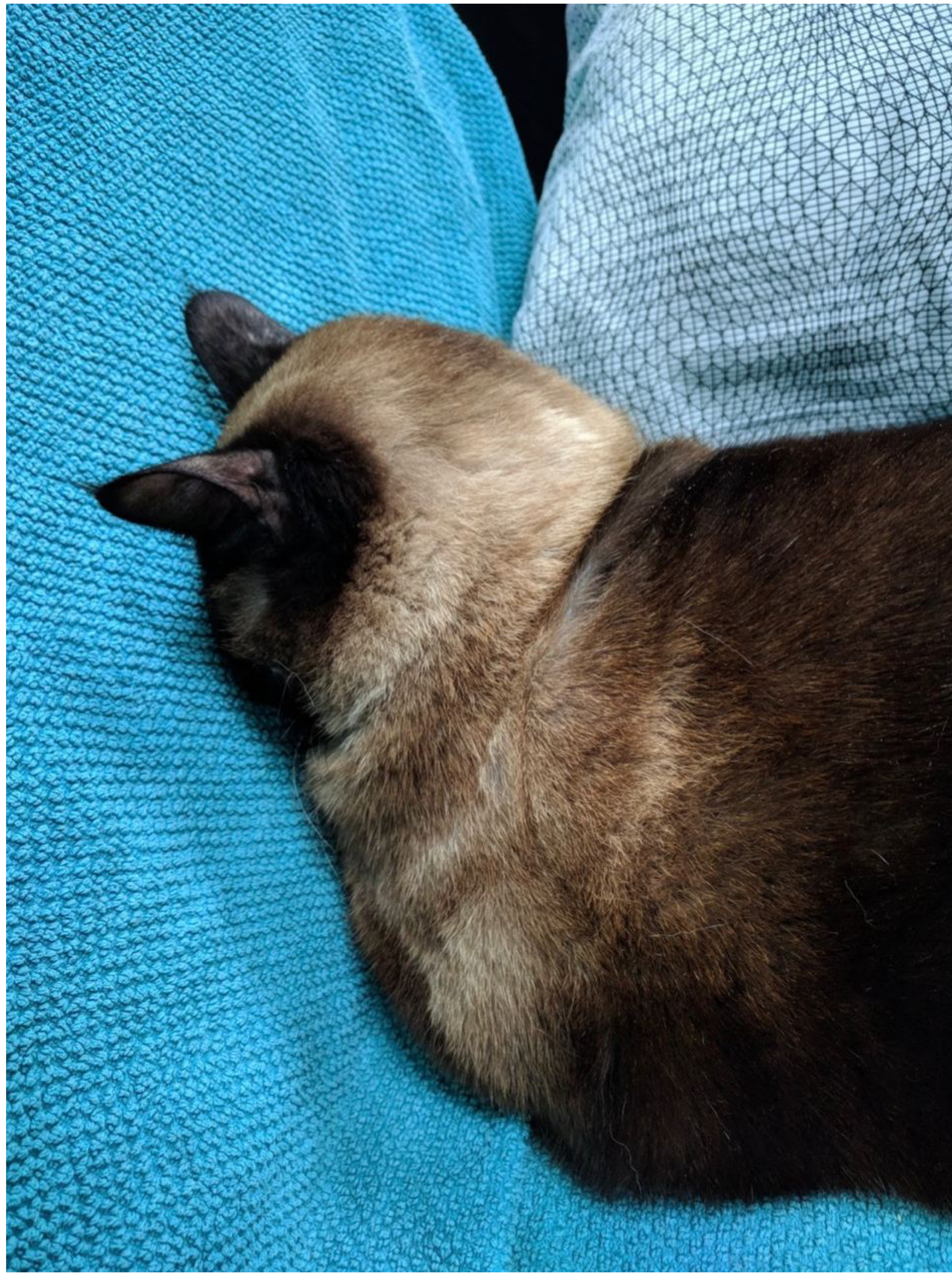


SU(n)

- Extension of gauge principles to non-Abelian groups SU(n)
  - In particle physics, in particular SU(2) and SU(3)
  - These theories are also named Yang-Mills theories
- SU(n) transformations $\psi \rightarrow \psi' = U(x)\psi = e^{\frac{1}{2} i g \alpha^a(x) T^a}\psi$
  - U is a unitary $n \times n$ matrix
  - T^a are $n^2 - 1^*$ linear independent hermitian $n \times n$ matrices (so called generators),
    *The condition $\det(U) = \pm 1$ removes one generator.
  - $\alpha^a(x)$ are real functions
  - $\alpha^a(x)T^a$ (with summation over a) describe all possible rotations
  - $[T^a, T^b] = i f^{abc} T^c$ with structure constants f^{abc}

A group G consists of elements a, inverse elements a^{-1}, a unit element 1, and a multiplication rule with the following properties:
1) If a, b in G → $c = a \cdot b$ in G
2) $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
3) $a \cdot 1 = 1 \cdot a = a$
4) $a \cdot a^{-1} = a^{-1} \cdot a = 1$

A group G is called abelian, if
5) $a \cdot b = b \cdot a$

Covariant derivatives

- Analogue to QED: invariance under local SU(n) transformations by introducing covariant derivatives

$$\partial_\mu \;\rightarrow\; D_\mu = \partial_\mu + igT^a A_\mu^a$$

QED:

$$\partial_\mu \;\rightarrow\; D_\mu = \partial_\mu + iqA_\mu(x)$$

$$A_\mu^a \;\rightarrow\; A_\mu^a - \frac{1}{g}\partial_\mu \alpha^a(x) - f^{abc}\alpha^b(x)A_\mu^c$$

$$A_\mu \;\rightarrow\; A_\mu - \frac{1}{q}\partial_\mu \alpha$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{i}{q}[D_\mu, D_\nu]$$

SU(3): Gell-Mann Matrices

- Totally anti-symmetric structure constant tensor :
  - $f^{123} = 1, f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \frac{1}{2}, f^{458} = f^{678} = \frac{\sqrt{3}}{2}$
- Typical representation of generators T^a via traceless, hermitian matrices $\lambda^a = \frac{1}{2}T^a$ (“Gell-Mann-Matrices”)

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Lagrange density of QCD

- Invariance under local SU(3): QCD
  - Analogue to electric charge in QED: 3 “colour” charges i = red, green, blue
  - 8 vector fields A_μ^a: 8 gluons carry colour charge and colour anti-charge
  - Additional terms in field-strength tensor (from non-zero commutator): self interaction of gluons by their colour charge
  - Analogue to QED: gluons have to be massless to not break gauge invariance

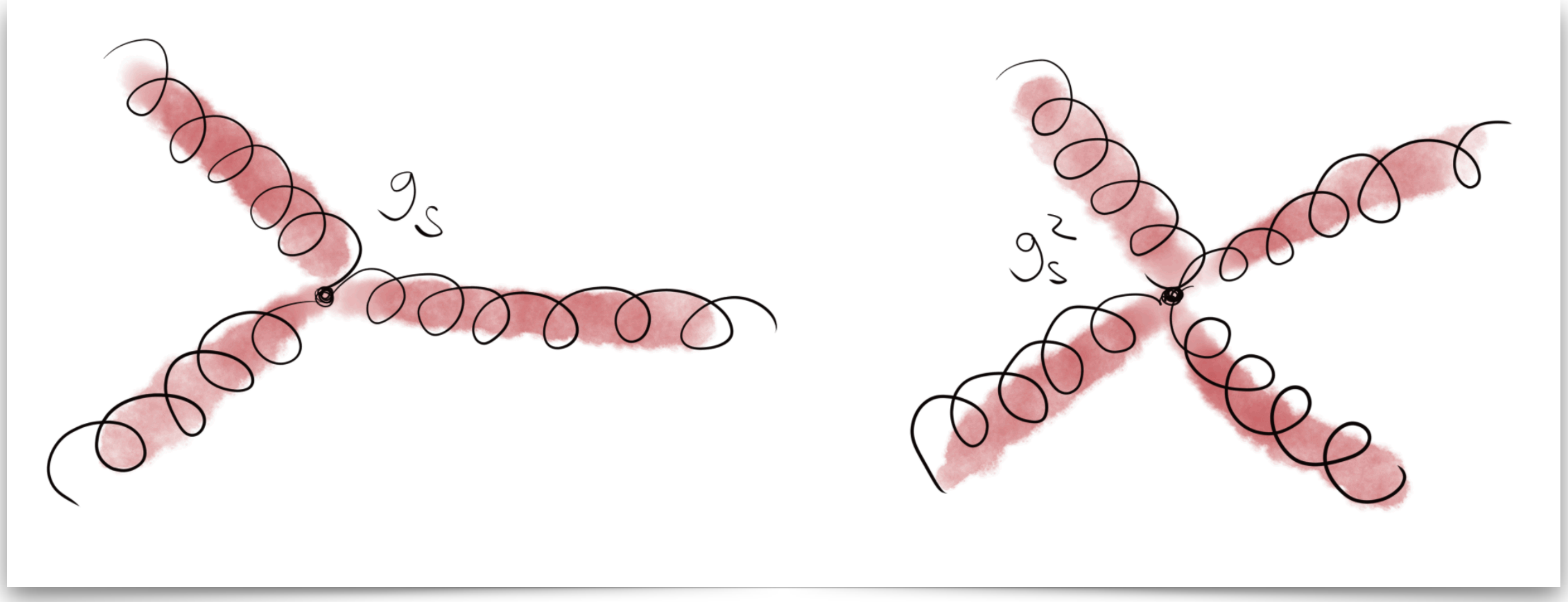
$$
\begin{aligned}
|1\rangle &= (r\bar{b} + b\bar{r})/\sqrt{2} & |5\rangle &= -i(r\bar{g} - g\bar{r})/\sqrt{2} \\
|2\rangle &= -i(r\bar{b} - b\bar{r})/\sqrt{2} & |6\rangle &= (b\bar{g} + g\bar{b})/\sqrt{2} \\
|3\rangle &= (r\bar{r} - b\bar{b})/\sqrt{2} & |7\rangle &= -i(b\bar{g} - g\bar{b})/\sqrt{2} \\
|4\rangle &= (r\bar{g} + g\bar{r})/\sqrt{2} & |8\rangle &= (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}
\end{aligned}
$$

$$
\mathcal{L}_{\mathrm{QCD}} = \underbrace{\bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi}_{\text{mass and kin. term of quark}} - \underbrace{g_S \bar{\psi}\left(\gamma^\mu T^a A_\mu^a\right)\psi}_{\text{quark-gluon coupling}} - \underbrace{\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}}_{\text{gluon kin. energy and gluon self interaction}}
$$

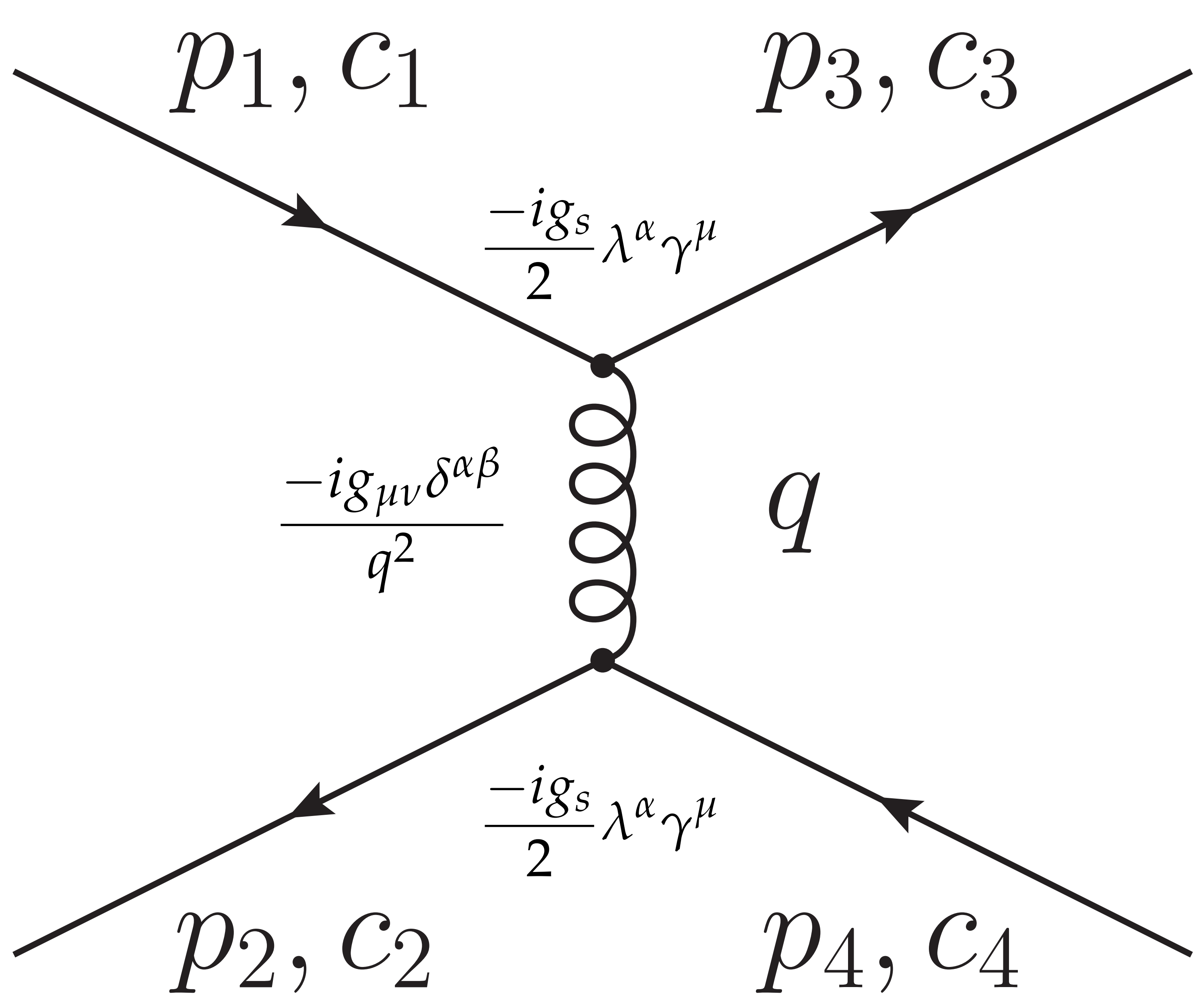
Gluon self interactions

$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$

- Gluon self interactions are qualitatively new interactions compared to QED from $\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$ term:
  - higher order terms with $\sim g_S (A^a_\mu)^3$ and $\sim g_S^2 (A^a_\mu)^4$ do not exist in QED (no photon self interactions)



Feynman rules for QCD



$$-iM = [\bar{u}_3 c_3^\dagger]\left[-i\frac{g_s}{2}\lambda^\alpha\gamma^\mu\right][u_1 c_1]\left[\frac{-ig_{\mu\nu}\delta^{\alpha\beta}}{q^2}\right][\bar{v}_2 c_2^\dagger]\left[-i\frac{g_s}{2}\lambda^\beta\gamma^\nu\right][v_4 c_4]$$

$$M = \frac{-g_s^2}{4}\frac{1}{q^2}[\bar{u}_3\gamma^\mu u_1][\bar{v}_2\gamma_\mu v_4](c_3^\dagger\lambda^\alpha c_1)(c_2^\dagger\lambda^\alpha c_4)$$

color factor $$f = \frac{1}{4}(c_3^\dagger\lambda^\alpha c_1)(c_2^\dagger\lambda^\alpha c_4)$$

Color factors in QCD

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

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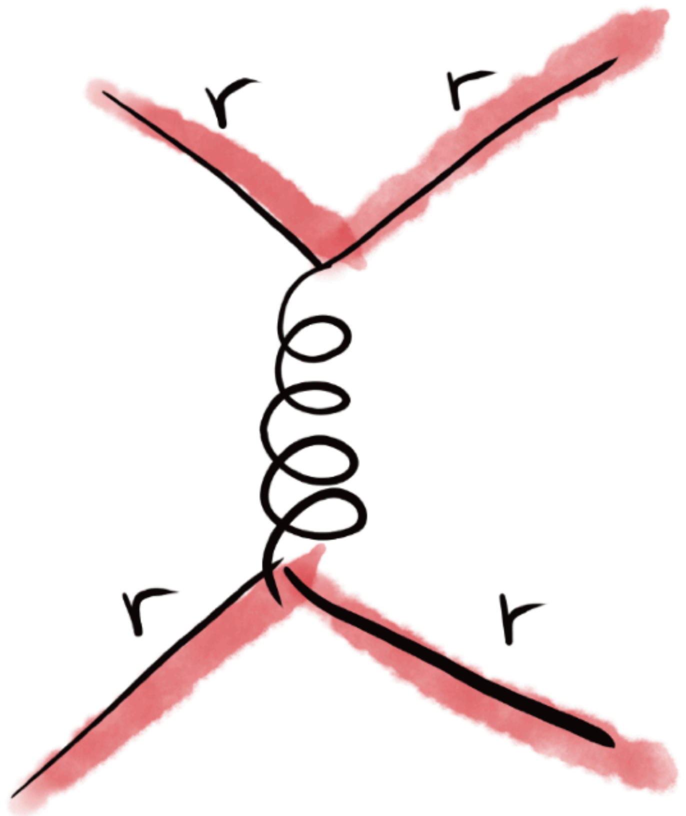
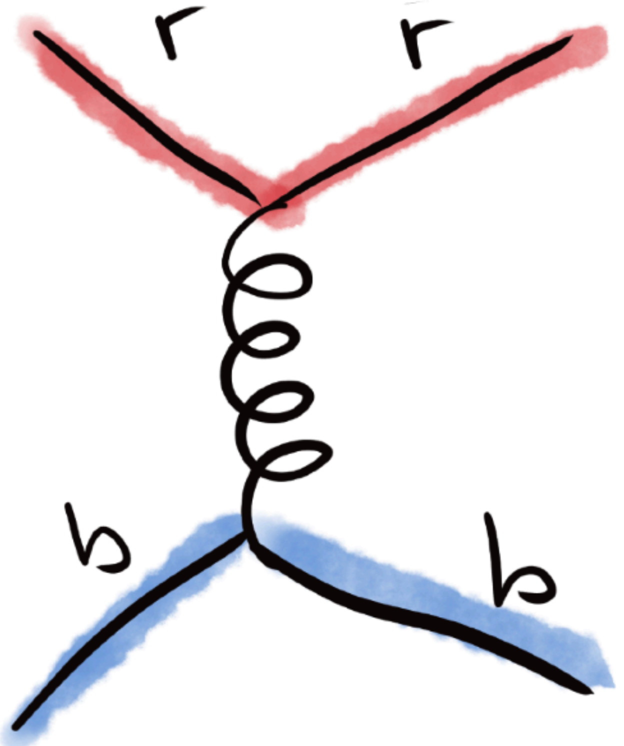
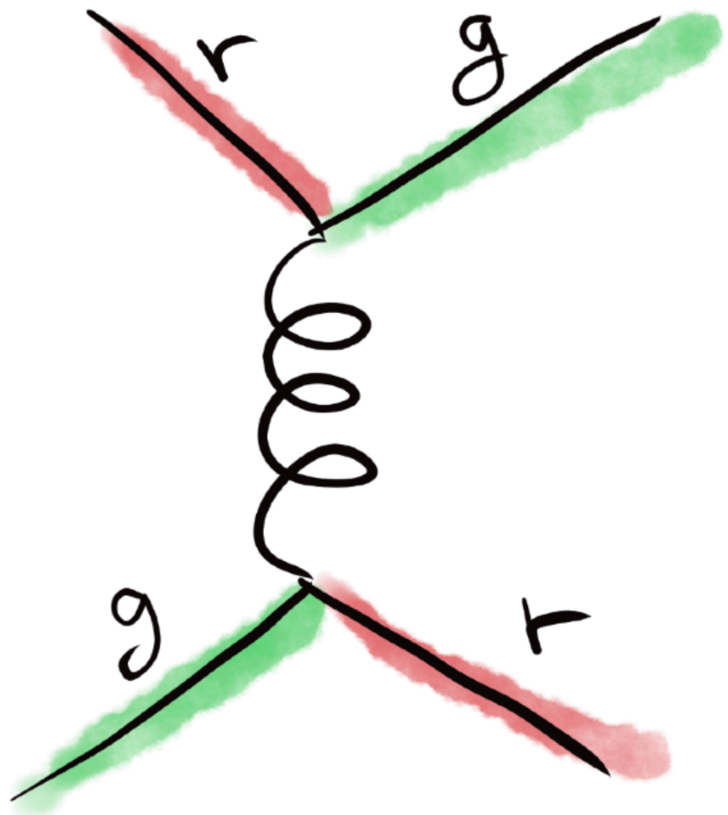


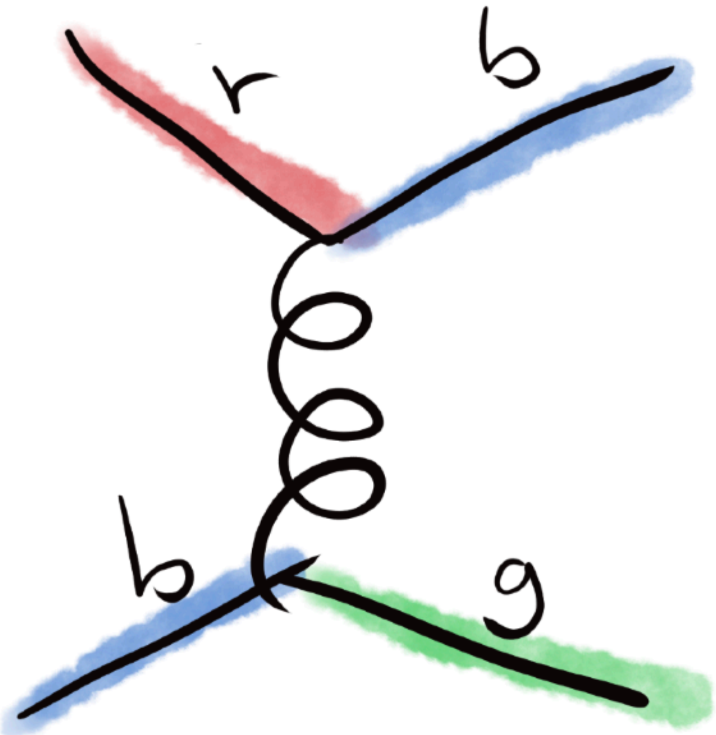
$$\sum_a \lambda^a_{rr} \lambda^a_{rr} = \lambda^3_{11} \lambda^3_{11} + \lambda^8_{11} \lambda^8_{11}$$



$$\sum_a \lambda^a_{rr} \lambda^a_{bb} = \lambda^8_{11} \lambda^8_{33}$$



$$\sum_a \lambda^a_{12} \lambda^a_{21} = \lambda^1_{12} \lambda^1_{21} + \lambda^2_{12} \lambda^2_{21}$$



$$\sum_a \lambda^a_{12} \lambda^a_{23} = 0$$

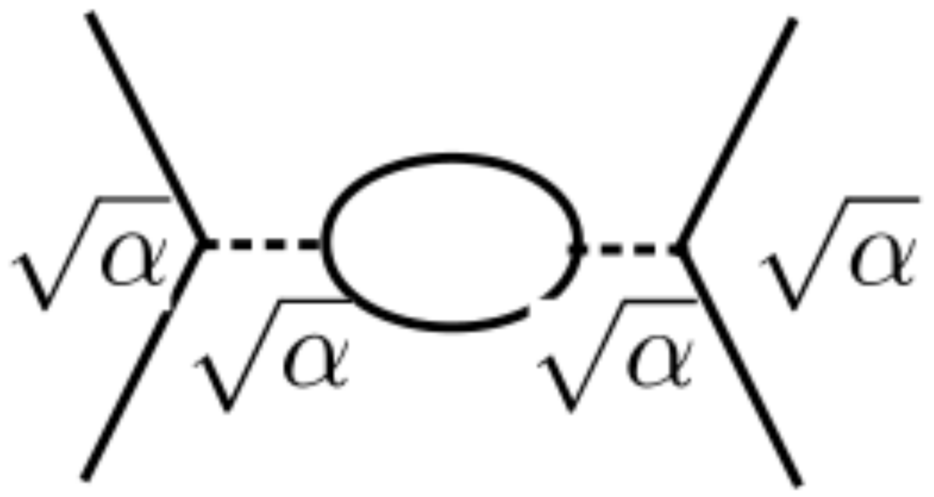
Color must be conserved!

Summary QCD

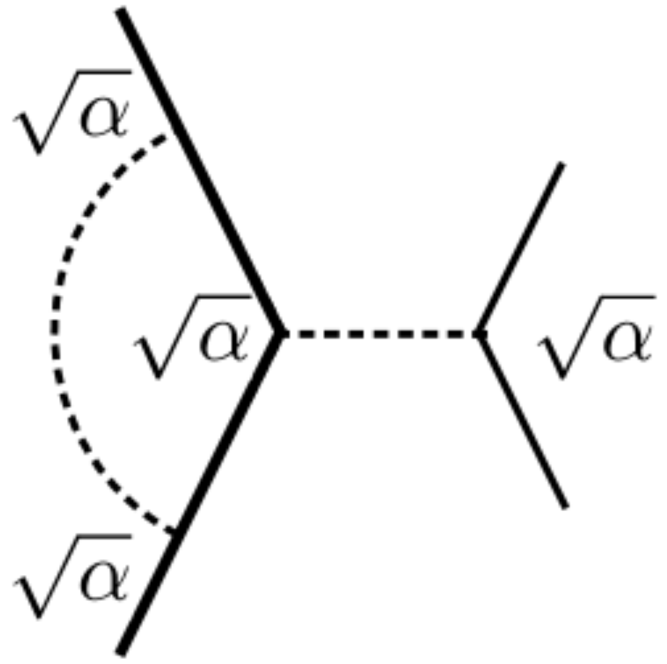
- SU(3) describes the exact color symmetry of the strong interaction
- 8 Gell-Mann Matrices $\lambda \rightarrow$ 8 bosons (gluons)
- Strong coupling constant g_S
- No fundamental reason why QCD cannot be realized as U(3) in nature. This would result in an additional colorless 9th gluon… our world would be very different!

Not the whole picture…

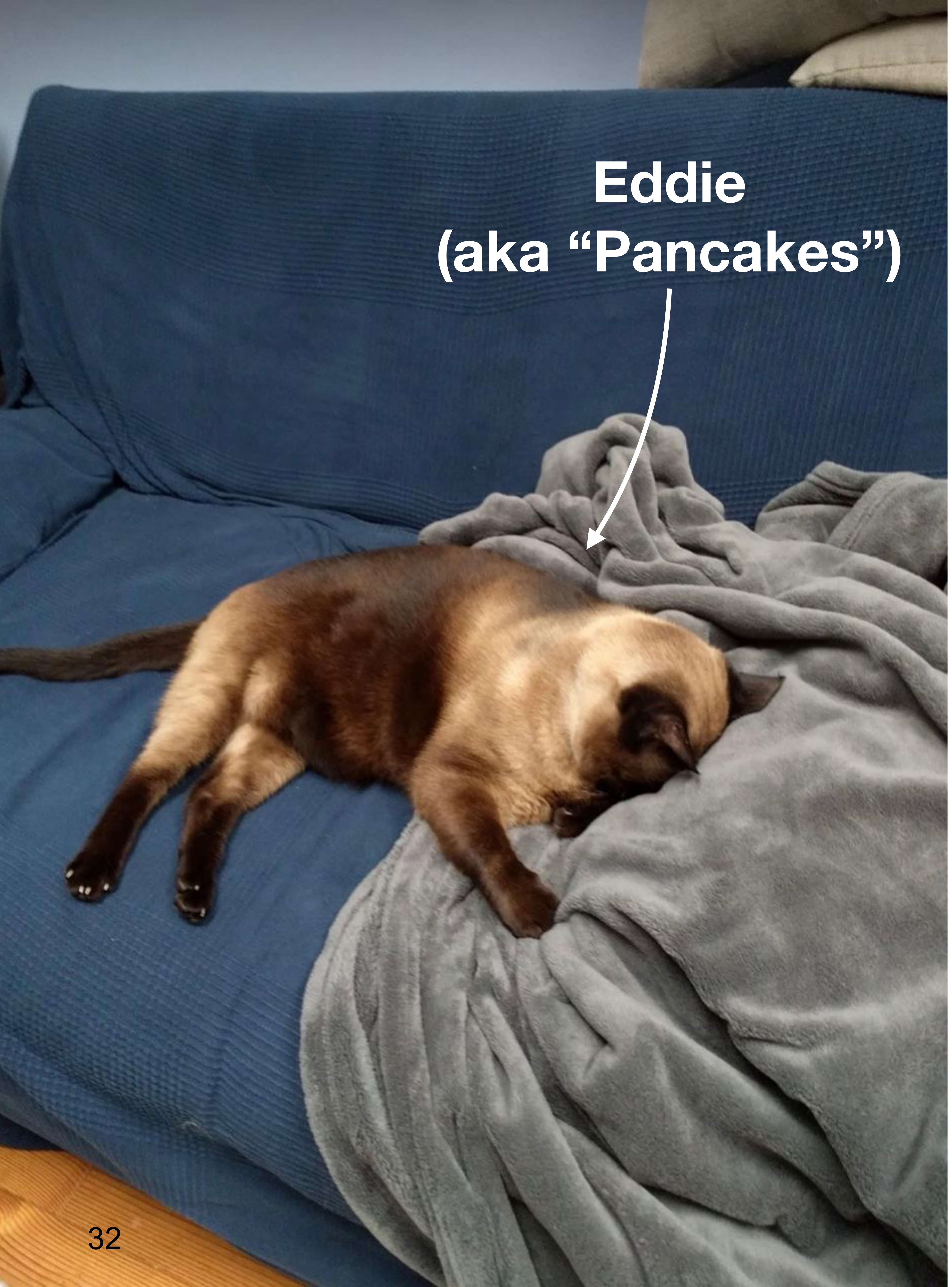
- Dynamics of a theory not entirely described by Lagrange density
  - Fields are quantised: effects due to quantum corrections (often called radiative corrections) occur
  - Taken into account in perturbation series
- 'Good' quantum-field theories, like the Standard Model, are
  - Anomaly free: symmetries of the Lagrangian not destroyed by quantum corrections
  - Renormalizable: divergencies in quantum corrections absorbed in redefined parameters of the Lagrangians



Modifies effective particle masses ('running masses')



Modifies effective couplings ('running couplings')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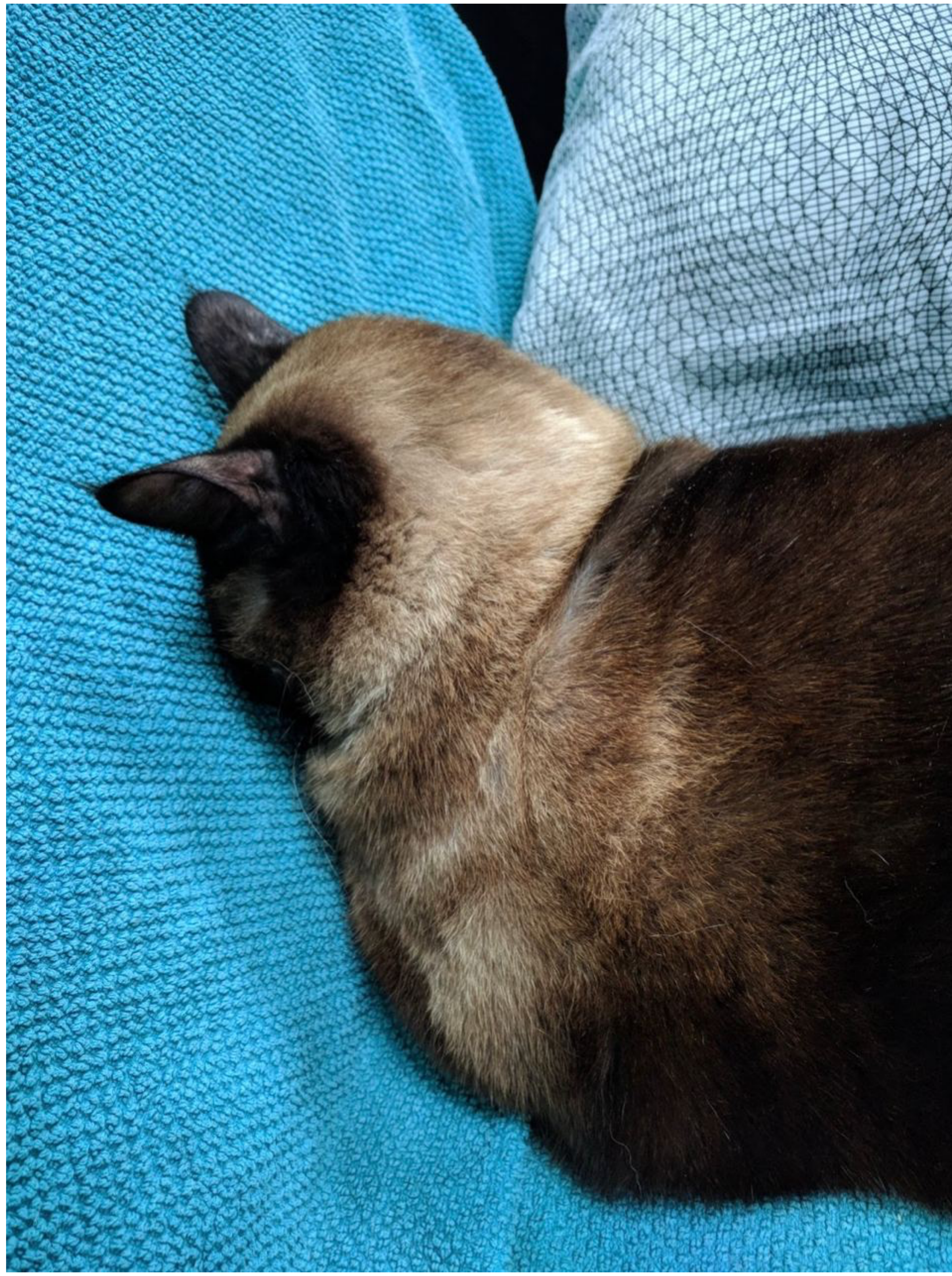
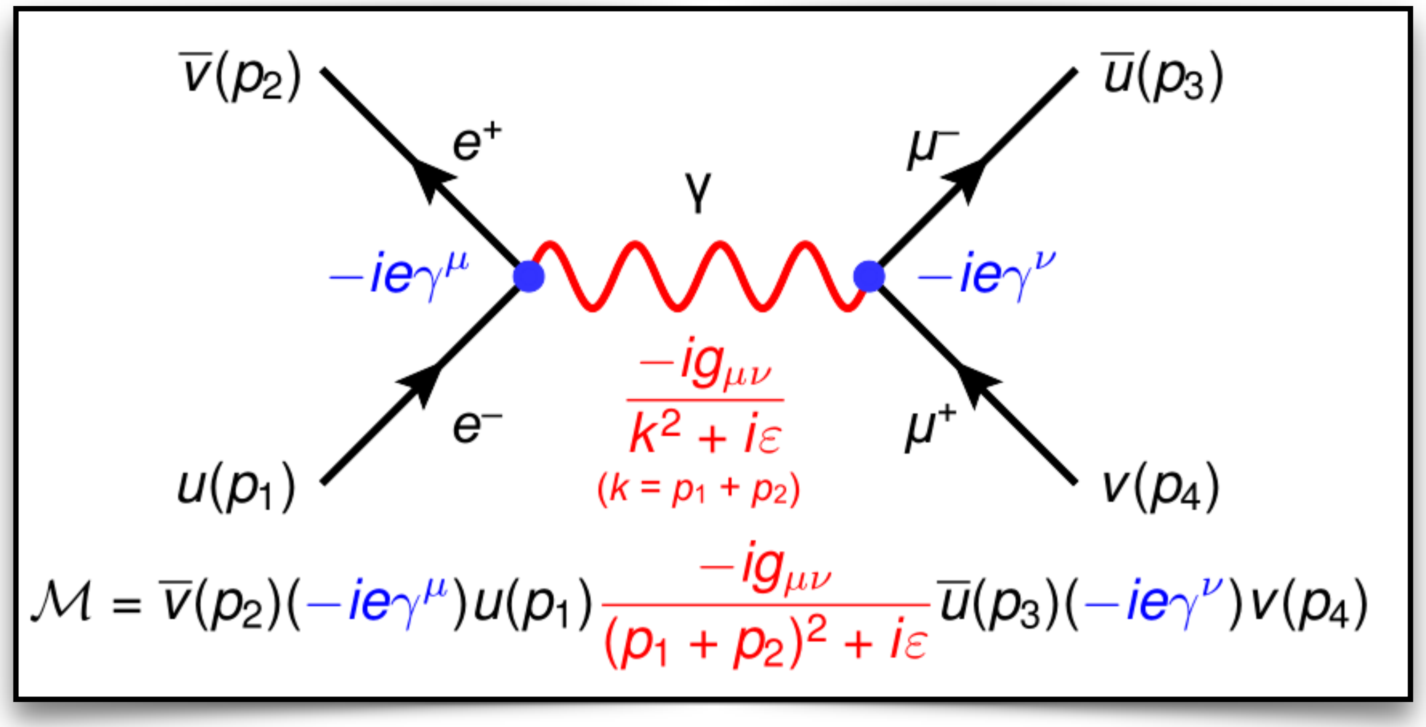


Parity conservation in QED (and QCD)

- Consider QED process $e^+e^- \to \mu^+\mu^-$



- Feynman rules for QED:

electrons — muons

$$-i\mathcal{M} = [\bar{v}_e(p_2)ie\gamma^\mu u_e(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\bar{u}_m(p_3)ie\gamma^\nu v_m(p_4)]$$

$$= -\frac{e^2}{q^2}j_e^\mu g_{\mu\nu}j_m^\nu = -\frac{e^2}{q^2}j_e j_m$$

- Recall parity transformation from last lecture:

$(ab)^\dagger = b^\dagger a^\dagger$ $\quad\gamma^{0,\dagger} = \gamma^0$

$$u \to u' = \hat{P}u = \gamma^0 u$$
$$\bar{u} = u^\dagger\gamma^0 \to (\hat{P}u)^\dagger\gamma^0 = (\gamma^0 u)^\dagger\gamma^0 = u^\dagger\gamma^{0,\dagger}\gamma^0 = \bar{u}\gamma^0$$

Parity conservation in QED (and QCD)

- $j_e = \bar{u}\gamma^\mu u \rightarrow \hat{P}j_e = \begin{cases} \hat{P}j_e^0 = \bar{u}\gamma^0\gamma^0\gamma^0 u \overset{\gamma^0\gamma^0=1}{=} \bar{u}\gamma^0 u = j_e^0, & \text{if } k = 0 \\ \hat{P}j_e^k = \bar{u}\gamma^0\gamma^k\gamma^0 u \overset{\gamma^0\gamma^\mu = -\gamma^\mu\gamma^0}{=} -\bar{u}\gamma^k\gamma^0\gamma^0 u \overset{\gamma^0\gamma^0=1}{=} \bar{u}\gamma^k u = -j_e^k, & \text{if } k = 1,2,3 \end{cases}$

  and analogue for j_m

  - → the time like component is unchanged, the space-like changes sign

- $j_e j_m = j_e^0 j_m^0 - j_e^k j_m^k \rightarrow \hat{P}j_e j_m = j_e^0 j_m^0 - (-j_e^k)(-j_m^k) = j_e j_m$
  - **→ QED matrix element is invariant under parity transformation!**

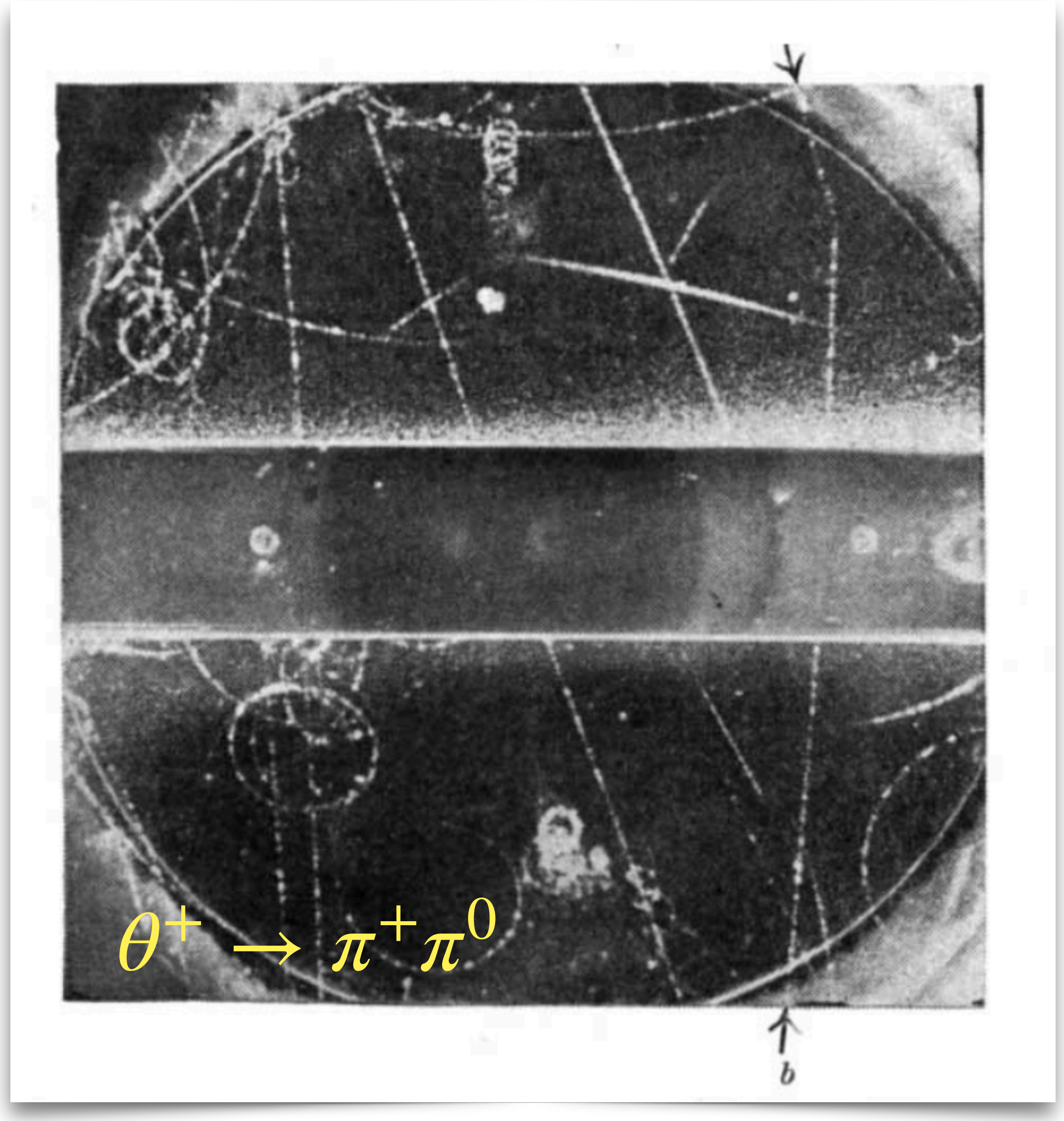
- QCD matrix element has the same form and conserves parity as well

“It doesn't matter how beautiful your theory is.
If it disagrees with experiment, it's wrong.”

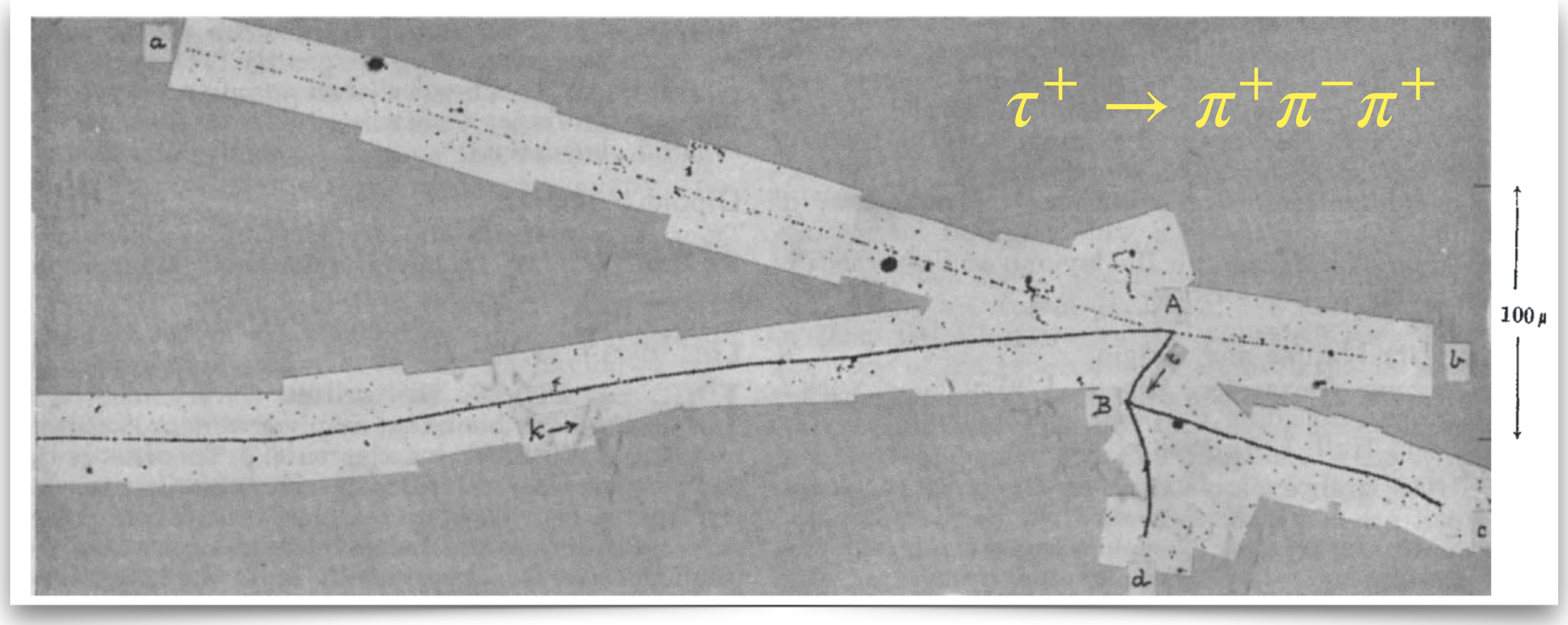
(R. Feymann)

$\tau - \theta$ puzzle

- Reminder (Lecture 2):
  - In the early 1950s, experiments observed two new particles in cosmic rays:
    - $\theta^+ \to \pi^+\pi^0$ and $\tau^+ \to \pi^+\pi^-\pi^+$
    - The puzzle was: The mass and lifetime of the two particles was identical…
    - The parity of the particle was different though since all pions have spin=0 and negative parity ($\hat{P}(q) = -P(\bar{q}) = 1$)
  - **Proposal by Yang and Lee: Parity is not conserved in these decays and τ and θ are the same particle!**



Credit: G. D. Rochester &et al. Nature 160 (1947) 855-857



Credit: Brown et al., Nature 163 (1949) 80

The Wu-Experiment

- Experiment proposed by Yang and Lee (Nobel price1957)
- Experiment performed by Chien-Shiung Wu at the US National Bureau of Standards (now NIST)
  - Wu was the world-expert for beta decay spectroscopy, the NBS team were world-leading cryogenic experts (most physicists considered the experiment impossible)
  - This experiment was genius and technically very challenging!
- Result was totally unexpected:

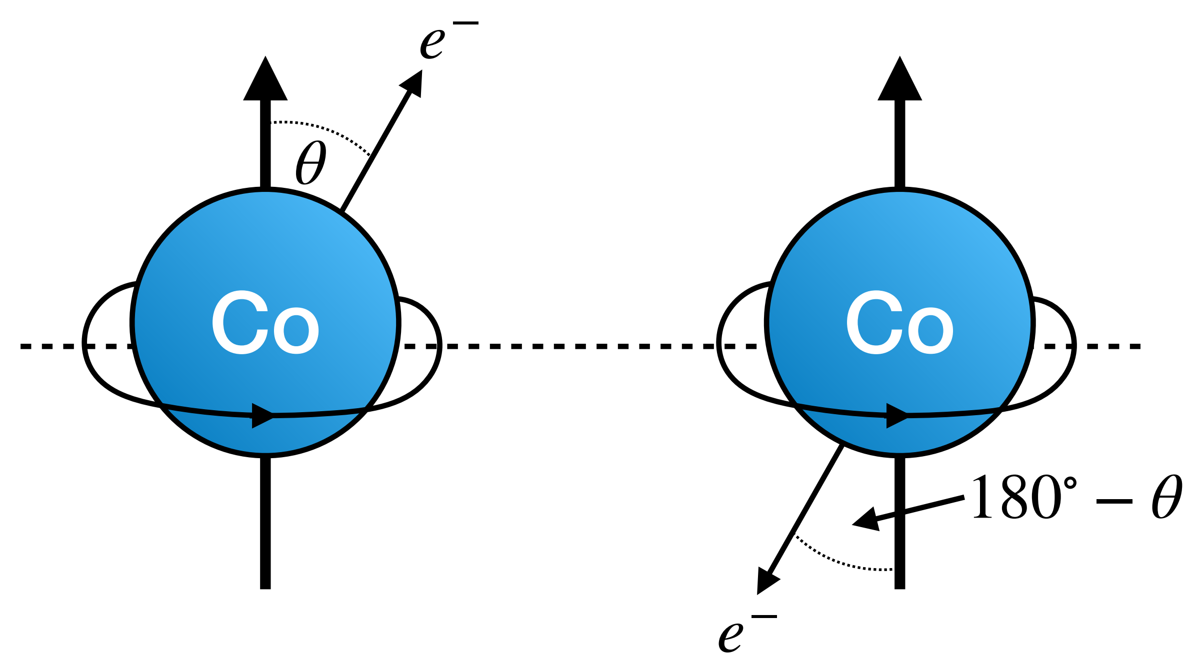
  Parity is violated in weak interactions!

Credit: Smithsonian Institution

The Wu-Experiment

- Measurement of angular distribution of emitted e^- in β-decays of polarized
  $^{60}Co \rightarrow {}^{60}Ni^* + e^- + \bar{\nu}_e$

  followed by $^{60}Ni^* \rightarrow$
- Theory: Check that the number of electrons emitted in direction of ^{60}Co spin equals those emitted in opposite direction
- Experiment: Measure electrons at fixed angle, but switch direction of ^{60}Co spin



$$\vec{J} \rightarrow \vec{J}' = P(\vec{J}) = \vec{J}$$

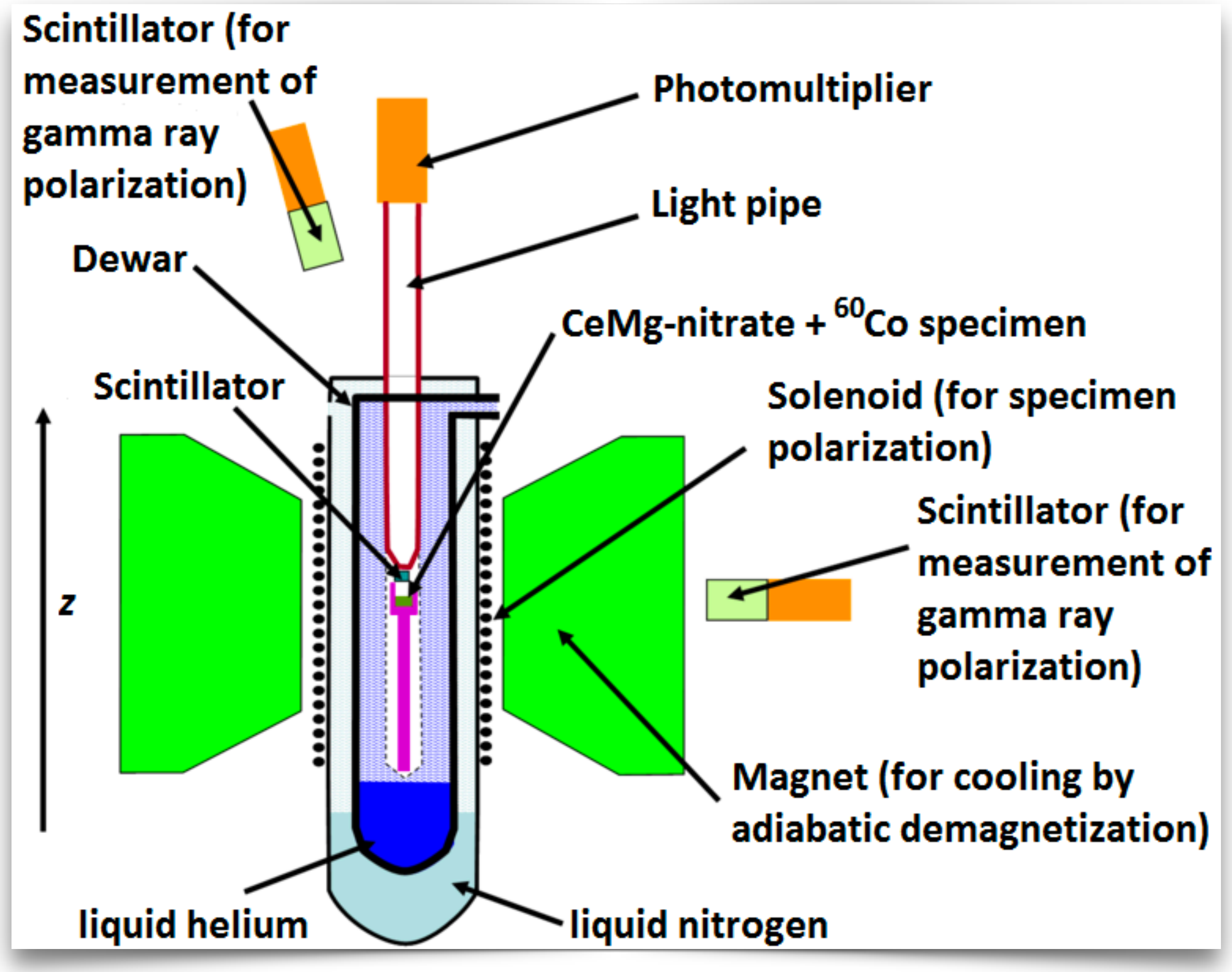
axial vector

$$\vec{p} \rightarrow \vec{p}' = P(\vec{p}) = -\vec{p}$$

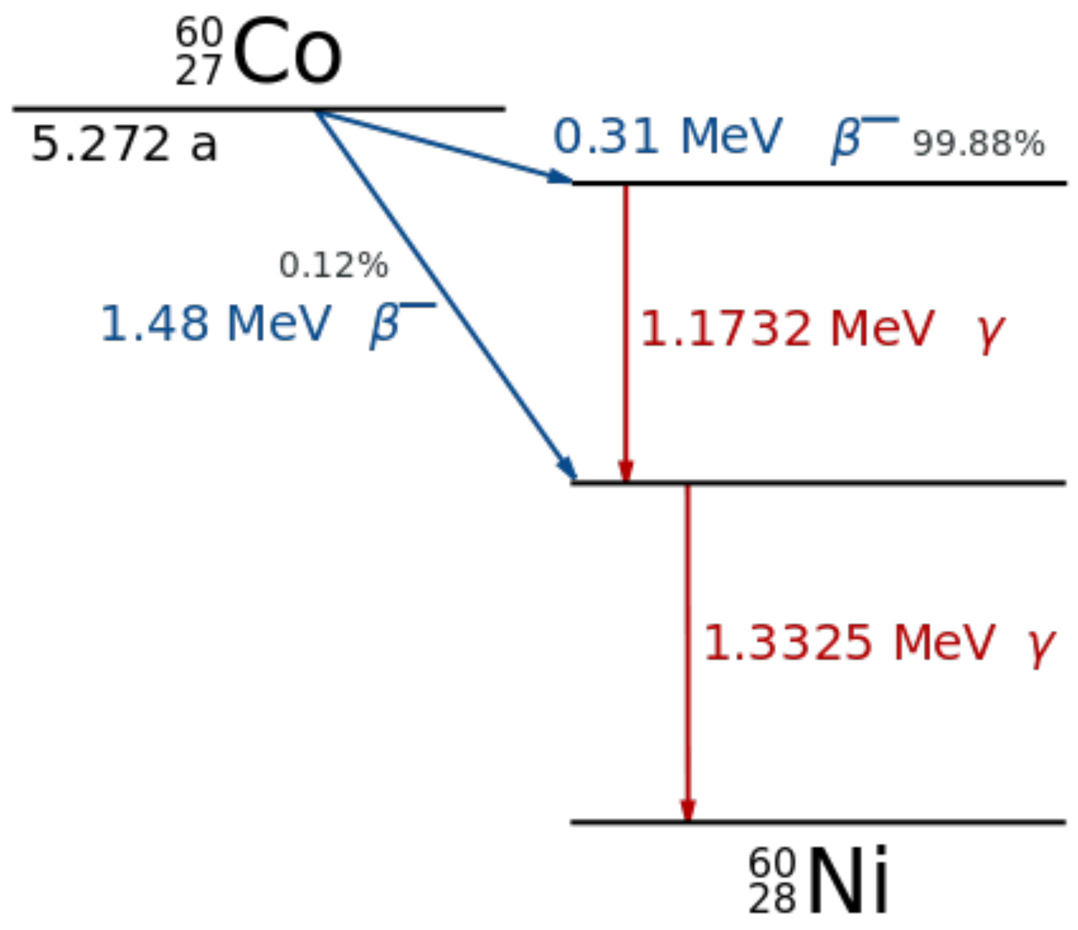
(polar) vector

The Wu-Experiment

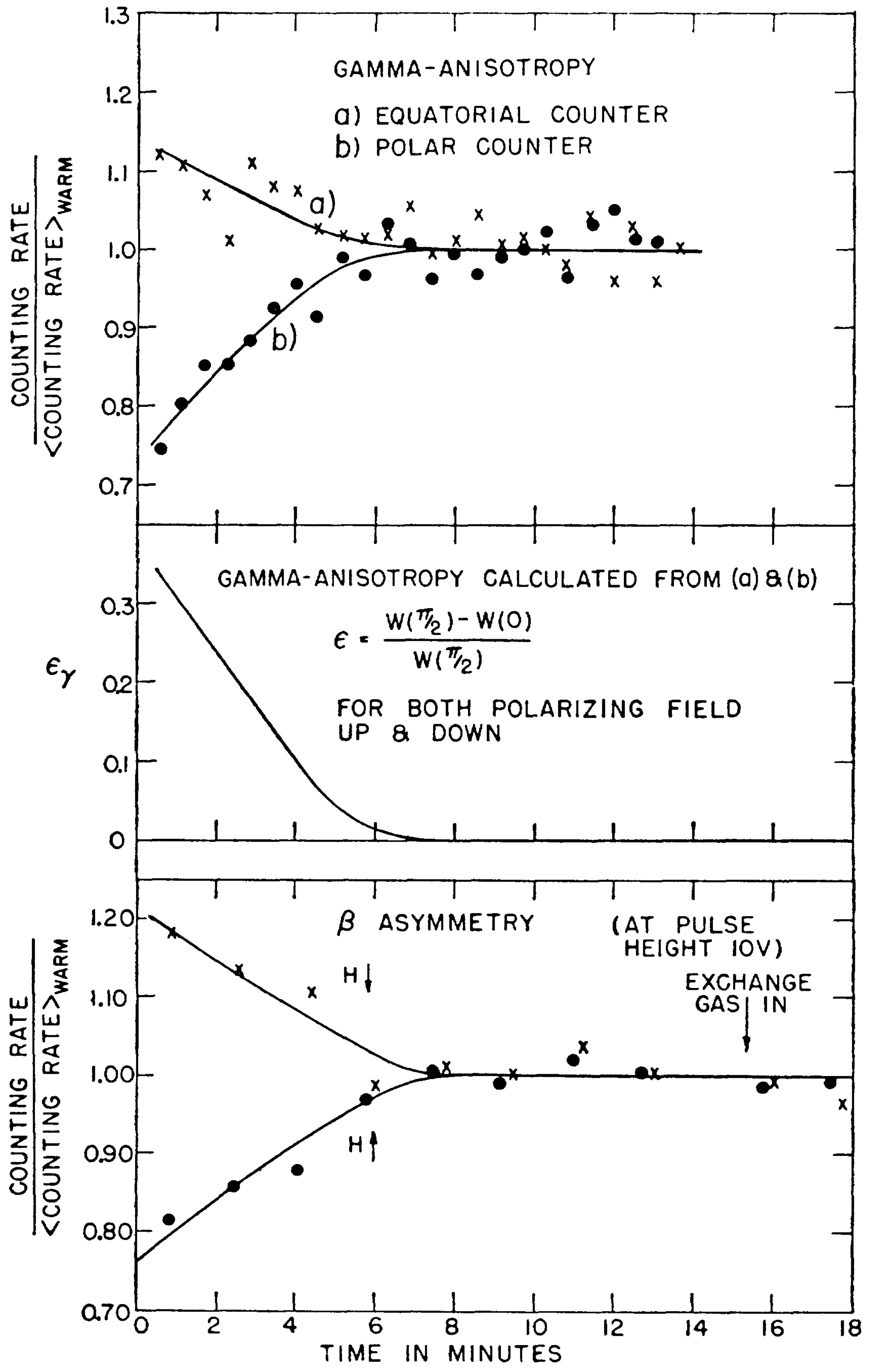
- Requirements:
  - very high degree of polarization using strong B-field → Wu and her team achieved ~60% polarization
  - extremely cold temperatures to reduce thermal movement using thermal demagnetization → magnetic field must be switched off during measurement
  - very precise knowledge of polarization fraction, measured using angular spectrum of γ decays of excited Ni nuclei → this is pure QED, known to conserve parity
  - … plus many more small and large experimental tricks



Source: Pen88, with English translation by Stigmatella Aurantiaca



The Wu-Experiment

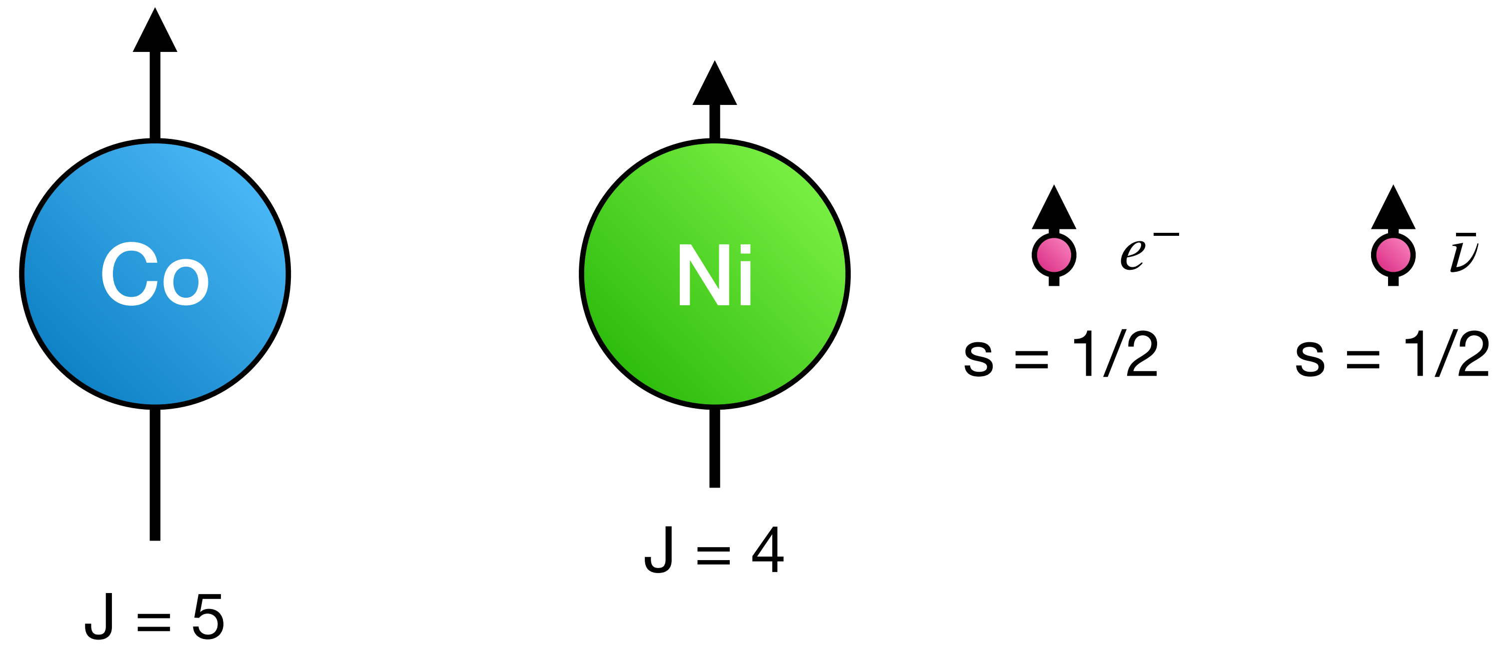


measured photon anisotropy (related to polzarization fraction)

measured electron rates for different polarizations

warming up with time →

- "According to Lee and Yang the present experiment indicates (…) that conservation of parity is violated."
- As ΔJ=+1 both electron and anti-neutrino spin have to point in the same direction
- Electrons are emitted opposite to ^{60}Co spin, they must have negative helicity and antineutrinos positive helicity

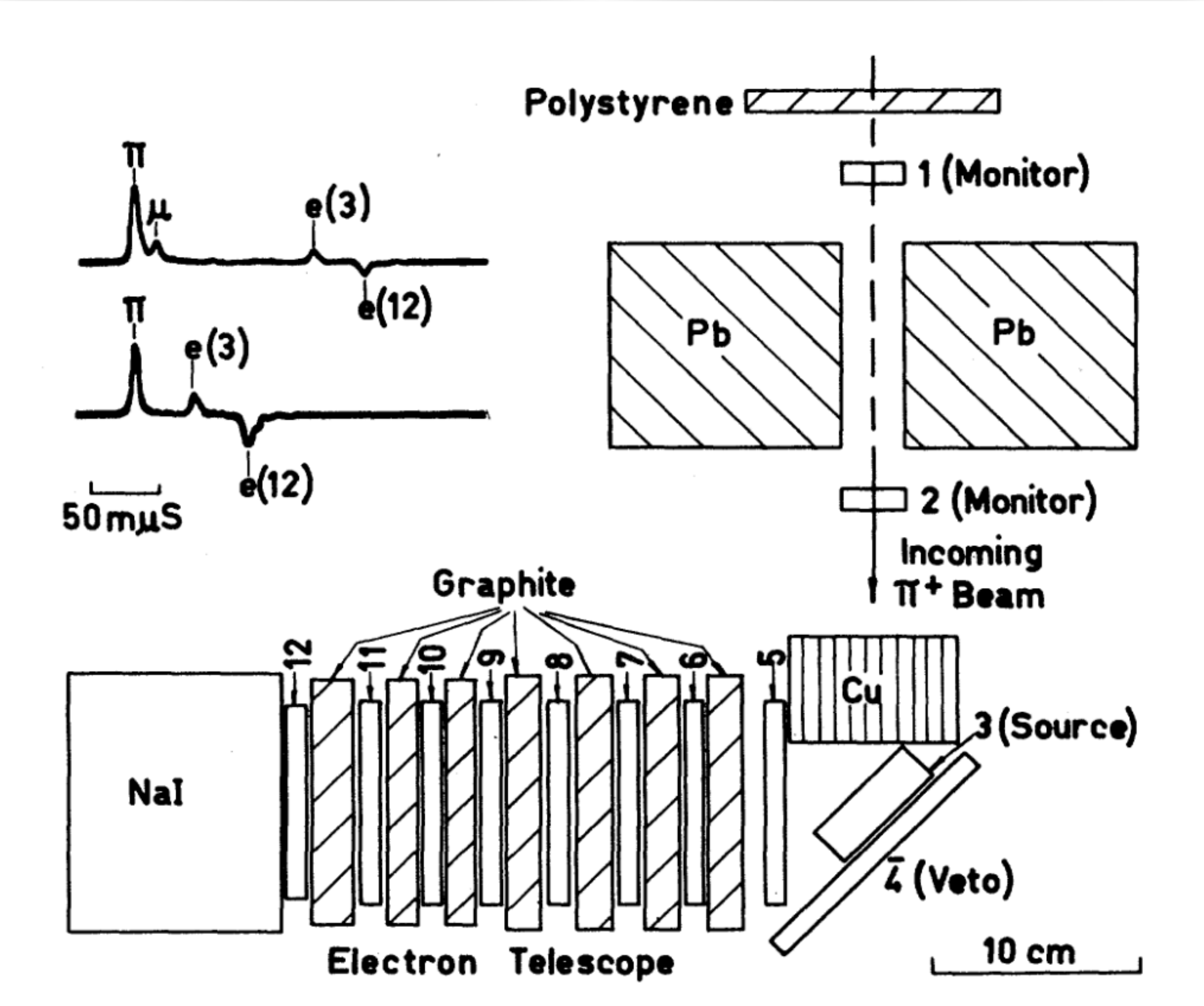


- "Goldhaber experiment" later confirmed that neutrinos are left-handed; anti-neutrinos are right-handed

C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, "Experimental Test of Parity Conservation in Beta Decay", Phys. Rev. **105**, 1413 (1957)

Pion decay

- In 1958, the charged pion decay into electrons $\pi^+ \rightarrow e^+ + \nu_e$ had not been observed
  - $\pi^+ \rightarrow \mu^+ + \nu_\mu$ should be much less likely since ΔQ is much smaller for this decay
- 600 MeV protons on a fixed target ($123\bar{4}$ configuration) produces 127 MeV pions (almost at rest)
- Count two pulse (π-e) vs three pulse (π-μ-e) events

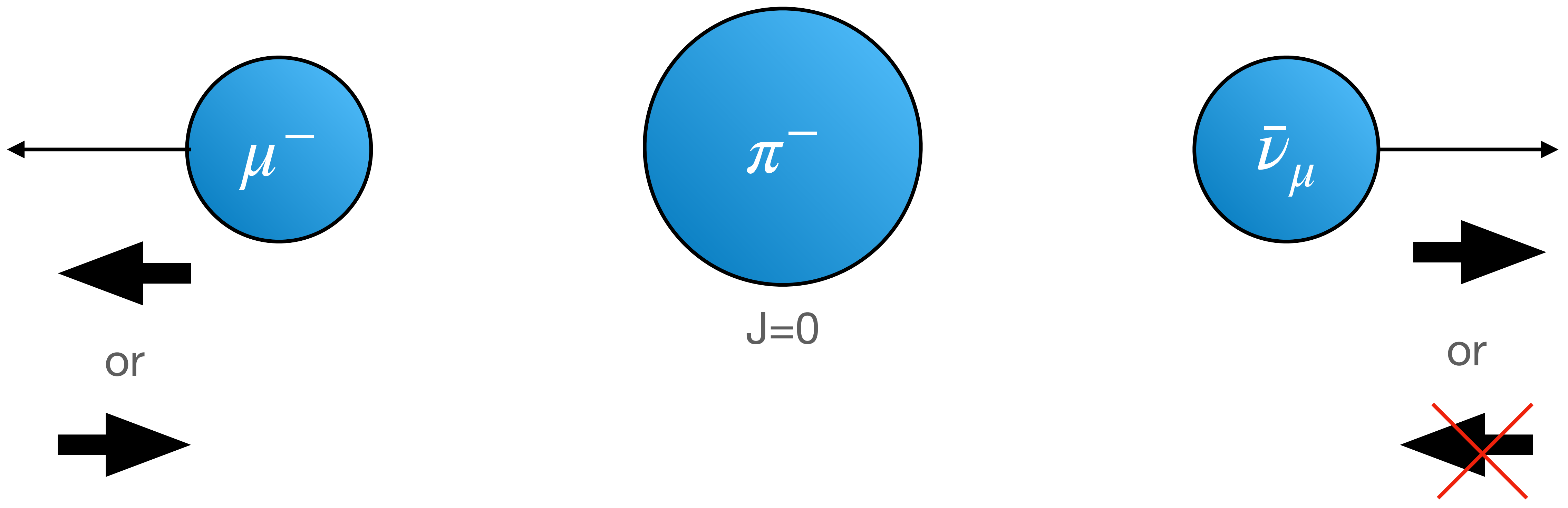


Source: Phys. Rev. Lett. 1 (1958) 247-249

Pion decay

- $$\frac{BF(\pi \to e\nu)}{BF(\pi \to \mu\nu)} = 1.23 \times 10^{-4}$$ (today)
- Two-body decay with both decay products exactly back-to-back in pion restframe
- Pion has spin 0; Muon, electron, and neutrinos have spin projections $\pm 1/2$
  - Must be opposite to result in spin=0 configuration

Since muons are massive, right-handed helicity has a (small) left-handed chiral component



Forbidden: Anti-neutrinos must have right-handed helicity and (because they are massless) have right-handed chirality

LH chiral component of RH helicity spinor: $\mathscr{M} \propto 1 - \frac{|\vec{p}_{\mu,e}|}{E_\pi + m_{\mu,e}} = \frac{m_{\mu,e}}{m_\pi + m_{\mu,e}}$ for pion decay at rest

Summary

- Symmetries as basic principle of physics theories
- Principle of local gauge invariance
  - Postulate invariance of Lagrange density under local gauge symmetry → all interactions (and gauge bosons as mediators)
  - QED: symmetry under U(1) gauge transformation → photon exchange
  - QCD: symmetry under SU(3) gauge transformation → gluon exchange
- Feynman rules:
  - set of rules how to matrix elements
  - can be read off (at leading order) from Lagrange density
  - represented by Feynman graphs
- Experimental observation of parity violation in the Wu experiment

Summary

	QED	QCD	Weak
Energy	✓	✓	✓
Charge	✓	✓	✓
Baryon Number	✓	✓	✓
Lepton Number	✓	✓	✓ *
Isospin (I)	✓	✓	✗ (ΔI=1/2 or 1)
Strangeness (S)	✓	✓	✗ (ΔS=0 or 1)
Charm (C)	✓	✓	✗ (ΔC=0 or 1)
Parity (P)	✓	✓	✗ (maximal in CC)
charge-conj. (C)	✓	✓	✗
CP	✓	✓	✗ (small)
CPT	✓	✓	✓

What questions do you have?